











PROCEEDINGS  
OF THE  
ROYAL SOCIETY OF EDINBURGH.



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OF  
THE ROYAL SOCIETY  
OF  
EDINBURGH.

VOL. XXXIV.

NOVEMBER 1913 TO JULY 1914.

231812

EDINBURGH :  
PRINTED BY NEILL AND COMPANY, LIMITED.  
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# PROCEEDINGS

OF THE

## ROYAL SOCIETY OF EDINBURGH.

SESSION 1913-14.

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EDINBURGH :

PUBLISHED BY ROBERT GRANT & SON, 107 PRINCES STREET, AND  
WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON.

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Price Six Shillings and Sixpence.



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2. METHOD OF PUBLICATION.—As soon as the final revise of a Transactions paper has been returned, or as soon as the sheet in which the last part of a Proceedings paper appears is ready for press, a certain number of separate copies or reprints, in covers bearing the title of the paper and the name of the author, are printed off and placed on sale. The date of such separate publication will be printed on each paper.

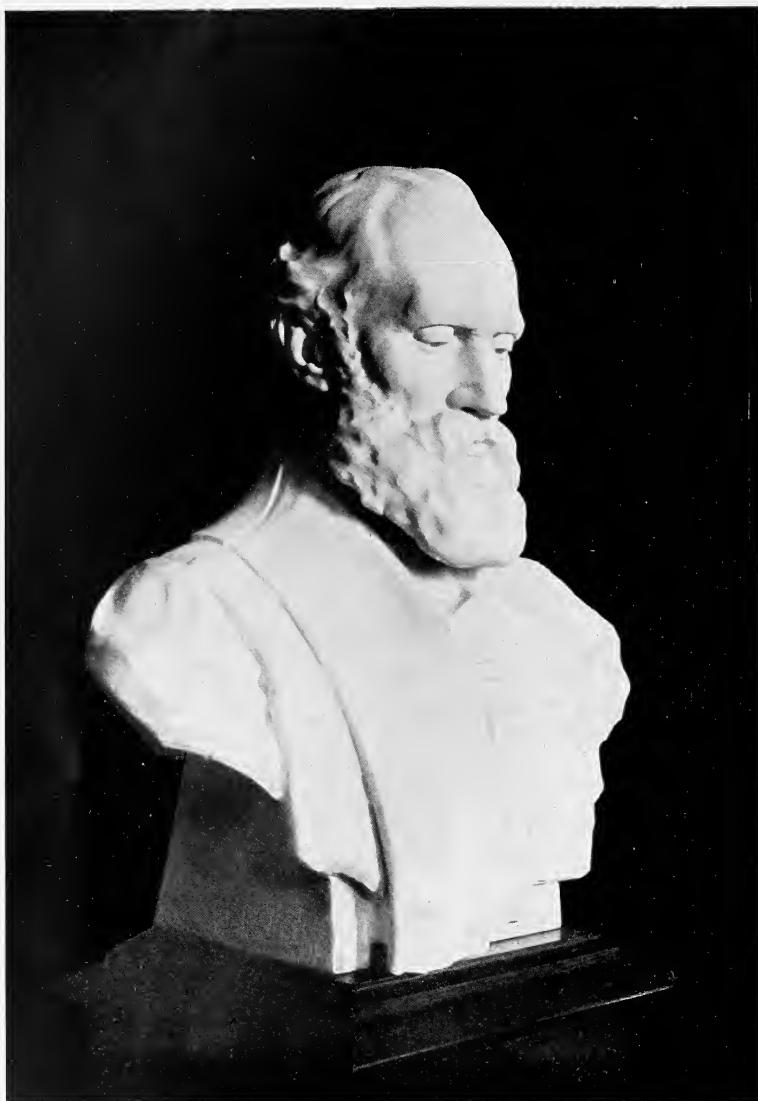
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5 SPECIAL DISCUSSION OF PAPERS ACCEPTED FOR PUBLICATION.—Where a paper has been accepted for publication, the Council may, with the consent of the author, select this paper for Special Discussion. In the case of such papers advanced proofs will be sent to the members of the Society desiring copies, and copies will be supplied to the author for distribution. A paper selected for Special Discussion will be marked with an asterisk (\*) and placed first on the Billet for the day of reading. Any following papers for that day may be adjourned or held as read if the discussion prevents their being read.

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THE RIGHT HON. LORD KELVIN, G.C.V.O., F.R.S.,  
President of the Royal Society of Edinburgh, 1873-1878, 1886-1890,  
and 1895-1907.



## PROCEEDINGS

OF THE

# ROYAL SOCIETY OF EDINBURGH.

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VOL. XXXIV.

1913-14.

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### Presentation of Bust of Lord Kelvin.

THE General Statutory Meeting of the Royal Society of Edinburgh was held in 24 George Street on Monday, 27th October 1913, at 4.30 p.m.

Principal Sir William Turner, K.C.B., President, in the chair.

Before the transaction of the usual business, the bust of Lord Kelvin (see frontispiece), which had been gifted to the Royal Society of Edinburgh by Lady Kelvin, was presented by Professor Crum Brown, acting for Lady Kelvin, and received by the President for the Society.

Professor Crum Brown said:—

“MR PRESIDENT, FELLOWS OF THE ROYAL SOCIETY OF EDINBURGH, LADIES AND GENTLEMEN,—Lady Kelvin, knowing the great interest Lord Kelvin always took in the Royal Society of Edinburgh, and knowing also the high admiration and warm affection of the Fellows of the Society for their late President, has, with thoughtful kindness, expressed the wish to give this beautiful bust, by Mr Shannan, to remain in the rooms of the Society as a permanent memorial, and has asked me to present it in her name. I feel very highly honoured by Lady Kelvin’s request. I have had the great privilege of intimate acquaintance with Lord Kelvin since my boyhood, and it is impossible for me to tell how much I owe to him.

“I shall not attempt a review of Lord Kelvin’s work or character, but I may remind you of his supreme love of truth and of his intense interest in everything, however apparently trivial, connected with the constitution or with the working of the physical universe. These were the prime motives to his work, and he carried it out in the same spirit. Having formulated a problem, he followed the straightest road to its solution. Of course he encountered difficulties: these he did not evade, he surmounted them. To do so he had often to invent and construct special instruments of wholly

novel type. These were always marked by singular ingenuity, and designed so that they do the work for which they were made with the greatest possible accuracy. Lord Kelvin was a great mathematician. We all remember the "green books," always at hand, in which he worked out the mathematical analysis of the data obtained in his experiments, and of anything else he wished to subject to mathematical treatment. He was never at a loss to find the mathematical key. He made no show of abstruse formulæ. In his mathematical as in his experimental work he took the most direct and the simplest way consistent with accuracy. Lord Kelvin was no intellectual miser. When, in the course of his scientific work, he came across something which could be so applied as to be of practical use, he developed this application, and thus became the inventor of truly scientific instruments, differing in character from those he made for purely scientific purposes only in this, that they are also used and very highly prized by those who are not necessarily scientific, who perhaps do not care about the dissipation of energy or vortex motion. These practical men come, by using Lord Kelvin's inventions, to see that pure science is not vain; they come to know something of the tree from its fruit. Lord Kelvin was quite free from selfishness or jealousy. He rejoiced in his own work and discoveries; he rejoiced also in the discoveries of others. I recollect very well his enthusiasm over the work of Becquerel, of Crookes, of Dewar, of Graham Bell, and of many others. In the questions of first importance to man, where science gives no help, Lord Kelvin was a humble and devout disciple. In Lady Kelvin's name I hand over to the Royal Society of Edinburgh, through you, Sir, as President, this beautiful work of art and striking likeness of Lord Kelvin, one of the greatest discoverers in pure science, a true benefactor of mankind, our honoured President and dear friend."

After the bust was unveiled, Sir William Turner received it in the name of the Society with the following words:—

"I feel sure that no more appropriate Fellow of the Society could have been chosen to act as spokesman on this occasion than our dear colleague and friend, Professor Crum Brown. He has given so admirable a summary of Lord Kelvin's character and intellectual power as one of the great scientific men of the age that I need not attempt to follow him in that direction. But, speaking as the President of the Society, and speaking in regard to the man who immediately preceded me in the presidential chair, I think it might be useful and instructive to say a few words about Lord Kelvin as Fellow of the Royal Society of Edinburgh. I find that Lord Kelvin joined the Society in 1847. He remained a Fellow for sixty years.

Two years after he joined the Society, he made his first communication, which was printed in our *Transactions* for the year 1849. It is interesting to note that this communication was on the subject of heat, and for ten years after that date he produced a series of most important memoirs on heat and other forms of activity, showing himself to be one of the most active-minded and original-minded men engaged in physical science. Our *Transactions* are a valuable record of all the early work which he gave to the world; and he looked upon the Society as the medium through which his ideas were to be submitted to the consideration of his fellow men of science.

"I can only refer to the numerous communications Kelvin made to the Society; and it is interesting to note that there was a communication from him in our *Proceedings* for 1906, the year before he died. This was a great feature in Lord Kelvin's intellectual career—he had an active mind to the end. The last communication published in our *Proceedings* was on the initiation of deep-sea waves. The sea and the deep sea exercised indeed an important influence over his practical career. As we all know, it was through Lord Kelvin's investigations that the laying and the commercial working of the Atlantic cable were brought about, and his improved compass has been a boon to all seamen. In 1873 Kelvin was elected our President for the period of five years. In 1886 he was for a second time chosen for a similar period. He had served for four years of the second period when the Council of our Society received an informal intimation from the Council of the Royal Society of London that they wished Lord Kelvin to be their President. It was felt that it would be difficult to discharge the duties of this office if he remained President of the Edinburgh Royal Society. Accordingly it was suggested that we might be able to surrender Lord Kelvin to the Royal Society of London. This our Council agreed to do; and in 1890 Lord Kelvin became their President. When in 1895 he retired from his Presidentship in London, he was for the third time appointed our President, and he continued in this office till his death in 1907. We can at once understand how Lady Kelvin should feel desirous that, so far as marble can perpetuate personality and expression, there should be such a perpetual memorial of her great husband in the building of the Society which he had adorned in the double capacity of Fellow and President. I ask Professor Crum Brown, as the mouthpiece of Lady Kelvin on this occasion, to be good enough to convey to her Ladyship our most devoted and hearty thanks for this admirable bust of her late husband, which will form one of the precious possessions of the Society."

I.—Opening Address by the President, Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., delivered at the First Ordinary Meeting for the Session, held on November 3, 1913.

GENTLEMEN,—For the high honour you have done me in electing me to the Presidency of this, the premier scientific Society of Scotland, I offer you my grateful thanks. I am proud indeed that you should have deemed me not unworthy to succeed the eminent men who have heretofore occupied this chair. My complacency, however, is tempered, if not subdued, by the consciousness of my own limitations. But if I cannot, like my predecessors, add lustre to the office I hold, I can at least endeavour to devote all my energies to the performance of its duties.

It is matter of sincere congratulation that our Society continues to prosper, and to keep up its reputation by the number and value of its contributions to the stock of knowledge. During the past session no fewer than 46 papers were communicated. Of these 19 dealt with chemical and physical subjects; 19 were zoological; 3 botanical; 2 geological; while pure mathematics, engineering, and anthropology were each represented by one paper. In addition to these papers, two addresses were delivered at the request of the Council—one being physiological and the other astronomical.

During the session, I regret to say, our Society has sustained not a few losses—twenty-three of our fellow-members having died. Of this number, some will be long remembered by us not only for the distinction of their own careers, but for the active part they took in conducting the affairs of the Society.

RAMSAY HEATLEY TRAQUAIR, M.D., LL.D., F.R.S., F.G.S. . . . Dr Traquair became a Fellow of the Society in 1874, and served many years on the Council—his first term of office being from 1875 to 1878, and his last from 1904 to 1910, when he acted as Vice-President. He communicated many important papers to the Society, and was awarded the Makdougall-Brisbane and Neill Medals. Dr Traquair died on 24th November 1912. . . . [See Obituary Notice, *Proceedings*, vol. xxxiii. pp. 336–341.]

JOHN WILLIAM SHEPHERD, Glasgow, was elected in 1897. He died on 26th November 1912.

ANDREW JAMIESON, M.Inst.C.E. . . . He was elected a Fellow of the Society in 1882, and died on 4th December 1912. . . . [See Obituary Notice, *Proceedings*, vol. xxxiii. pp. 334, 335.]

SIR GEORGE HOWARD DARWIN, K.C.B., M.A., LL.D., F.R.S., second son of the famous naturalist, was born in 1845, and died on 7th December 1912. After a brilliant career at Cambridge, he became barrister in 1874, but subsequently returned to Cambridge and devoted himself to mathematical science, and in 1883 was elected Plumian Professor of Astronomy and Experimental Philosophy. He is the author of many important and suggestive papers, a number of which appeared in the *Proceedings* and *Philosophical Transactions* of the Royal Society, of which Society he became a Fellow in 1879, and was the recipient of the Copley and Royal Medals. The great merits of his original researches have been recognised by many Universities at home and abroad, and by learned Academies and Institutions all the world over, who have enrolled his name among their hon. members. He was elected an Honorary Fellow of this Society in 1897.

LIEUT.-COL. FREDERICK BAILEY obtained his commission in the Royal Engineers in 1859, and went to India in 1864, where he served in the Bhutan Expedition of 1864-5, for which he obtained a medal. In 1871 he was attached to the Indian forest service, in which department he remained for close on twenty years, having during that long period occupied several very important posts. So highly were his services appreciated by the Indian Government that he was eventually appointed Inspector-General of Forests. In 1890 temporary illness compelled him to return to this country. About this time the importance of Forestry as a branch of University education had been recognised by the institution of a Lectureship on the subject in the University of Edinburgh, and Lieut.-Col. Bailey was called upon to become first lecturer. He threw himself with characteristic zeal into his work, and soon gained the confidence of his students and the admiration of his colleagues. It is chiefly due to his indefatigable exertions that a degree in Forestry was eventually instituted by the University Court. Nor can it be doubted that it is to his energy and enthusiasm that the subject of Forestry now occupies so prominent a position not only in the University of Edinburgh but in Scotland generally. Lieut.-Col. Bailey also found additional outlets for his energies as an active member of the Royal Scottish Arboricultural Society, and as Secretary of the Royal Scottish Geographical Society. This latter post he occupied with conspicuous ability for many years, until the increasing work of his Lecture-

ship compelled him in 1903 to resign. For four years longer he retained his position at college, when, to the regret of his colleagues, failing health obliged him to retire. Lieut.-Col. Bailey was elected a Fellow of the Society in 1894, and served one term on the Council (1896-99). His death took place on 21st December 1912.

JOHN M'ARTHUR, F.C.S., Sussex. . . . He was elected to the Fellowship of the Society in 1888, and died on 19th December 1912. . . . [See Obituary Notice, *Proceedings*, vol. xxxiii. p. 333.]

ROBERT M. FERGUSON, Ph.D., LL.D., who died on 31st December 1912, at the advanced age of 84, was elected a Fellow in 1868, and served three terms on the Council. He acted also as the representative of the Society on George Heriot's Trust. . . . [See Obituary Notice, *Proceedings*, vol. xxxiii. pp. 342-345.]

A. BEATSON BELL, advocate, of Kilduncan, Kingsbarns, Fife, and a former Chairman of H.M. Prison Commissioners for Scotland, died on 6th January 1913, in his 81st year. He was elected a Fellow in 1886, and served three terms on the Council. Mr Bell was all his life much interested in educational matters, having acted as a director of Edinburgh Academy, a governor of Donaldson's Hospital, and a governor of the Trust for Education in the Highlands. He held various other important positions, such as director of the Royal Institution for the Home Relief of Incurables, and director of the Royal Sick Children's Hospital. He was also a Fellow of the Royal Scottish Society of Arts, and served as President of that Society from 1897 to 1899.

GEORGE ALEXANDER GIBSON, D.Sc., M.D., LL.D., F.R.C.P.E., was elected a Fellow in 1881, and served one term on the Council. He died on 18th January 1913, in his 59th year. I need not attempt to appraise Dr Gibson's position as a medical man—we know that he was a brilliant member of his profession, respected and beloved by all who knew him. It is hardly too much to say that the premature death of this distinguished physician has been mourned by the whole community, and has been felt as a great loss to this Society.

SIR WILLIAM WHITE, K.C.B., F.R.S., London, was elected a Fellow in 1890. Sir William, who has been called "the father of the modern British Navy," had a most notable career. With no advantages of fortune or social position, he worked his way in the Naval service from the humble position of a shipwright apprentice to the important position of Director of

Naval Construction. Under his superintendence some two hundred and fifty ships of various types were added to our Navy at a cost of about one hundred millions sterling, and for the work of construction of that great fleet Sir William was ultimately responsible. He died on 27th February 1913, in the 68th year of his age.

J. J. KIRK DUNCANSON, M.D., studied medicine at Edinburgh and various medical schools on the Continent. He graduated M.D. at the University of Edinburgh in 1871, was elected to the Fellowship of our Society in 1890, and died on 12th March 1913.

WALTER WHITEHEAD, F.R.C.S.E., was formerly Professor in Clinical Surgery in Victoria University, Manchester; past-President of the British Medical Association; and author of many important works in surgery. He was elected a Fellow in 1881, and died on 19th August 1913.

JAMES GORDON MACGREGOR, D.Sc., LL.D., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, passed away very suddenly on 21st May 1913. He was elected to our Fellowship in 1880, and served one term on the Council. His genuine character had endeared him to a wide circle of friends, who could not but appreciate his kindly, frank manner and engaging simplicity; while the whole-hearted zeal with which he devoted himself to his duties gained the admiration of his colleagues. Professor MacGregor was a sterling man, whose premature death was deeply regretted.

WILLIAM COLIN MACKENZIE, M.D., F.R.C.S., Melbourne, Australia, who was elected a Fellow of the Society in 1905, was Demonstrator in Anatomy in the University of Melbourne.

JOHN PENNY, M.B., C.M., D.Sc., Cumberland, elected 1900, died 19th June 1913. He was a distinguished medical graduate of Edinburgh University, who afterwards specialised in the Department of Public Health, and obtained the degree of D.Sc. Although as a medical officer of health his time was largely occupied, he yet engaged actively in research, and contributed a number of important papers to various medical publications.

WILLIAM GAYTON, M.D., M.R.C.P., M.R.C.S., etc., was elected to the Fellowship of the Society in 1900, and died in August 1913. He was Medical Superintendent of the N.W. Fever Hospital, and for thirteen years of Homerton Small-pox Hospital. Dr Gayton was the author of various papers on vaccination and small-pox.

JAMES MCCUBBIN, B.A., Kilsyth, was elected a Fellow of the Society in 1899, and died on 2nd September 1913. He was latterly Rector of the Burgh Academy, Kilsyth.

HUGH MARSHALL, D.Sc., F.R.S., Professor of Chemistry, Dundee University College, was elected a Fellow in 1888, and died on 6th September 1913, at the early age of 46. He had a distinguished career at the University of Edinburgh, graduating as D.Sc., in his 21st year, a triumph which, so far as I know, is unique. He was for a number of years assistant to Professor Crum Brown, and Lecturer in the University on Mineralogy and Crystallography, until his appointment to the chair of Chemistry at Dundee in 1908. Although his time was much occupied in teaching, Professor Marshall yet found time to engage in original research, and published various valuable papers on chemical and crystallographical subjects. The quality of this research work was attested by the award of the Gunning "Joseph Black" Prize of the University, and of the Keith Prize and Medal of this Society, as also by his election to the Fellowship of the Royal Society of London.

ALEXANDER MACFARLANE, M.A., D.Sc., LL.D., Ontario, Canada, was elected in 1878, and died in September 1913, aged 62. He greatly distinguished himself as a student of mathematics in the University of Edinburgh, where he graduated as D.Sc.—his thesis, "On Electric Sparks in Air," appearing subsequently in the *Transactions* of this Society. For some time he acted as assistant to the late Professor Chrystal, and in 1885 was appointed Professor of Physics in Texas University. He latterly devoted much attention to the study and development of vector algebras, his latest communication on the subject having been read before the Congress of Mathematicians which met at Cambridge in 1912.

SIR WALTER NOEL HARTLEY, D.Sc., F.R.S., was elected in 1877, and died on 11th September 1913. He was Hon. Fellow of King's College, London, and for some time Professor of Chemistry in the Royal College of Science for Ireland. He was author of works on air and its relations to life, and on water, air, and disinfectants, and communicated a number of papers to the Royal Society of London, the Fellowship of which he attained in 1884. He contributed also to the *Journal* of the Iron and Steel Institute, the *Transactions* of the Chemical Society, and to the publications of various other scientific institutions. Amongst the honours conferred upon him in recognition of his work he was awarded the Longstaff Medal of the Chemical Society for researches in spectro-chemistry.

JOHN MACMILLAN, M.A., D.Sc., M.B., C.M., etc., Edinburgh, was elected in 1876, and died on 7th October 1913. He was a brilliant student, first at St Andrews, where he graduated as M.A., and afterwards at the University of Edinburgh, where he obtained the degree of B.Sc. Later on he entered upon the study of medicine at the same university, and graduated M.B., C.M., subsequently passing as B.Sc. in Public Health, and finally obtaining the doctorate of science. With such an academic career Dr Macmillan could hardly fail to make his mark in his profession, and by his medical brethren he was held in the highest esteem. As Lecturer in Medical Jurisprudence in the Extra-mural School of Medicine, Edinburgh, he was much appreciated by his pupils; while as a practitioner he endeared himself to his patients by his unfailing kindness and sympathy.

SIR JOHN BATTY TUKE, M.D., D.Sc., LL.D., was elected in 1874, and served three terms on the Council; he died on 13th October 1913. Born in 1835, in Yorkshire, he came early to Edinburgh, and graduated in medicine in 1856. Shortly afterwards he went to New Zealand, where he was attached to the colonial forces as surgeon, becoming senior medical officer on the outbreak of the Maori War in 1860. On his return to this country he devoted himself especially to the treatment of mental diseases, and soon attained eminence in his profession. He occupied many important positions as a medical man, and was twice elected to represent the Universities of St Andrews and Edinburgh in Parliament. During his term of office he naturally took great interest in all educational matters, and for these services, as well as for his eminence as a physician, he obtained honorary degrees from Trinity College, Dublin, and the University of Edinburgh. As member of Parliament for the Universities of Edinburgh and St Andrews he was of great service in pressing the claims of the Society upon the Government during the negotiations in regard to the removal of the Society from the Royal Institution to its present premises in George Street.

WILLIAM DONALDSON, M.A., was elected in 1896, and died on 16th October 1913. For over thirty years he was headmaster and controller of Viewpark School—a private educational institute in this city. He was devoted to his profession, and held in high esteem by all who knew him—and by none more so than his pupils.

## II.—Observations on the Auditory Organ in the Cetacea. By Principal Sir Wm. Turner, K.C.B., D.C.L., F.R.S.

(Read December 1, 1913. MS. received December 2, 1913.)

EARLY in September of this year I received from the Falkland Islands a box, dispatched by Mr G. Millen Coughtrey, a former student of the University, now an *employé* in the New Zealand Whaling Company. It contained a number of specimens which illustrated the anatomy of the auditory apparatus in the Cetacea. The whales were caught in the South Atlantic, mostly at South Shetland, though some were from Graham's Land, at which place he had been whaling last season.

### EXTERNAL AUDITORY MEATUS AND EARWAX.

The Cetacea do not possess an auricle or pinna of the ear. A small external opening capable of admitting a probe may be seen, when carefully looked for, at the side of the head, behind the outer angle of the eye. It is the orifice of the external auditory meatus, which penetrates the cutis and the thickness of the blubber to reach the tympano-petrosus bone in which the essential parts of the organ of hearing are situated. The length of the meatus varies in different species. The lumen of the meatus may easily be overlooked, but it widens in its course, especially as it approaches the tympanic bone. It is usually destroyed in removing the blubber,\* and has not received much attention in cetological literature.

The presence in it of a ceruminous secretion, the earwax, has, however, been occasionally noted. Thomas Buchanan, surgeon in Hull nearly a century ago,† described and figured in 1828 dissections of the meatus and tympanum in the Greenland whale, *Balaena mysticetus*. He saw in the meatus an unctuous cerumen of a greyish-blue colour, "but in no great quantity." He thought that the collapsed state of the orifice, the great length of the meatus, its winding course, a valve-like obstruction about its middle, and the unctuous secretion tended to prevent the passage of sea water down the auditory canal, in which none was present in the specimens

\* Robert Gray, "Auricular Opening and External Auditory Meatus in *Balaena mysticetus*," *Journal Anat. and Phys.*, vol. xxiii., 1889.

† *Physiological Illustrations of the Organ of Hearing*, London, 1828. Hull at that time was the great shipping port of the whaling industry.

he dissected. Carte and Macalister described\* the meatus in *Balaenoptera rostrata* as lined by a pseudo-mucous membrane of modified cuticle, arranged in three longitudinal folds, and filled with a dark, greyish sebaceous substance produced in ceruminous glands, the openings of which were visible on the mucous membrane. The most recent account of the meatus and its contents has been given by D. G. Lillie.† He described in *Balaenoptera musculus* the opening of the meatus, its course to the tympanum, where the lumen widened to  $1\frac{1}{2}$  inch diameter, and its relation to the membrana tympani. The meatus contained a solid plug of wax, the base or deep end of which was cup-like and moulded on the convex sac-like surface of the membrana tympani, which projected into the deep end of the meatus. The cup was about 1 inch deep and  $1\frac{1}{2}$  inch in breadth. The plug of wax was about 5 inches long, and its outer part formed a thin flattened rod which lay in the inner half only of the meatus. Lillie stated

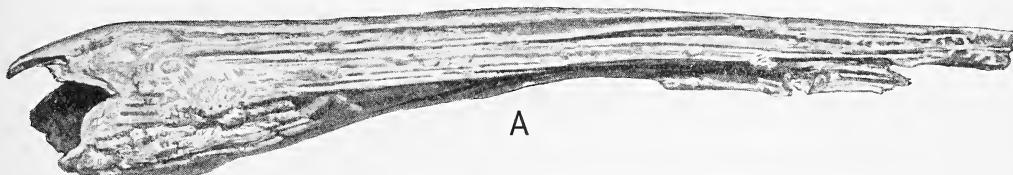


FIG. 1.—Plug of earwax from meatus of *Megaptera longimana*, slightly reduced in size.

that the meatus appeared to be full of water, in which the wax and the tympanic sac were immersed.

Mr Coughtrey's collection contained several good specimens of plugs of dark, yellowish-brown earwax.

*Megaptera longimana*.—A plug from each auditory meatus of a hump-backed whale, captured January 1913, was sent. One was complete, the other was not so perfect: they were 150 and 159 mm. (6 and  $6\frac{1}{4}$  inches) long respectively. The tympanic end, 22 mm. (about  $\frac{3}{4}$  inch) broad and 10 mm. thick, was hollowed into a cup 22 mm. deep, which without doubt had been in close apposition with the convex sac-like tympanic membrane that had occupied the deep expanded part of the meatus (fig. 1). The plug gradually diminished in diameter, and at the opposite end it was flattened, only 12 mm. broad and 1 mm. thick. The surfaces of the plug were marked with shallow ridges and furrows which extended in its long diameter.

A much smaller plug, 112 mm. long, was included in the collection. The tympanic end, not cup-like, had apparently been broken, its transverse diameter was 12 mm., and it rapidly narrowed to a point at the opposite

\* *Trans. Roy. Soc. London*, 1867.

† *Proc. Zool. Soc. London*, p. 769, 1910, with figures and plate.

end. The specimen was not labelled, but had probably been from the meatus of another Megaptera.

*Balaenoptera sibbaldi*.—A single plug of earwax from one meatus of a Blue Whale, captured in South Shetland in 1912, was sent. It had been injured at the tympanic end, and only a portion of the cup-like cavity had been preserved. The plug was 50 cm. (nearly 20 inches) long, 26 mm. (1 inch) broad, and 12 mm. thick at its deep end. It gradually diminished in breadth and thickness, so that the opposite outer end, though 20 mm. ( $\frac{3}{4}$  inch) broad, was only 3 mm. thick, and possessed a flattened, ribbon-like aspect (fig. 2, A). The surfaces of the plug were fluted longitudinally, and

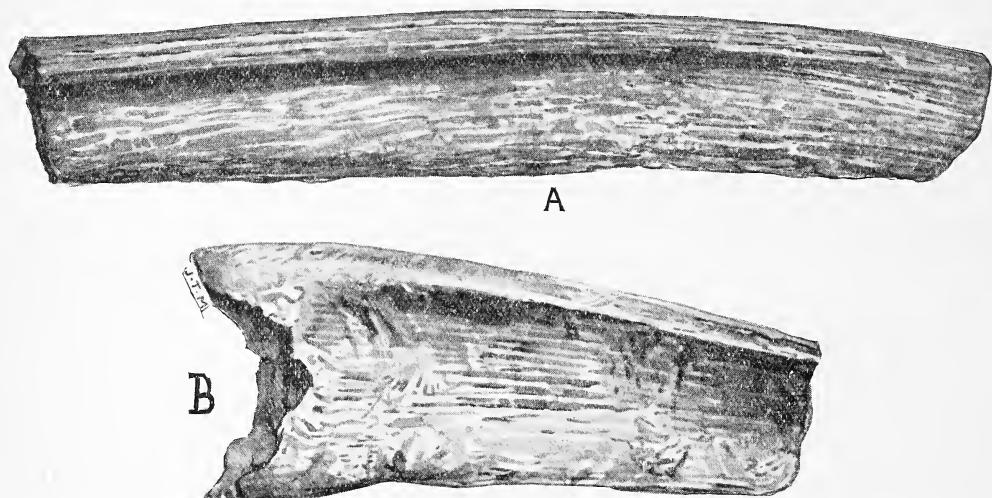


FIG. 2.—Earwax from meatus of *Balaenoptera sibbaldi*, natural size. A, outer fourth of plug with thin flattened end to the right; B, tympanic end with cup-like depression.

had doubtless been adapted to ridges and furrows on the surface of the lining membrane of the meatus. Coughtrey had noted that the plug gave a very good impression of the canal in which it was situated. The tympanic end of a second plug, 80 mm. long, 34 mm. in greatest breadth, and 15 mm. thick, had been preserved. The cup-like cavity was nearly complete and was 15 mm. in depth (fig. 2, B).

The length of the auditory meatus in the Cetacea bears a proportion to the thickness of the blubber on the side of the head. If the wax plug were in every case of equal length with the meatus, it would be a gauge to the thickness of the blubber, but in the specimen dissected by Lillie the plug was not equal in length to the meatus. I am not acquainted with any exact measurement of the thickness of the blubber on the side of the head in Megaptera. Sir John Struthers in his account of the Tay

Megaptera\* gave 4 inches in the fore part of the carcase and 3 inches further back as the thickness of the blubber, but the length of the wax club in the South Shetland Megaptera indicated a greater thickness on the side of its head. In the Longniddry *B. sibbaldi* which I dissected in 1869-70† the blubber on the top of the beak and cranium was 8 to 15 inches thick, whilst in front of the dorsal fin it was 12 to 16 inches, and behind that fin 14 to 21 inches.

#### TYMPANO-PETROUS BONES.

The collection contained the following specimens:—

*Megaptera longimana*.—(a) Right and left tympano-petrosus bones of the Humpbacked Whale, from a specimen captured in 1913 near Bryde Island, Graham's Land; the tympanics were 109 and 113 mm. ( $4\frac{1}{4}$  and  $4\frac{1}{2}$  inches) respectively in length. (b) Left tympano-petrosus tympanic, 108 mm. long. (c) Right and left tympanics, 106 mm. long. (d) Right tympanic, 107 mm. long.

*Balaenoptera sibbaldi*.—(a) Three pairs of tympano-petrosus bones of the Blue Whale from South Shetland, captured 1912, and (b) a single left tympanic. The tympanics as a rule varied in length from 121 to 129 mm., but one pair measured exceptionally 146 and 148 mm. (about  $5\frac{3}{4}$  inches).

*Balaenoptera musculus*.—A pair of tympano-petrosus bones from a whale captured at Admiralty Bay, South Shetland, in November 1912, is labelled B.W., i.e. Blue Whale. The examination of these specimens satisfied me that the tympanic differed from that bone in *B. sibbaldi* in several particulars, as follows: It possessed a deep groove on the outer surface parallel and close to the posterior border, which gave to that border a more definite character than was the case in *B. sibbaldi*. On the other hand, it did not possess the long broad groove parallel to and bounding the outer side of the lower border, which gave rise in *sibbaldi* to a very prominent keel; the Eustachian end of the tympanic cleft was also less scooped out than in *sibbaldi*. The tympanic was 125 mm. long, 89 high, and 77 in greatest breadth. In one of the pair the opisthotic process of the petrous was entire, 435 mm. long by 135 mm. in greatest breadth, almost identical in its dimensions with those of a large *B. sibbaldi*.

Differing from Sibbald's Whale in several particulars, the tympanics more closely corresponded in their characters with *B. musculus*, so that I am disposed to regard this pair of specimens as from that species.

\* *Anatomy of the Humpbacked Whale*, Edinburgh, 1889.

† Account of the great Finner Whale, stranded at Longniddry, *Trans. Roy. Soc. Edin.*, vol. xxvi., 1870.

This series of tympanics are of importance in showing that the Balænopteridæ, *Megaptera longimana*, *Balaenoptera sibbaldi*, and *B. musculus*, which frequent the North Atlantic and are captured in Scottish waters, are also denizens of the South Atlantic. The University Museum also contains a pair of tympanic bones from *Balaenoptera borealis*, Rudolphi's whale, the Sye Whale, from the South Atlantic,\* captured in 1911, which is also a Scottish species.

Many naturalists have described with more or less detail the tympano-petrosous bones in the whalebone and toothed whales. I have also figured in my descriptive Catalogue† the characters of the tympanics in a large number of species. The additions to the collection through the recent gift of Mr Coughtrey have enabled me to study more completely the relations of the tympanic and petrous bones to each other, to the chain of tympanic ossicles, and the approximate arrangement of the membrana tympani and the external auditory meatus. Carte and Macalister had previously given a careful description of the tympano-petrosous in *Balaenoptera rostrata*, and Struthers had recorded their characters in *Megaptera longimana*, but the dissections of D. G. Lillie of the region in *Balaenoptera musculus* are much more complete, as he had the advantage of studying the bones along with the associated soft parts. The University collection contains specimens of these bones in both the whalebone and toothed whales; they corresponded with each other in general characters, though with modifications in detail, which expressed specific and generic differences. In no specimen, however, had the external meatus, the tympanic membrane, and the Eustachian tube been preserved.

The following description is based on the characters of the tympano-petrosous bones in *Balaenoptera sibbaldi*,‡ though with specific modifications it applies generally to the baleen whales. The tympanic bone was keeled on its inferior surface. Its outer surface was convex and marked by an oblique, strong, relatively wide groove-like depression which divided it into an anterior and a posterior part, the latter of which was the larger. The upper border of this surface was sinuous and was connected by a broad posterior pedicle to the under-border of the long, flattened, winglike opisthotic process of the petrous. The anterior surface of the posterior pedicle was hollowed, smooth, directed towards the tympanic cavity and the meatus,

\* Catalogue, p. 58 (*C. Bpt.*, 5). I described a Scottish specimen in *Journ. Anat. and Phys.*, April, 1882.

† *The Marine Mammals in the Anatomical Museum of the University of Edinburgh*, London, 1912; also "The Right Whale of the North Atlantic (*Balaena biscayensis*)" in *Trans. Roy. Soc. Edin.*, vol. xlvi, part 4, 1913.

‡ The University Museum now contains eighteen tympanic bones of Sibbald's Whale.

and had evidently been invested by the tympanic membrane (fig. 3, AU), where it formed the cul-de-sac which projected into the auditory meatus;

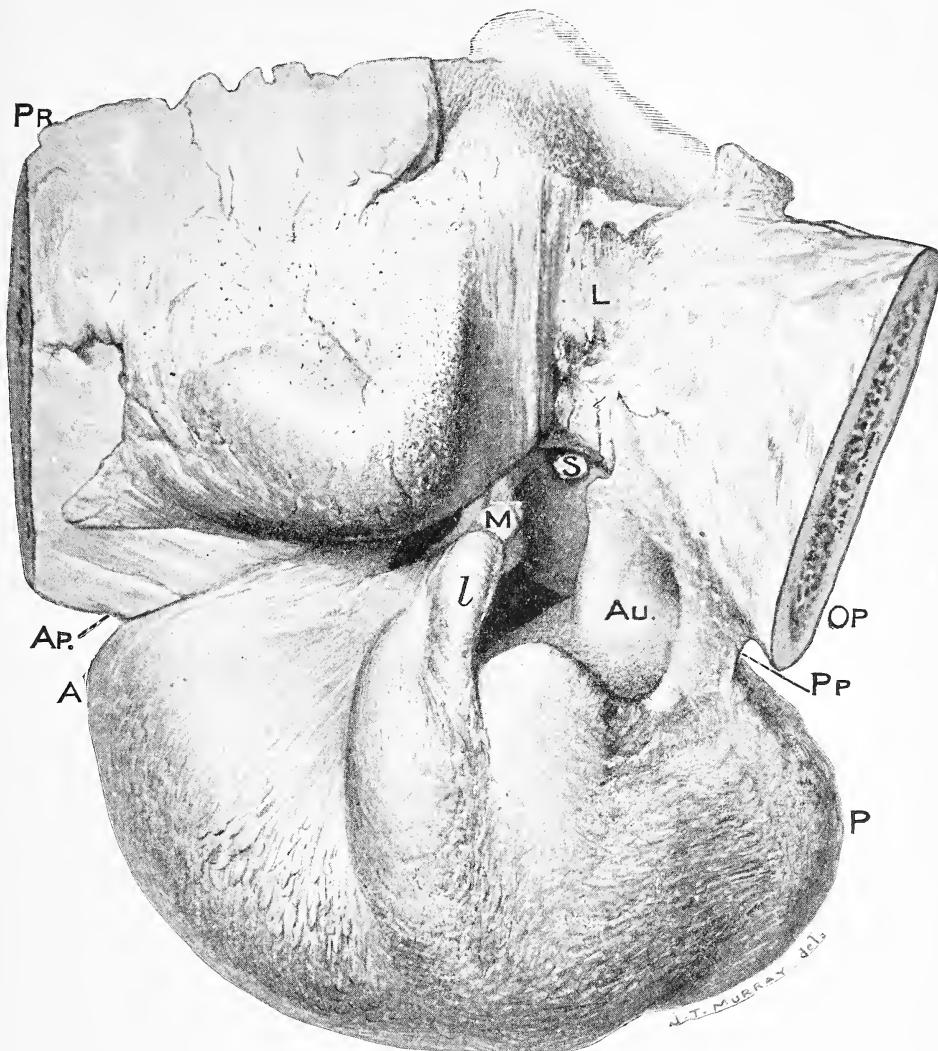


FIG. 3.—Outer aspect of left tympano-petrosous bones of *Balaenoptera sibbaldi*, natural size. A, anterior end, P, posterior end of tympanic; Au, indicates the region adjoining the external auditory meatus covered by the tympanic membrane and the place of projection into meatus of tympanic membrane; l, lip-like process of sinuous border with which malleus is fused; M, head of malleus; S, head of stapes in funnel-like depression on inner wall of tympanum; L, labyrinthine part of petrous; PR, pre-otic, and OP, opisthotic parts of petrous cut across; AP, anterior, and PP, posterior tympano-petrosous peduncles. The incus is not figured, as it would have obscured the stapes.

the smooth surface was bounded by a rough area, to which had doubtless been attached the deep end of the wall of the meatus. The sinuous border

in front of the posterior pedicle was at first concave, then somewhat elevated, and was succeeded by a narrow, deep groove, which formed the posterior boundary of a strong curved process. I have elsewhere \* named this process lip-like or mallear, for the malleus was fused with it; it ascended towards but did not touch the under surface of the labyrinthine part of the petrous, and ended in a free rounded edge (fig. 3, l). At its front was the wide groove-like depression which separated the posterior from the anterior part of the outer surface of the tympanic. The upper border of this part of the surface was connected by a broad anterior pedicle † to the pre-otic division of the petrous. The labyrinthine or proper periotic division of the petrous was relatively small; it lay between and gave origin to the pre-otic and opisthotic divisions of the bone, and it formed the roof and inner wall of the tympanic cavity (fig. 3, L).

The gap between the anterior and posterior pedicles, the sinuous border, and the edge of the labyrinthine roof had without doubt been associated with the membrana tympani, and through the large part of the gap behind the lip-like process the sac-like prolongation of this membrane had projected into the lumen of the meatus, the wall of which had been attached to the margin of the gap.

Buchanan, in his *Illustrations* (*op. cit.*), figured dissections of the dilated tympanic end of the meatus in *Balaena mysticetus*, and showed the sac-like surface of the tympanic membrane, which formed a convex projection into its lumen and was enclosed by its wall. He stated the sac to be divided internally into a major and a minor concavity by a valve-like membranous process from the wall, to the whole length of which the slender process of the malleus was attached, whilst the handle was connected with the outer edge of the osseous tympanum. Buchanan adopted the view of Sir Everard Home, that the membrana tympani had a muscular layer, to which he added a reticulated nervous plexus situated subjacent to the cuticle. Knox ‡ saw in *Balaenoptera rostrata* a bag-like projection of the membrana tympani projecting into the auditory meatus; he stated that in a foetal *Balaena mysticetus* the membrane, though thick, is not muscular. The presence of muscular and nerve fibres in the membrane is not now accepted. Lillie, in his excellent description and figure of the membrana tympani in *B. musculus*, showed that it formed a sac, not unlike the finger of a glove, about 3 inches long and  $\frac{3}{4}$  inch in diameter, which projected into the dilated

\* *Marine Mammals, op. cit.* pp. 20, 74.

† The anterior and posterior pedicles in *Balaenoptera rostrata* were thin plates of bone and were very easily fractured.

‡ *Catalogue of Anatomical Preparations of the Whale*, Edinburgh, 1838.

lumen of the auditory meatus, where its somewhat rounded end fitted into the cup-like base of the plug of wax. By its opposite end it was continuous with the membranous lining of the tympanic cavity. A ligament about 1 inch long and 5 mm. broad sprang from the upper part of the sac, passed towards the tympanic cavity under the malleo-incal joint, and became attached to the manubrium of the malleus. The tympanic sac and the ligament were together about 4 inches long.\*

The Tympanic Ossicles are frequently missing in museum specimens, and I have carefully looked for them in the bullæ of the whales in the

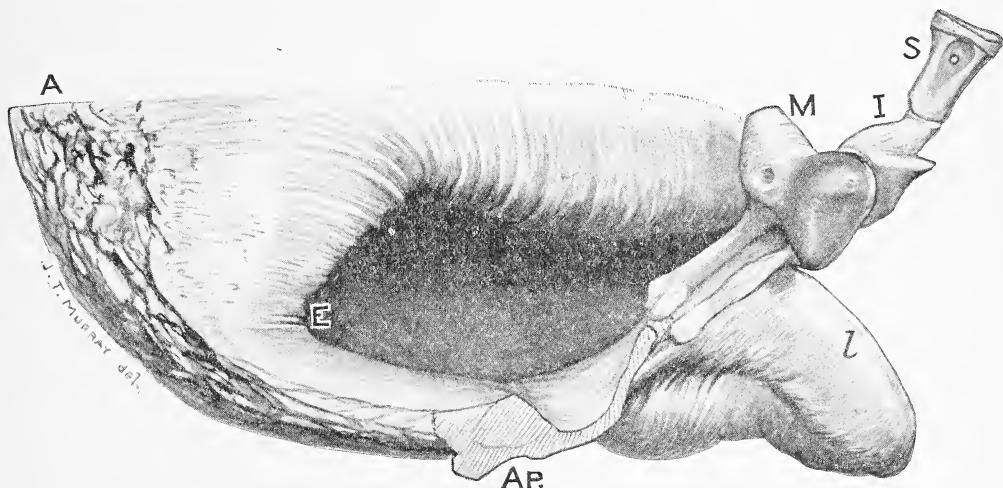


FIG. 4.—Chain of left tympanic ossicles, tympanic cavity and cleft of *Balaenoptera sibbaldi*, natural size seen from above. A, anterior end of tympanic; E, Eustachian end of cleft; AP, anterior tympano-petrosus peduncle cut across; l, lip-like process of sinuous border; M, malleus with two processes fused with anterior border of lip; I, incus; S, stapes: their several articulations are represented.

University collection. The incus, owing to the diarthrodial joints between it and the malleus and stapes being apt to give way, is seldom present, even when the malleus and stapes with their firmer attachments have been preserved.

The Ossicles in *B. sibbaldi* will now be described. The upper end of the Malleus consisted of a rounded head with a groove separating it from the part of the bone which had on its inner aspect two articular surfaces for the incus set at an angle to each other. The diameter of the conjoined head and articular part was 16 mm. From the lower part of the head, a process descended, 18 mm. long, which was fused in its entire length with the anterior border of the lip-like process of the sinuous border of the tympanic (fig. 4). A second descending process, parallel and close to the

\* I have figured in *Marine Mammals* a similar sac in *Hyperoodon*.

preceding, was similarly attached to the lip. These processes were brittle, and if the tympanic was roughly handled they easily broke and the malleus became detached and lost. The Incus had a body, 8 mm. in diameter, on which were two concave articulations for the malleus. A short, sharp process projected from the posterior surface and nearly reached the roof of the tympanic cavity. From the inner aspect sprang a longer curved process which, together with the body, measured 14 mm.; at its free end was an oval facet for the stapes. The Stapes had a corresponding facet on its head, from which a pair of short relatively thick legs arose, to end in the plate-like foot of the stirrup. A very thin layer of bone passed between the legs and was pierced by a minute foramen. The stapes, 11 mm. long, occupied a funnel-shaped depression in the inner or petrosal wall of the tympanum, and its oval foot, 8 mm. in diameter, was attached to the fenestra ovalis of the cochlea (fig. 4).

As the fusion of the malleus with the tympanic gave to the Cetacea an exceptional character as compared with other mammals, I may state the species in which I have noted this arrangement. In the whalebone whales I saw it in *Balaena mysticetus* and *biscayensis*, in *Balaenoptera musculus*, *sibbaldi*, *borealis*, *rostrata*, and in *Megaptera longimana*. In the toothed whales I saw it in *Hyperoodon*, *Phocæna*, *Globicephalus*, *Grampus*, *Delphinus*, *Tursiops*, and apparently in *Monodon*. In several other species in the University Museum the malleus was not in place and had not been preserved. It should be stated that previous observers have noted the fusion of the malleus with the tympanic in certain species. Knox saw it\* in *Balaenoptera sibbaldi* and *rostrata*; Carte and Macalister spoke of it in *B. rostrata* as a process of the tympanic bone, from the margin of whose centre it projected; Dwight described it in *B. musculus* as co-ossified with the tympanic by a processus longus, which had a deep groove anteriorly; Lillie in *B. musculus* as fused to the inner edge of the lip of the tympanic.

Different views have been expressed regarding the morphology of the processes of the malleus. By some, the process fused with the tympanic lip has been regarded as the manubrium or handle of the bone. Possibly the two parallel processes which I have figured in *B. sibbaldi* were only a twin-like arrangement of this process. Others, again, have considered the fused process to be the long slender (*gracilis*) process of the human anatomist. Buchanan, whilst recognising the handle as always attached to the outer

\* Knox, *Catalogue*, pp. 14, 21, who named the species *Balaena maximus borealis* and *minimus*; Carte and Macalister, *op. cit.*; Dwight in *Memoirs of Boston Society of Nat. Hist.*, vol. ii.; Lillie, *Proc. Zool. Soc. London*, 1910; Turner in *Marine Mammals* and in *Memoir on Balaena biscayensis*, *Trans. Roy. Soc. Edin.*, vol. xlvi, 1913.

edge of the osseous tympanum, described in *B. mysticetus* the valvular fold of the tympanic membrane as attached to the whole length of the gracilis, or slender process. Lillie, again, considered that the only attachment of the membrane to the malleus in *B. musculus* was through the connection of the ligament to a short process, which he regarded as the manubrium, whilst the fused process was the processus gracilis. The development of these processes requires to be studied before their morphology can be precisely determined.

The inner surface of the tympanic bone was separated from the outer by the tympanic cavity, the upper internal border of the former surface was thick, rounded, and striated, where it turned over into the cavity (fig. 4). This border was distant from the sinuous upper border of the outer surface by the width of the tympanic cleft, which extended forwards from the posterior pedicle to the anterior or Eustachian notch of the cleft. In the whalebone whales the cleft was approximately horizontal, though in the genus *Balaena* it had a deep notch at its anterior end.\* In the toothed whales the cleft inclined downwards at this end and opened by a mouth immediately above the anterior end of the inferior surface. The Eustachian tube had not been preserved in my specimens. Mr Lillie has been more fortunate, and he has described in *B. musculus* a sac-like prolongation of the tympanic membrane through the Eustachian notch into the pterygoid fossa, from which the Eustachian tube proper arose as a relatively narrow canal, about one foot long, which extended forwards to open into the naso-pharyngeal chamber.

Observations have been made in several toothed whales on the arrangement of the membrana tympani and its prolongation forwards into the sinuses in the cranio-facial bones. Buchanan described and figured the membrane in the Narwhal (*Monodon*) as nearly circular, concave towards the meatus, convex towards the tympanic cavity; the manubrium of the malleus was, he said, attached to it. In the University Museum is the tympanic of a well-grown foetal Narwhal about 5 feet 5 inches long.† The lip-like process of the sinuous border was prominent, and the gap between it and the posterior peduncle was occupied by the dried tympanic membrane, which did not bulge outwards towards the meatus. The malleus incus and stapes were present. The malleus had been attached to the lip-like process, but owing to the fragility of the bone it had broken away. The auditory arrangements in the Porpoise (*Phocæna communis*) were

\* See the figures in my Memoir on the North Atlantic *Balaena biscayensis*, *Trans. Roy. Soc. Edin.*, 1913.

† *Proc. Roy. Soc. Edin.*, vol. ix. p. 102, 1876.

figured by Monro *secundus*,\* who described a concave tympanic membrane at the bottom of the meatus; a communication between the cavity of the tympanum and other cavities analogous to the human frontal, sphenoidal and maxillary sinuses; an Eustachian tube which connected the tympanic cavity with the nasal chamber. The prolongation of the tympanic membrane into the air sinuses in these bones of the skull, as well as into the palate bone, has been described by Rapp† in the porpoise, and by Claudius‡ in *Delphinus delphis*, together with the relations of the ossicles to the tympanic membrane and the communication of the Eustachian tube with the cavity.

The Petrous in the large whales is a heavy massive bone interlocked with the base of the skull, consisting of three definite divisions, a central labyrinthine part which contained the cochlea, vestibule, and semicircular canals, in which the auditory nerve was distributed; a short anterior pre-otic process and a long posterior opisthotic process, as an example of which *Balaenoptera sibbaldi* may be described (fig. 3). The Labyrinthine division had a rough upper surface in relation to the basis craniæ; a smooth under surface characterised by a large bluntly conical projection, which was directed outwards towards the tympanic, but separated from it by the cleft in the tympanic bulla. This surface also formed the roof and inner wall of the tympanic cavity; in it was a funnel-like depression in which the stapes was situated and was attached by its foot to the fenestra ovalis of the cochlea. The inner aspect of the petrous was prolonged and perforated by the large canals for the passage to the labyrinth of the divisions of the auditory nerve, and by smaller foramina and canals.

The Pre-otic was an irregular conical mass which projected forwards to end in a more or less pointed process, it was continuous with the labyrinthine division, and was connected with the tympanic by the anterior pedicle; it occupied a depression in the squamous temporal above the pterygoid fossa. The Opisthotic, so-called mastoid, was a long flattened wing-like plate continuous with the labyrinthine division, and connected with the tympanic by the posterior pedicle. In one of my specimens of *B. sibbaldi* it was 17 inches (432 mm.) long, and in another  $5\frac{1}{4}$  inches (134 mm.) broad. It was locked into a groove between the squamous-temporal and the ex-occipital. In a Megaptera the greatest length was 235 mm., and the greatest breadth 100 mm.; in *B. musculus*, 200 by 70 mm.; in *B. rostrata*, 70 by 35 mm.

\* "On Fishes," p. 45, Edinburgh, 1785.

† *Die Gehörwerkzeuge der Cetaceen*, Tübingen, 1836. *Die Cetaceen*, Stuttgart, 1837.

‡ *Physiologische Bemerkungen über das Gehörorgan der Cetaceen*, Kiel, 1858.

The auditory apparatus in the Cetacea has been modified in adaptation to the aquatic life of an air-breathing mammal, which can respire only during the relatively short period when the nasal opening or blowhole is above the surface of the water. There can be no doubt that the tympanic cavity contains air, which it obtains during inspiration through the communication of its Eustachian tube with the naso-pharyngeal chamber. The immersion of the side of the head in the water renders unnecessary the development of an external auricle, capable of being turned in different directions to receive aerial sound waves, and the question naturally arises, how can sound waves be conveyed so as to impress the nerve apparatus in the whale's labyrinth?

On this matter different opinions have been expressed. Buchanan considered \* that the Eustachian tube and not the external meatus "conducted the pulsations of sound into the tympanum," causing vibratory movements of its membrane and corresponding action in the chain of ossicles; whilst the meatus, through the width of its tympanic end, facilitated the vibratory movements of the sac-like prolongation of the membrane into it. This view has not, however, been accepted.

Others have regarded the vibrations as excited by the aerial sound waves propagated down the external meatus, which directly impressed the membrane and were then conveyed by the chain of ossicles to the fenestra ovalis and labyrinth. As against this view it should be kept in mind that in the Cetacea the period is short and infrequent during which the external aperture is exposed to the air; waves of sound could be transmitted only intermittently and not to much purpose. Sufficient evidence now exists that in the whalebone whales the meatus is blocked with a large plug of wax; the lumen, therefore, cannot be occupied with air to permit the transmission through this medium of sound waves. On the other hand, the wax-plug is a solid body closely moulded in these whales on the sac-like membrane of the tympanum. As such it would doubtless transmit, as the cranial bones themselves can do, sound waves generated in the surrounding water, which would produce vibratory movements of the tympanic membrane and the chain of ossicles. In the baleen whales sufficient pressure exists in the air of the tympanum to produce the convex pouch-like projection of the membrane into the auditory meatus.

Some years ago Claudio wrote an interesting memoir on this subject,† and argued that in the Cetacea the sound waves were not directly transmitted by the Eustachian tube, the meatus auditorius, or through

\* *Op. cit.*

† *Ueber das Gehörorgan der Cetaceen*, Kiel, 1858.

the bones of the head to the nerves in the labyrinth; but that the waves detached themselves from the bones and thus impressed the air contained in the tympanic cavity and in the sac-like projection of its membrane in the baleen whales, and its prolongations into the accessory sinuses in the dolphins. The waves might then act in two ways, either through the fenestra ovalis and fenestra rotunda, by impressing the lymph in the divisions of the labyrinth and through it the end organs of the auditory nerve, or by setting in movement the chain of ossicles which have as their fixed point of attachment the malleo-tympanic interossification.

Claudius, therefore, thought that the sound waves reached the head of a Cetacean through the water in which it lived; that they were transmitted by the bones of the head to the air in the tympanic cavity, and that the waves generated in it directly caused vibrations through the fenestra ovalis and rotunda in the lymph in the labyrinth, as well as along the chain of bones, and impressed the nerve end organs. This view of the mode of excitation of the auditory nerve seems to be the most satisfactory.

*(Issued separately December 31, 1913.)*

III.—Note on a Siliceous Sponge of the Order Hexactinellida from South Shetland. By Principal Sir William Turner, K.C.B., D.C.L., F.R.S.

(Read December 1, 1913. MS. received December 2, 1913.)

THE specimen was presented to me by Mr G. Millen Coughtrey, who obtained it in Admiralty Bay, South Shetland, lat. 62° S., and long. 58° W., in 1912. It was procured in about 20 fathoms, and was brought to the surface when the ship's anchor was weighed. It consisted of white, delicate, thread-like spicules collected into two tufts or bundles. At the first glance the threads might easily have been mistaken for white hair, but they would not burn; neither were they calcareous, for they were not acted on by mineral acids. From their vitreous appearance they were obviously siliceous and indeed were not unlike spongework. Their aspect and composition led me to regard them as belonging to a siliceous sponge, but the body of the sponge was wanting. In its absence one had to rely on the characters of the tufts and spicules in attempting to determine the genus of the sponge.

One tuft was about 40 cm. (16 inches) long, and 3½ cm. at its greatest transverse diameter. The thread-like spicules were compacted and interlaced together at the proximal and mid parts of the tuft, but at the distal end it was somewhat dishevelled. It contained many hundred spicules and seemed as if it had belonged to one sponge. The smaller tuft was not so compact and might possibly have been divided into two parts, one for each of two smaller sponges.

The threads were the basalia or basal spicules of the sponge, which had grown downwards from the base of its body and had penetrated the mud on the floor of the sea in which the sponge lived. In weighing the ship's anchor the tufts of basal spicules had been drawn up at the same time. The spicules were so brittle that it was difficult to pick a single one out of the tuft without breaking it, but with care I obtained examples 30 cm. (12 inches) long. The spicules were smooth on the surface, translucent, and the transverse diameter ranged from 35 to 92  $\mu$ . In many of the smaller sizes the appearance of a narrow canal in the long axis of the spicule was seen, but in the wider spicules no structural differentiation was observed. One end of the spicule was frequently

attenuated to a fine point, though often it was broken abruptly; the opposite end was sometimes broken, at others it terminated in a very minute knob (scarcely visible to the naked eye), which, when magnified, was seen to be the rounded end of the basal spicule from which four hook-like processes equal in length arose; they were recurved in their direction, almost parallel to the long axis of the spicule, and formed an anchor-like arrangement which assisted in fixing the sponge in the mud (figure).

A close examination of the tufts showed that a number of the spicules, more especially near the proximal part of the tuft, had attached to them globular blackish specks, 300 to 320  $\mu$  in diameter, which contrasted in colour with the white spicules. Under low magnification they looked like very minute grains, and had with strong transmitted light a greyish-blue



Anchor-like end of basal spicule of siliceous sponge, South Shetland.

tint. Neither Canada balsam nor treatment with acetic acid and glycerine made the centre translucent, but the thinner periphery permitted greyish-blue-tinted siliceous microscopic flakes or scales to be seen, which, when superimposed on each other, gave opacity to the object. Owing to their hardness and brittleness, attempts failed to make sections through them. They seemed to be aggregations of siliceous plates attached to the spicule, the purport of which was difficult to explain.

A brown substance was occasionally seen to surround some of the basal spicules in the proximal part of the tuft. The largest example was 6 mm. long and was fusiform. From its structure it was essentially a fragment of the body of the sponge which had adhered to the basal spicules. In part it contained nuclear-looking bodies embedded in a granular protoplasm, but a number of ray-like spicules were also present. Many of these were four-rayed tetracts, which radiated horizontally from a common centre, and the largest specimens measured 0.5 mm. between the tips of opposite rays.

Others were much smaller, 0·2 mm. between their tips, and in many five or six rays could be seen, one or two of which had been broken across at or near the common centre; the type, therefore, was hexact. In the middle of the centre was a very minute circle, from which a line passed along the axis of each ray almost to its tip. The rays were sometimes smooth, but more usually slightly roughened near the tip and faintly serrated at the sides. Mingled with the ray-like spicules were numerous disc-shaped Diatoms, which varied in their dimensions from 77 to 96  $\mu$  across the face of the disc. They had doubtless lived in the mud in which the basal tuft had been anchored.

As the siliceous sponges have been described by Professor F. E. Schulze in an elaborate memoir in his *Challenger Report* on the Hexactinellida,\* I have examined the text and figures so as to identify, if possible, the species from the characters presented by the basal spicules. The length and thickness of the tufts and the number of spicules varied materially in different genera and species, but in the genera *Hyalonema* and *Pheronema* species existed in which the basal tuft of spicules attained a considerable length. Schulze gave as a character of *Hyalonema* a tuft consisting of long and strongly developed basal spicules which projected downwards from the centre of the lower end of the body, and the spicules themselves, either wholly, or for the most part, had four-toothed anchors. He stated that in *H. sieboldii* the total length of the body and tuft varied from 50 to 80 cm., and as the body occupied 10 to 15 cm. of that length, the basal tuft varied from 30 to 60 cm., and broke up at the lower end in a brush-like manner. This species inhabits the seas of Japan. In *H. affine* the tuft was 47 cm. long, but only 8 mm. broad. Wyville Thomson dredged in the sea north of the Butt of Lewis, from a depth of 450 to 500 fathoms, *Hyalonemata* in which the root tuft measured 40 cm. or more. In *Pheronema carpenteri*, obtained in the sea north of Scotland, as well as off the coast of Brazil, a number of slender tufts, only 1 to 2 mm. in breadth, the spicules of which were 30 to 40 cm. long, interlaced abundantly in the felt-work of the basal tuft. Wyville Thomson considered that in the larger specimens the tufts may measure several decimetres. Schulze stated that other species of this genus also possessed long basal spicules.

Schulze has figured numerous examples of four- and six-rayed spicules in the bodies of the sponges in the Hexactinellida group. In my specimen the rayed spicules found in the basal tuft did not properly belong to it, but had accidentally become intermingled with the basal spicules. Schulze specially referred to the lower end of the body of *Hyalonema* as containing

\* Vol. xxi, part 53, 1886.

four-rayed, tetract spicules. In the length of the tuft and in the numerous spicules which composed it, this sponge had also affinities with *Hyalonema*, though none of Schulze's figures had so bulky a tuft. *Hyalonema sieboldii*, however, seems to be the sponge which most closely corresponds with it in this particular.

Two other questions of interest arise out of this specimen, viz.: the locality and the depth from the surface. Schulze, in his map, showed the distribution of the order Hexactinellida, and localised a small species, *Rossella antarctica* (Carter), obtained by Sir James Ross in 1839–43, as far south as lat.  $74\cdot5^{\circ}$ , at a depth of 300 fathoms; also a small species, *Polyrhabdus oviniformis* (Schulze), obtained by the *Challenger* in lat.  $62\cdot26^{\circ}$ , in 1975 fathoms. With these exceptions no other specimen of this order, which from the size of the basal tuft was obviously a large species, had previously been obtained so near the Antarctic circle as  $62^{\circ}$  S. In his chapter on the bathymetrical distribution of the Hexactinellida, Schulze gave several species as dredged from a depth at and near 100 fathoms; but the depth of only 20 fathoms, given by Mr Coughtrey for the South Shetland specimen, localises *Hyalonema* in a shallower sea than had previously been recorded.

(Issued separately December 31, 1913.)

IV.—Some Factorable Minors of a Compound Determinant.  
By Professor W. H. Metzler.

(MS. received April 17, 1913. Read November 3, 1913.)

If we start with a determinant  $A$  of order  $n$ , and, using exclusive umbral notation, take the minor

$$M \equiv \begin{pmatrix} (n \mid m \mid k) \\ \alpha & 1 \end{pmatrix} \quad \begin{pmatrix} (n \mid m \mid k) \\ \alpha & 2 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} (n \mid m \mid k) \\ \alpha & \lambda \end{pmatrix} \lambda = m_k$$

\* of the  $(n-k)$ th compound of A, Sylvester \* has shown that

$$M = A \begin{array}{c} (m-1)k \\ \cdot \end{array} \left| \begin{array}{c} (n \mid m) \\ \cdot \\ (n \mid m) \\ \cdot \end{array} \right|^{(m-1)k-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

Besides this, Muir † has considered another type of minor which breaks up into factors. It may be obtained from  $M$  by putting  $k=m-1$ , and in place of the combinations  $(n | m | m-1), \dots (n | m | m-1)$  indicating the selections of rows for the elements we take the combinations  $\underline{12} \dots \underline{m-1}$ ,  $23 \dots m$ ,  $34 \dots \underline{m+1}, \dots \underline{mn+1} \dots \underline{2n-2}$ , where for definiteness of statement we suppose  $a=1$ , and  $(n | m) \equiv \underline{\underline{12}} \dots \underline{\underline{m}}$ . Thus the theorem given in Muir is

$$= A \cdot \begin{vmatrix} 23 & \dots & \overline{m+1} \\ (n|m) & & \\ \hline 1 & & \end{vmatrix} \cdot \begin{vmatrix} 34 & \dots & \overline{m+2} \\ (n|m) & & \\ \hline 1 & & \end{vmatrix} \cdots \begin{vmatrix} m & \dots & \overline{2m-3} \\ (n|m) & & \\ \hline 1 & & \end{vmatrix} \quad (B)$$

In both these theorems the combinations indicating the selection of row numbers are definite. In Sylvester's theorem they are the same as the selection of the column numbers. In Muir's the first one is the same, and the rest may be obtained by a definite sliding process.

The object of the present paper is to show that there are a large number of other minors which break up into factors, and to give a general theorem (C) which includes these two as special cases.

Theorems (A) and (B) are readily proved by the method used by the

\* *Philosophical Magazine*, 1851.

<sup>†</sup> *A Treatise on Determinants*, Art. 93.

author in 1897.\* Starting with a particular case of the general theorem (C), where  $m=5$  and  $k=3$ , and using a similar method, we have on multiplying

$$M \equiv \left( \begin{array}{c|c|c|c|c|c|c|c|c|c|c} --- & ---4 & ---5 & -34 & -35 & -45 & 234 & 235 & 245 & 345 \\ \hline 123 & 124 & 125 & 134 & 135 & 145 & 234 & 235 & 245 & 345 \end{array} \right)$$

by

$$N \equiv \left( \begin{array}{c|c|c|c|c|c|c|c|c|c} 45 & 35 & 34 & 25 & 24 & 23 & 15 & 14 & 13 & 12 \\ \hline 45 & 35 & 34 & 25 & 24 & 23 & 15 & 14 & 13 & 12 \end{array} \right),$$

where the positions indicated by the dashes in M may be filled with any numbers from the set  $12 \dots n$ , as long as (1) no two numbers in the same element are alike, for in that case every element in that row of M would be zero; (2) the row numbers in the  $i$ th row of M and the complementary with respect to  $m$  of the column numbers in the  $i$ th column of M have no numbers in common, for if they had then the corresponding element in the principal diagonal of the product would be zero, and therefore the product zero; the product

$$M \cdot N = A^{10} \cdot \left| \begin{array}{c} ---45 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} ---345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} ---345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} 12345 \\ 12345 \end{array} \right|^4$$

For the product has every element on one side of the principal diagonal zero, and therefore it equals the product of the elements along the principal diagonal.

By Sylvester's theorem

$$N = A^6 \cdot \left| \begin{array}{c} 12345 \\ 12345 \end{array} \right|^4,$$

and dividing out the common factor from both sides we have

$$M = A^4 \cdot \left| \begin{array}{c} ---45 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} ---345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} ---345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right| \cdot \left| \begin{array}{c} -2345 \\ 12345 \end{array} \right|,$$

where of course the numbers in the places indicated by the dashes are the same with which we started in M.

It will be observed that the 4 in the row numbers of the second row of M, the 5 in the third row, the 3 and 4 in the fourth, the 3 and 5 in the fifth, etc., are what make all the elements on one side of the principal diagonal of the product vanish and the minor break up into factors. This is true independent of the numbers (under the restrictions named) in the places indicated by the dashes.

\* Metzler, "Compound Determinants," *American Journal of Mathematics*, vol. xx. No. 3.

The row of  $M$  which has the three arbitrary row numbers is the same (viz. the first) as the column of  $M$  which has the column numbers 1 2 3. That is, the three arbitrary row numbers are associated with the column numbers 1 2 3, and it is obvious that they might have been associated with any of the ten combinations of the numbers 1 2 3 4 5 taken three at a time. Thus, for instance, associating them with the combination 2 3 4, we have the minor

$$= A^4 \cdot \left| \begin{array}{c} 1 - - - 5 \\ 1 2 3 4 5 \end{array} \right| \cdot \left| \begin{array}{c} 1 - - 4 5 \\ 1 2 3 4 5 \end{array} \right| \cdot \left| \begin{array}{c} 1 - - 4 5 \\ 1 2 3 4 5 \end{array} \right| \cdot \left| \begin{array}{c} 1 - 3 4 5 \\ 1 2 3 4 5 \end{array} \right| \cdot \left| \begin{array}{c} 1 - 3 4 5 \\ 1 2 3 4 5 \end{array} \right| \cdot \left| \begin{array}{c} 1 - 3 4 5 \\ 1 2 3 4 5 \end{array} \right|.$$

If in M we put in the place of the dashes the same numbers as those in the column numbers with which they are associated, the six factors other than  $A^4$  are all equal, and we have Sylvester's theorem.

If  $k=4$  and we take the minor

$$M \equiv \left( \begin{array}{c|c|c|c|c} 1 & 2 & 3 & 4 & - \\ \hline 1 & 2 & 3 & 4 & - \\ 1 & 2 & 3 & 4 & 5 \\ \hline & 1 & 2 & 4 & 5 \\ & 1 & 3 & 4 & 5 \\ \hline & - & - & - & - \end{array} \right),$$

which equals

$$A \cdot \begin{vmatrix} 1 & 2 & 3 & 4 & - \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 3 & - & - \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & - & - & - \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & - & - & - & - \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix},$$

and put in the place of the dashes 8, 7 8, 6 7 8, 5 6 7 8, respectively, so that

$$M \equiv \left( \begin{array}{|c c c c|} \hline 1 & 2 & 3 & 4 \\ \hline 1 & 2 & 3 & 4 \\ \hline 1 & 2 & 3 & 5 \\ \hline 1 & 2 & 3 & 5 \\ \hline 1 & 2 & 4 & 5 \\ \hline 1 & 3 & 4 & 5 \\ \hline 2 & 3 & 4 & 5 \\ \hline 5 & 6 & 7 & 8 \\ \hline \end{array} \right),$$

we have

$$M = A \cdot \begin{vmatrix} 1 & 2 & 3 & 4 & 8 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 3 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix} \cdot \begin{vmatrix} 1 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 \end{vmatrix},$$

which is an example of Muir's theorem.

In the general case the theorem is simple, though its statement is a little cumbersome.

Take any one of the  $m_k = \lambda$  combinations  $(n \mid m \mid k)$ ,  $\begin{matrix} (n \mid m \mid k) \\ \alpha \quad 1 \\ \alpha \quad 2 \end{matrix}, \dots$   $(n \mid m \mid k)$ , say the  $\beta$ th or  $\begin{matrix} (n \mid m \mid k) \\ \alpha \quad \beta \end{matrix}$  to start with, and arrange the set into groups as follows:—

1st group containing 1 combination consisting of  $(n \mid m \mid k)$ .

2nd group containing  $(m-k)_1$  combinations, consisting of those which have in common the first  $(k-1)$  only of the numbers of  $(n \mid m \mid k)$ .

$(h+1)$ st group containing  $(m-k+h-1)_h$  combinations, consisting of those which have in common the first  $(k-h)$  only of the numbers of  $\begin{pmatrix} n \\ \alpha & \beta \end{pmatrix}^h$ .

$(k+1)$ th and last group containing  $(m-1)_k$  combinations, consisting of those which do not contain the first number in  $\begin{pmatrix} n \\ \alpha & \beta \end{pmatrix}^k$ .

Let

$$\begin{pmatrix} n \\ \alpha & \beta \end{pmatrix}^h \begin{pmatrix} k \\ 1 \end{pmatrix}^{(h)} \quad (i=0, 1, 2, \dots, (m-k+h-1)_h - 1)$$

represent the combinations of the  $(h+1)$ st group. By giving  $h$  in this the values from 0 to  $k$ , it will represent the combinations of each group.

Let the first combinations in the 1st, 2nd,  $\dots$   $(k+1)$ th groups, arranged in this order, be represented by

$$\begin{pmatrix} n \\ \alpha & \beta_0 \end{pmatrix}, \begin{pmatrix} n \\ \alpha & \beta_1 \end{pmatrix}, \dots, \begin{pmatrix} n \\ \alpha & \beta_k \end{pmatrix},$$

respectively, and let

$$\Delta \frac{p-1}{\prod_{i=0}^h} \frac{k}{\prod_{i=0}^h} \left( \begin{vmatrix} \begin{pmatrix} n \\ \alpha & \beta_h+i \end{pmatrix} \\ \begin{pmatrix} n \\ \alpha & \beta \end{pmatrix}^h \begin{pmatrix} k \\ 1 \end{pmatrix}^{(h)} \begin{pmatrix} c_i^{(h)} \\ \end{pmatrix} \end{vmatrix} \right) \quad (p=(m-k+h-1)_h)$$

represent the determinant whose elements along the principal diagonal arranged in the above order are

$$\begin{vmatrix} \begin{pmatrix} n \\ \alpha & 1 \end{pmatrix} \\ \begin{pmatrix} n \\ \alpha & 1 \end{pmatrix} \end{vmatrix}, \begin{vmatrix} \begin{pmatrix} n \\ \alpha & 2 \end{pmatrix} \\ \begin{pmatrix} n \\ \alpha & 2 \end{pmatrix} \end{vmatrix}, \dots, \begin{vmatrix} \begin{pmatrix} n \\ \alpha & \lambda \end{pmatrix} \\ \begin{pmatrix} n \\ \alpha & \lambda \end{pmatrix} \end{vmatrix}.$$

Then the determinant

$$\begin{aligned} \mathbb{D} &\equiv \Delta \frac{p-1}{\prod_{i=0}^h} \frac{k}{\prod_{i=0}^h} \left( \begin{vmatrix} \begin{pmatrix} n \\ \alpha & \beta_h+i \end{pmatrix} \\ (b_i^{(k-h)}) (c_i^{(h)}) \end{vmatrix} \right) \\ &= \frac{p-1}{\prod_{i=0}^h} \frac{k-1}{\prod_{i=0}^h} \left( \begin{vmatrix} \begin{pmatrix} n \\ \alpha \end{pmatrix} \\ (b_i^{(k-h)}) (c_i^{(h)}) (\begin{pmatrix} n \\ \alpha \end{pmatrix} \mid \begin{pmatrix} m \\ \beta_h+i \end{pmatrix}) \end{vmatrix} \right) \cdot A^{(m-1)k} \quad (C) \end{aligned}$$

where  $(b_i^{(k-h)})$  represents any set of  $(k-h)$  of the numbers 1 2 3  $\dots$   $n$ , as long as (1)  $(b_i^{(k-h)}) (c_i^{(h)})$  has no two numbers alike; and (2) no numbers in  $(b_i^{(k-h)})$  and  $\begin{pmatrix} n \\ \alpha & \beta_h+i \end{pmatrix}$  are alike.

For if we multiply D by

$$\Delta \frac{p-1}{\prod_0^i} \cdot \frac{k}{\prod_0^h} \left( \begin{vmatrix} (n|m|k) \\ \alpha \beta_h + i \\ (n|m|k) \\ \alpha \beta_h + i \end{vmatrix} \right),$$

which by Sylvester's theorem is equal to

$$A^{(m-1)_{k-1}} \cdot A_{(n|m)}^{(m-1)_k},$$

$$(n|m)$$

the resulting determinant will have every element on one side of the diagonal zero, and therefore equal to the product of the elements along the diagonal. Dividing out the common factor from both sides, we get the result given.

SYRACUSE UNIVERSITY,  
March 1913.

(Issued separately December 31, 1913.)

## V.—The Theory of Bigradients from 1859 to 1880.

By Thomas Muir, LL.D.

(MS. received June 9, 1913. Read November 17, 1913.)

My last communication in reference to the history of bigradients (*Proc. Roy. Soc. Edin.*, xxx. pp. 396–406) brought the record up to the year 1859. The present paper continues it to the year 1880.

BRUNO, F. FAÀ DI (1859).

[THÉORIE GÉNÉRALE DE L'ÉLIMINATION, . . . . x+224 pp. Paris.]

In his section on the highest-common-divisor (pp. 47–52) Bruno, denoting the  $r^{\text{th}}$  Sturmian remainder of

$$a_0x^m + a_1x^{m-1} + \dots, \quad b_0x^{m-1} + b_1x^{m-2} + \dots$$

by  $R_r$ , finds for it the expression

$$\lambda_r \sum_{l=0}^{l=m-r+1} \left\{ x^{m+r-l-1} \begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_{2r-1} & a_{2r+l} \\ \cdot & a_0 & a_1 & \dots & a_{2r-2} & a_{2r+l-1} \\ \cdot & \cdot & a_0 & \dots & a_{2r-3} & a_{2r+l-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_0 & b_1 & b_2 & \dots & b_{2r-1} & b_{2r+l} \\ \cdot & b_0 & b_1 & \dots & b_{2r-2} & b_{2r+l-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & b_{r-1} & b_{r+l} \end{vmatrix} \right\}$$

where  $\lambda_r$  and  $R_r/\lambda_r$  are the “allotrious factor” and “simplified residue” of Sylvester (1853). He must, however, have overlooked the question of sign, for the example which he gives, namely,

$$\frac{1}{b_0^2} \left\{ \begin{vmatrix} a_0 & a_1 & a_2 & x^2 \\ b_0 & b_1 & b_2 & \end{vmatrix} + \begin{vmatrix} a_0 & a_1 & a_3 & x \\ b_0 & b_1 & b_3 & \end{vmatrix} + \begin{vmatrix} a_0 & a_1 & a_4 & \\ b_0 & b_1 & \cdot & \\ \cdot & b_0 & b_3 & \end{vmatrix} \right\}$$

as the first Sturmian remainder of

$$a_0x^4 + a_1x^3 + \dots + a_4, \quad b_0x^3 + b_1x^2 + b_2x + b_3$$

is incorrect in this particular.

In the section on the properties of the resultant (pp. 68-81) he recalls Richelot's theorem of 1840, that if  $w$  be a common root of the equations

$$a_0x^m + a_1x^{m-1} + \dots = 0, \quad b_0x^n + b_1x^{n-1} + \dots = 0$$

whose resultant is  $R$ , then we have

$$\begin{aligned} \frac{\partial R}{\partial a_0} : \frac{\partial R}{\partial a_1} : \dots : \frac{\partial R}{\partial a_m} &:: w^m : w^{m-1} : \dots : 1 \\ \frac{\partial R}{\partial b_0} : \frac{\partial R}{\partial b_1} : \dots : \frac{\partial R}{\partial b_n} &:: w^n : w^{n-1} : \dots : 1. \end{aligned}$$

This is not brought forward as a theorem in determinants, but for comparison, when  $n=m$ , with Jacobi's theorem of 1835 to the effect that *the signed primary minors associated with the elements of any row of Bezout's condensed eliminant*

$$\left| \begin{array}{c|c|c|c} |a_0b_1| & |a_0b_2| & \dots & \\ |a_0b_2| & |a_0b_3| + |a_1b_2| & \dots & \\ \dots & \dots & \dots & \dots \end{array} \right|_m$$

are proportional to

$$w^{m-1}, w^{m-2}, \dots, w, 1.$$

In the section on common roots (pp. 81-84) he obtains such a root when it is solitary by taking any one of these three series of proportionals and dividing one member of the series by the member immediately following. When the number of such roots is  $k$  he has recourse to the Sturmian remainders previously found, stating for comparison Lagrange's set of conditions :\*

$$R = 0, \quad \frac{\partial R}{\partial a_m} = 0, \quad \frac{\partial^2 R}{\partial a_m^2} = 0, \quad \dots, \quad \frac{\partial^{k-1} R}{\partial a_m^{k-1}} = 0.$$

### TRUDI, N. (1862).

[TEORIA DE' DETERMINANTI, . . . . xii + 268 pp. Napoli.]

To Trudi is due the first methodical exposition of bigradients, a nineteen-page chapter of the first part (*Teoria*) of his text-book being specially devoted to them, and several chapters of his second part (*Applicazioni*) making constant use of them.

The nineteen-page chapter or section (§ xi., pp. 94-112) bears the heading "Matrici e determinanti a due scale." It contains, first of all, careful explanations of the various expressions which he finds necessary to use in

\* LAGRANGE. Réflexions sur la résolution algébrique des équations. *Nouv. Mémoires Acad. . . . Berlin*, 1770, 1771: or *Oeuvres complètes*, iii. pp. 205-421 (227-229).

a special sense while dealing with such determinants; for example, *scala*, *scala di grado r*, *scala diretta*, *scala inversa*, *scala completa*, etc. He next gives an account of the simple properties of bigradient arrays, or, as he calls them, *two-scale matrices*, and introduces a notation for them, writing

$$\begin{vmatrix} (a_0)_r \\ (b_0)_s \end{vmatrix}$$

to denote the bigradient array in which the elements  $a_0, a_1, \dots, a_m$  are repeated  $r$  times, and the elements  $b_0, b_1, \dots, b_n$  are repeated  $s$  times,  $a_0$  and  $b_0$  when furthest to the left being in the first column, and  $a_m$  and  $b_n$  when furthest to the right being in the last column. Clearly, the notation would have been less imperfect if written

$$\begin{vmatrix} (a_0, \dots, a_m)_r \\ (b_0, \dots, b_n)_s \end{vmatrix}.$$

For example, the bigradient array

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & \cdot & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & \cdot \\ \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ \cdot & \cdot & \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & \cdot & \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot & \cdot \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot & \cdot & \cdot \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

might with fair appropriateness be denoted by

$$\begin{vmatrix} (a_0, \dots, a_7)_3 \\ (b_0, \dots, b_4)_6 \end{vmatrix},$$

the only weak point then being that the introduction of the 6 is uncalled for, on account of the necessary equality of  $m+r$  and  $n+s$ , either of which specifies the number of columns in the array. It is a convenience, however, to have both the outside suffixes 3, 6 in front of us, because their sum gives the number of the rows, a sum we should otherwise have to know from  $m+2r-n$ . Instead of all the determinants of such an array being viewed, as hitherto, of equal prominence, Trudi only concerns himself with the first two of the ten, namely, those which have in common the first eight columns of the array. These  $n-r+1$  determinants he designates not very happily "the successive determinants of the array." The name "principal" which he gives to the first determinant of all may be advantageously translated "leading."

The number of bigradient arrays associated with the two sets of elements

$$a_0, a_1, \dots, a_m \quad b_0, b_1, \dots, b_n$$

is evidently  $n$ : thus, in the case where  $m, n = 7, 4$  the arrays are

$$\left\| \begin{matrix} (a_0, \dots, a_7)_1 \\ (b_0, \dots, b_4)_4 \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_7)_2 \\ (b_0, \dots, b_4)_5 \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_7)_3 \\ (b_0, \dots, b_4)_6 \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_7)_4 \\ (b_0, \dots, b_4)_7 \end{matrix} \right\|$$

the last, where  $r=n$ , being square, and therefore preferably written in the form

$$\left| \begin{matrix} (a_0, \dots, a_7)_4 \\ (b_0, \dots, b_4)_7 \end{matrix} \right|.$$

These and other preliminaries being settled, he is in a position to deal with an important theorem on the subject of what we may call *the condensation of a bigradient array*. The proof given is, unfortunately, not at all so simple as it might have been. We shall therefore substitute for it one of our own, which Trudi himself would probably have devised had he been aware of Cayley's work of 1845. Taking, first, a case in which  $m=n$ , say the case

$$\left\| \begin{matrix} a_0 & a_1 & a_2 & a_3 & a_4 & \cdot & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & \cdot \\ \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 \\ \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot \end{matrix} \right\| \text{ or } \left\| \begin{matrix} (a_0, \dots, a_4)_3 \\ (b_0, \dots, b_4)_3 \end{matrix} \right\|$$

we multiply the determinant

$$\left| \begin{matrix} 1 & \cdot & \cdot & \cdot & \cdot & -b_0 \\ \cdot & 1 & \cdot & \cdot & -b_0 & -b_1 \\ \cdot & \cdot & 1 & -b_0 & -b_1 & -b_2 \\ \cdot & \cdot & \cdot & a_0 & a_1 & a_2 \\ \cdot & \cdot & \cdot & a_0 & a_1 & a_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & a_0 \end{matrix} \right|$$

by the given array in column-by-column fashion, obtaining (*Hist.*, ii. p. 34)

$$(a_0)_3 \left\| \begin{matrix} (a_0, \dots, a_4)_3 \\ (b_0, \dots, b_4)_3 \end{matrix} \right\| = \left\| \begin{matrix} a_0 & a_1 & a_2 & a_3 & a_4 & \cdot & \cdot & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & \cdot & \cdot \\ \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & \cdot \\ \cdot & \cdot & \cdot & |a_0b_1| & |a_0b_2| & |a_0b_3| & |a_0b_4| & \cdot \\ \cdot & \cdot & \cdot & |a_0b_2| & |a_0b_3| + |a_1b_2| & |a_0b_4| + |a_1b_3| & |a_1b_4| & \cdot \\ \cdot & \cdot & \cdot & a_0b_3 & |a_0b_4| + |a_1b_3| & |a_1b_4| + |a_2b_3| & |a_2b_4| & \cdot \end{matrix} \right\|$$

Of the seven identities included in this the first two are Trudi's, and these he writes in combination, thus—

$$\left| \begin{array}{cccccc} a_0 & a_1 & a_2 & a_3 & a_4 & \cdot & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & \cdot \\ \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 \\ \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot \end{array} \right| = \left| \begin{array}{cccc} |a_0b_1| & |a_0b_2| & |a_0b_3| & |a_0b_4| \\ |a_0b_2| & |a_0b_3| + |a_1b_2| & |a_0b_4| + |a_1b_3| & |a_1b_4| \\ |a_0b_3| & |a_0b_4| + |a_1b_3| & |a_1b_4| + |a_2b_3| & |a_2b_4| \end{array} \right|,$$

meaning thereby that the determinants got by leaving out the 7<sup>th</sup> column on the left and the 4<sup>th</sup> on the right are equal to one another, and also those determinants got by leaving out the 6<sup>th</sup> column on the left and the 3<sup>rd</sup> on the right.\*

He then draws attention to the fact that the two-line determinants involved in the array on the right are principal minors of the array

$$\begin{array}{ccccc} a_0 & a_1 & a_2 & a_3 & a_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 \end{array}$$

and he formulates a mnemonic rule like Sylvester's (*Hist.*, ii. p. 340) for the formation of the condensed array. His own illustrative examples are

$$\begin{aligned} & \left| \begin{array}{cccccc} a & b & c & d & \cdot & \cdot \\ \cdot & a & b & c & d & \cdot \\ \cdot & \cdot & a & b & c & d \\ \cdot & \cdot & t & u & v & x \\ \cdot & t & u & v & x & \cdot \\ t & u & v & x & \cdot & \cdot \end{array} \right| = \left| \begin{array}{ccc} au - bt & av - ct & ax - dt \\ av - ct & ax - dt & bx - du \\ + bv - cu & & \end{array} \right| \\ & \left| \begin{array}{cccc} a & b & c & d \\ \cdot & a & b & c & d \\ \cdot & t & u & v & x \\ t & u & v & x & \cdot \end{array} \right| = \left| \begin{array}{ccc} au - bt & av - ct & ax - dt \\ av - ct & ax - dt & bx - du \\ + bv - cu & & \end{array} \right|, \\ & \left| \begin{array}{cccc} a & b & c & d \\ t & u & v & x \end{array} \right| = \left| \begin{array}{c} au - bt, \\ av - ct, \\ ax - dt \end{array} \right|, \end{aligned}$$

\* With Cayley the assertion

$$\left| \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{array} \right| = \left| \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{array} \right|$$

included 6 equations, whereas with Trudi it only includes 3, namely, the first 3 of Cayley's 6; and with Cayley the assertion

$$\left| \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{array} \right| = \left| \begin{array}{ccc} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{array} \right|$$

was meaningless, whereas with Trudi it includes 2 equations. Since in the former case Trudi's 3 equations are known to necessitate the other 3, there is clearly no good reason for refusing to profit by the new usage. What is common to any two arrays which Trudi may equate is the excess of the number of columns over the number of rows: and evidently if his excess be  $\delta$ , the number of included equations is  $\delta+1$ .

where each array on the left is got from the one that precedes it by deleting the first row, the first column, and the last row; and each array on the right by merely deleting the last row. It is noted that the leading determinant of the condensed array is axisymmetric.

Lastly, it is pointed out that cases where  $m > n$  present almost no additional difficulty, as they are readily brought under the foregoing. Thus, if the case be

$$\begin{vmatrix} (a, b, c, d, e, f)_2 \\ (t, u, v)_5 \end{vmatrix},$$

we have only to take

$$\begin{array}{cccccc} a & b & c & d & e & f \\ 0 & 0 & 0 & t & u & v \end{array}$$

for our generating array and proceed exactly as before, the results being

$$\begin{aligned} \begin{vmatrix} a & b & c & d & e & f & \cdot \\ \cdot & a & b & c & d & e & f \\ \cdot & \cdot & \cdot & t & u & v & \cdot \\ \cdot & \cdot & \cdot & t & u & v & \cdot \\ \cdot & \cdot & t & u & v & \cdot & \cdot \\ t & u & v & \cdot & \cdot & \cdot & \cdot \end{vmatrix} &= \begin{vmatrix} \cdot & \cdot & at & au & av \\ \cdot & at & au+bt & av+bu & bv \\ at & au+bt & av+bu+ct & bv+cu & cv \\ au & av+bu & bv+cu & cv+du-et & dv-ft \\ av & bv & cv & dv-ft & ev-fu \end{vmatrix} \div a^3 \\ &= \begin{vmatrix} \cdot & \cdot & t & u & v \\ \cdot & t & u & v & \cdot \\ t & u & v & \cdot & \cdot \\ au & av+bu & bv+cu & cv+du-et & dv-ft \\ av & bv & cv & dv-ft & eu-fu \end{vmatrix}, \\ \begin{vmatrix} a & b & c & d & e & f \\ \cdot & \cdot & \cdot & t & u & v \\ \cdot & \cdot & t & u & v & \cdot \\ \cdot & t & u & v & \cdot & \cdot \\ t & u & v & \cdot & \cdot & \cdot \end{vmatrix} &= \begin{vmatrix} \cdot & \cdot & t & u & v \\ \cdot & t & u & v & \cdot \\ t & u & v & \cdot & \cdot \\ au & av+bu & bv+cu & cv+du-et & dv-ft \end{vmatrix}. \end{aligned}$$

The requisite division by  $a^3$  (in general  $a^{m-n}$ ) may be performed by removing the  $a$ 's one at a time, or by using the divisor in the form

$$\begin{vmatrix} a & b & c & \cdot & \cdot \\ \cdot & a & b & \cdot & \cdot \\ \cdot & \cdot & a & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{vmatrix}.$$

Another theorem of a similar kind but introduced for a different purpose, namely, for dilatation rather than condensation (pp. 129-131), is

$$\begin{vmatrix} (b_0, \dots, b_n)_r \\ (c_0, \dots, c_{n-1}) \end{vmatrix} = \frac{(-1)^\sigma}{b_0^{m-n+1}} \begin{vmatrix} (a_0, \dots, a_m)_{r+1} \\ (b_0, \dots, b_n) \end{vmatrix},$$

where the  $c$ 's are determinants defined by the postulated identity

$$\frac{a_0x^m + \dots}{b_0x^n + \dots} = q_0x^{m-n} + \dots + \frac{c_0x^{n-1} + \dots}{b_0x^n + \dots}$$

(see under *Recurrents*) and where  $\sigma = r+1+\frac{1}{2}(m-n)(m-n-1)$ . For example, when  $m=5, n=4, r=2$  the identity is

$$\left| \begin{array}{cccccc} b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & \cdot & c_0 & c_1 & c_2 & c_3 \\ \cdot & c_0 & c_1 & c_2 & c_3 & \cdot \\ c_0 & c_1 & c_2 & c_3 & \cdot & \cdot \end{array} \right| = \frac{(-1)^3}{b_0^2} \left| \begin{array}{cccccc} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & \cdot & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & \cdot \\ \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot & \cdot \end{array} \right|.$$

Trudi's proof consists in evolving the second member from the first, but here again it is simpler to use Cayley's multiplication-theorem of 1845. Thus, taking the second array as multiplier and the determinant

$$\left| \begin{array}{cccccc} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -q_1 & 1 & \cdot & \cdot & \cdot \\ \cdot & -q_1 & -q_0 & \cdot & 1 & \cdot & \cdot \\ -q_1 & -q_0 & \cdot & \cdot & \cdot & 1 & \cdot \\ -q_0 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{array} \right|$$

as multiplicand we at once find the product to be

$$\left| \begin{array}{cccccc} \cdot & \cdot & c_0 & c_1 & c_2 & c_3 & \cdot & \cdot \\ \cdot & \cdot & \cdot & c_0 & c_1 & c_2 & c_3 & \cdot \\ \cdot & \cdot & \cdot & \cdot & c_0 & c_1 & c_2 & c_3 \\ \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot & \cdot & \cdot \end{array} \right|,$$

which is equal to

$$-b_0^2 \left| \begin{array}{ccccc} c_0 & c_1 & c_2 & c_3 & \cdot & \cdot \\ \cdot & c_0 & c_1 & c_2 & c_3 & \cdot \\ \cdot & \cdot & c_0 & c_1 & c_2 & c_3 \\ \cdot & b_0 & b_1 & b_2 & b_3 & b_4 \\ b_0 & b_1 & b_2 & b_3 & b_4 & \cdot \end{array} \right| = -b_0^2 \left| \begin{array}{c} (b_0 \dots b_4)_2 \\ (c_0 \dots, c_3)_3 \end{array} \right|,$$

as was to be proved.

The use to which this second theorem is put (pp. 132-137) is in connection with the division-process for finding the highest-common-divisor of two integral functions, and, in particular, with the modification of the said process employed by Sturm in obtaining his so-called "remainders." From the general theorem\* connecting dividend, divisor, quotient, and remainder we know that the coefficients of the first remainder in such a process are proportional to the successive determinants of a bigradient array composed of the coefficients of the dividend and divisor. We thus also know that this remainder having been made the divisor and the previous divisor the dividend, the new remainder must be expressible in like fashion. In the second bigradient array thus arising, however, one of the two sets of elements is complicated, being in fact the successive determinants of the previous array : and what Trudi's "dilatation" theorem enables us to do is to supplant it by another array whose elements are simply the coefficients of the original functions. In this way the theorem finally reached is : *The coefficients of the  $r^{\text{th}}$  remainder  $R_r$  arising in the course of the performance of Sturm's division-process on*

$$a_0x^m + \dots + a_m, \quad b_0x^n + \dots + b_n$$

*are equal to the successive determinants of the array*

$$\left\| \begin{array}{c} (a_0, \dots, a_m)_r \\ (b_0, \dots, b_n) \end{array} \right\|$$

*divided by the product of the squares of the first coefficients of all the preceding remainders and by  $b_0^{m-n+1}$  and by the sign-factor  $(-1)^{\frac{1}{2}(m-n)(m-n-1)}$ . The  $r^{\text{th}}$  remainder, when divested of the threefold divisor here specified,  $a_r$ , say, Trudi follows Sylvester in calling the  $r^{\text{th}}$  residuo semplificato,† and denotes by  $\rho_r$ , so that  $R_r = \rho_r/a_r$ . For example, when the originating functions A and B are*

$$\begin{aligned} ax^4 + bx^3 + cx^2 + dx + e, \\ px^3 + qx^2 + rx + s, \end{aligned}$$

\* See under *Recurrents*.

† A most natural and helpful notation for such a remainder would be

$$\left\| \begin{array}{c} (a_0, \dots, a_m)_r \\ (b_0, \dots, b_n) \end{array} \right\| (x^{n-r}, \dots, x, 1).$$

Thus, in the case here used for purposes of illustration, the remainders would be written

$$\left\| \begin{array}{ccccc} a & b & c & d & e \\ \cdot & p & q & r & s \\ p & q & r & s & \cdot \end{array} \right\| (x^2, x, 1), \quad \left\| \begin{array}{ccccc} (a & b & c & d & e)_2 \\ (p & q & r & s)_3 \end{array} \right\| (x, 1), \quad \dots$$

the three Sturmian remainders in their "simplified" or disencumbered form are

$$\begin{array}{c} \left| \begin{array}{ccc|c} a & b & c & \\ \cdot & p & q & x^2 \\ p & q & r & \end{array} \right| + \left| \begin{array}{ccc|c} a & b & d & \\ \cdot & p & r & x \\ p & q & s & \end{array} \right| + \left| \begin{array}{ccc|c} a & b & e & \\ \cdot & p & s & \\ p & q & \cdot & \end{array} \right|, \\ \left| \begin{array}{ccccc|c} a & b & c & d & e & \\ \cdot & a & b & c & d & \\ \cdot & \cdot & p & q & r & x \\ \cdot & p & q & r & s & \\ p & q & r & s & \cdot & \end{array} \right| + \left| \begin{array}{ccccc|c} a & b & c & d & \cdot & \\ \cdot & a & b & c & e & \\ \cdot & \cdot & p & q & s & \\ \cdot & p & q & r & \cdot & \\ p & q & r & s & \cdot & \end{array} \right|, \\ \left| \begin{array}{ccccc|c} a & b & c & d & e & \cdot & \cdot \\ \cdot & a & b & c & d & e & \cdot \\ \cdot & \cdot & a & b & c & d & e \\ \cdot & \cdot & \cdot & p & q & r & s \\ \cdot & \cdot & p & q & r & s & \cdot \\ \cdot & p & q & r & s & \cdot & \cdot \\ p & q & r & s & \cdot & \cdot & \cdot \end{array} \right|, \end{array}$$

the removed encumbrances being the factors

$$\frac{1}{p^2} \left| \begin{array}{ccc|c} & p^2 & & \\ a & b & c & \cdot^2 \\ \cdot & p & q & \\ p & q & r & \end{array} \right|, \quad p^2 \left| \begin{array}{ccccc|c} a & b & c & d & e & \cdot^2 \\ \cdot & a & b & c & d & \cdot \\ \cdot & \cdot & p & q & r & \\ \cdot & p & q & r & s & \cdot \\ p & q & r & s & \cdot & \cdot \end{array} \right|,$$

respectively. The general expression for the factor,  $a_r$ , connecting the simplified and unsimplified forms of a remainder is readily got (pp. 138–139) in Sylvester's way by using the fact that *the product  $a_{r-1}a_r$  is equal to the square of the first coefficient of  $\rho_r$* . For, this is the same as saying that, if we denote the first determinant of

$$\left\| \begin{array}{c} (a_0, \dots, a_m)_r \\ (b_0, \dots, b_n) \end{array} \right\|$$

by  $D_r$ , we have

$$a_2a_2 = D_1^2, \quad a_2a_3 = D_2^2, \quad \dots$$

and these lead to

$$a_{2\mu} = \frac{1}{a_1} \cdot \frac{D_1^2 D_3^2 D_5^2 \dots D_{2\mu-1}^2}{D_2^2 D_4^2 \dots D_{2\mu-2}^2},$$

and

$$a_{2\mu+1} = a_1 \cdot \frac{D_2^2 D_4^2 \dots D_{2\mu}^2}{D_1^2 D_3^2 \dots D_{2\mu-1}^2},$$

where

$$a_1 = (-1)^{\frac{1}{2}(m-n)(m-n-1)} \cdot b_0^{m-n+}.$$

An important observation made in passing is that any simplified remainder can be condensed into a single determinant: for example, the three just given are equal to

$$\begin{array}{c} \left| \begin{array}{cccc} a & b & cx^2 + dx + e \\ . & p & qx^2 + rx + s \\ p & q & rx^2 + sx \end{array} \right| \text{ or } \left| \begin{array}{ccc} a & b & A \\ . & p & B \\ p & q & Bx \end{array} \right|, \\ \left| \begin{array}{ccccc} a & b & c & d & ex \\ . & a & b & c & dx + e \\ . & . & p & q & rx + s \\ . & p & q & r & sx \\ p & q & r & s & : \end{array} \right| \text{ or } \left| \begin{array}{ccccc} a & b & c & d & Ax \\ . & a & b & c & A \\ . & . & p & q & B \\ . & p & q & r & Bx \\ p & q & r & s & Bx^2 \end{array} \right|, \\ \left| \begin{array}{ccccc} a & b & c & d & e \\ . & a & b & c & d \\ . & . & a & b & c \\ . & . & . & p & q \\ . & . & p & q & r \\ . & p & q & r & s \\ p & q & r & s & : \end{array} \right| \cdot \left| \begin{array}{c} Ax^2 \\ Ax \\ A \\ B \\ Bx \\ Bx^2 \\ Bx^3 \end{array} \right|, \end{array}$$

where, be it remarked, the ultimate forms, namely, those explicitly involving A and B, are Cayley's of 1848.

Cayley's relation between any three consecutive "simplified remainders" is next given (pp. 140–142), the proof arising quite naturally and being mainly dependent on the equality  $a_{r-1}a_r = D_{r-1}^2$ . Thus, taking the equations that indicate the nature of the division-process, namely,

$$\begin{aligned} A &= Q_1 B - R_1 \\ B &= Q_2 R_1 - R_2 \\ R_1 &= Q_3 R_2 - R_3 \\ R_2 &= Q_4 R_3 - R_4 \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned}$$

and substituting  $\rho_r/a_r$  for  $R_r$ , we obtain

$$\begin{aligned} a_1 \cdot A &= a_1 Q_1 \cdot B - \rho_1 \\ a_1 a_2 \cdot B &= a_2 Q_2 \cdot \rho_1 - a_1 \rho_2 \\ a_2 a_3 \cdot \rho_1 &= a_1 a_3 Q_3 \cdot \rho_2 - a_1 a_2 \cdot \rho_3 \\ a_3 a_4 \cdot \rho_2 &= a_2 a_4 Q_4 \cdot \rho_3 - a_2 a_3 \cdot \rho_4 \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \end{aligned}$$

In this way there results the general equation

$$a_{r-1} a_r \cdot \rho_{r-2} = a_{r-2} a_r Q_r \cdot \rho_{r-1} - a_{r-2} a_{r-1} \cdot \rho_r,$$

and thence

$$D_{r-1}^2 \cdot \rho_{r-2} = a_{r-2} a_r Q_r \cdot \rho_{r-1} - D_{r-2}^2 \cdot \rho_r,$$

showing that  $\rho_{r-2}$  and  $\rho_r$  have different signs for any value of  $x$  that makes  $\rho_{r-1}$  vanish.

Proceeding from the above-noted Cayleyan mode of expressing the "simplified remainders," Trudi puts forward (pp. 145-152) another mode, each remainder now appearing as *a sum of a multiple of A and a multiple of B*; or, in Sylvester's words,\* as *a syzygetic function of A and B*. For example, the three remainders above given he considers in the form

$$\begin{array}{|ccc|c|} \hline & a & b & 1 \\ & . & p & . \\ & p & q & . \\ \hline & A + & & \\ & . & p & 1 \\ & p & q & x \\ \hline \end{array} \quad \begin{array}{|ccccc|c|} \hline a & b & c & d & x & A + & a & b & c & d & . & B, \\ . & a & b & c & 1 & & . & a & b & c & . & \\ . & . & p & q & . & & . & . & p & q & 1 & \\ . & p & q & r & . & & . & p & q & r & x & \\ p & q & r & s & . & & p & q & r & s & x^2 & \\ \hline \end{array} \quad \dots ;$$

and generally he writes

$$\rho_r = U_r A + V_r B,$$

where it is readily seen that as regards  $x$

$$\left. \begin{array}{l} U_r \text{ is of the degree } r-1, \\ V_r \dots \dots m-n+r-1, \\ U_r A \dots \dots m+r-1 \\ V_r B \dots \dots m+r-1 \end{array} \right\}$$

and where, as we know,

$$\rho_r \dots \dots n-r.$$

Observation also shows that the coefficients of the highest powers of  $x$  in  $U_r, V_r$  are  $-b_0 D_{r-1}, a_0 D_{r-1}$  respectively. By substituting the new forms for  $\rho_{r-2}, \rho_{r-1}, \rho_r$  in Cayley's relation

$$D_{r-1}^2 \cdot \rho_{r-2} - a_{r-2} a_r Q_r \cdot \rho_{r-1} + D_{r-1}^2 \cdot \rho_r = 0,$$

there is obtained

$$\begin{aligned} (D_{r-1}^2 \cdot U_{r-2} - a_{r-2} a_r Q_r \cdot U_{r-1} + D_{r-2}^2 \cdot U_r) A \\ + (D_{r-1}^2 \cdot V_{r-2} - a_{r-2} a_r Q_r \cdot V_{r-1} + D_{r-2}^2 \cdot V_r) B = 0; \end{aligned}$$

and, as Trudi proves that this can only hold when the coefficients of A and B vanish, it follows that each of the two series of functions

$$U_0, U_1, \dots, U_{n+1}, \quad V_0, V_1, \dots, V_{n+1}$$

\* It is worth noting that it was in this connection that the word "syzygetic" was first used, the full title of the memoir of 1853 (which clearly had considerable influence on Trudi) being "On a theory of the *syzygetic* relations of two rational integral functions, comprising an application to the theory of Sturm's functions, and that of the greatest algebraical common measure."

has one of the Sturmian properties which the  $\rho$ 's have been shown to possess.

As regards the highest-common-divisor (pp. 142–144) his result is: *In order that two functions may have a common divisor of the  $k^{\text{th}}$  degree, it is necessary and sufficient that the first determinant of each of the last  $k$  of their bigradient arrays shall vanish: and, when this holds, the coefficients of the divisor in question are the successive determinants of the  $(n-k)^{\text{th}}$  array.* For example, the functions being

$$a_0x^8 + a_1x^7 + \dots + a_8, \quad b_0x^5 + b_1x^4 + \dots + b_5,$$

their bigradient arrays are

$$\left\| \begin{matrix} (a_0, \dots, a_8)_1 \\ (b_0, \dots, b_5) \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_8)_2 \\ (b_0, \dots, b_5) \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_8)_3 \\ (b_0, \dots, b_5) \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_8)_4 \\ (b_0, \dots, b_5) \end{matrix} \right\|, \left\| \begin{matrix} (a_0, \dots, a_8)_5 \\ (b_0, \dots, b_5) \end{matrix} \right\|,$$

and the proposition states that if the first determinant of each of the last three arrays vanishes, the functions have the common cubic factor

$$\left\| \begin{matrix} (a_0, \dots, a_8)_2 \\ (b_0, \dots, b_5) \end{matrix} \right\| (x^3, x^2, x, 1).$$

At a later stage (p. 151) there is given the supplementary proposition that *the quotients resulting from dividing A and B by the said highest-common-divisor are, save for an unimportant factor in each case, the coefficients of B and A in Trudi's form of the  $(n-k+1)^{\text{th}}$  "simplified remainder"—that is to say, are  $V_{n-k+1}$  and  $U_{n-k+1}$  as before defined.*

The closely related question concerning the common roots of two equations he deals with at length in a section devoted to elimination (pp. 161–178). Starting with the proposition that,  $u$  and  $v$  being integral functions of  $x$ ,  $uA + vB$  must vanish for any common root of the equations  $A = 0$ ,  $B = 0$ , he next points out that  $u$  and  $v$  may be so chosen as to make  $uA + vB$  of a low degree in  $x$ , even of the degree zero. In the latter extreme case  $uA + vB$  must contain the eliminant as a factor, and if in addition it be of the proper degree in the coefficients of  $A$  and  $B$  it is the eliminant pure and simple. Attention is then called to the fact that the division-process for finding the highest-common-divisor of  $A$  and  $B$ , or the Sturmian modification of this process, supplies a series of pairs of functions like  $u$  and  $v$ , and in particular that the last "simplified remainder"  $D_n$ , as satisfying all the requirements mentioned, is the eliminant. The condition for the existence of more than one common root is investigated in like manner. If the number of the roots in question be  $k$ , the degree-number of  $uA + vB$  cannot be less than  $k$ . Founding on this, it is asserted that

functions of the form  $uA + vB$  whose degree-number is less than  $k$  must vanish identically, and that therefore in particular the last  $k$  "simplified remainders" of  $A$  and  $B$  must so vanish. In the next place, proof is adduced that the vanishing of these remainders is equivalent to the vanishing of their first coefficients: and finally, there is reached the following variant to the above proposition regarding the highest-common-divisor: *In order that the equations  $A=0, B=0$  may have  $k$  common roots, it is necessary and sufficient that  $D_n, D_{n-1}, \dots, D_{n-k+1}$  vanish: and, this being the case, the equation of the said common roots is  $\rho_{n-k}=0$ .* The fact that the vanishing of the first coefficients of the "simplified remainders" implies in each case the vanishing of all the coefficients following the first is merely commented on in passing. Attention, however, is more fully drawn to the important fact that the existence of the condensation-theorem makes it possible to put every proposition, which, like the foregoing, involves bigradients, into an alternative form. Thus, the condition that the equations

$$\begin{aligned} x^3 + a_1x^2 + a_2x + a_3 &= 0 \\ x^2 + b_1x + b_2 &= 0 \end{aligned} \left\{ \right.$$

may have two roots in common is, according to the said proposition, the vanishing of

$$\begin{vmatrix} 1 & a_1 & a_2 & a_3 & \cdot \\ \cdot & 1 & a_1 & a_2 & a_3 \\ \cdot & \cdot & 1 & b_1 & b_2 \\ \cdot & 1 & b_1 & b_2 & \cdot \\ 1 & b_1 & b_2 & \cdot & \cdot \end{vmatrix}, \quad \begin{vmatrix} 1 & a_1 & a_2 \\ \cdot & 1 & b_1 \\ 1 & b_1 & b_2 \end{vmatrix};$$

and this by the condensation-theorem is the same as the vanishing of

$$\begin{vmatrix} 1 & b_1 & b_2 \\ b_1 & b_2 + a_1b_1 - a_2 & a_1b_2 - a_3 \\ b_2 & a_1b_2 - a_3 & a_2b_2 - a_3b_1 \end{vmatrix}, \quad \begin{vmatrix} 1 & b_1 \\ b_1 & b_2 + a_1b_1 - a_2 \end{vmatrix}.$$

Bezout's "abridged method" and Sylvester's "dialytic" method, which resemble each other in involving elimination of successive powers of a common root, are only introduced by Trudi for purposes of corroboration. In connection with the former method there is noted Sylvester's theorem \* that the derived equations provide also an alternative way of obtaining the Sturmian "simplified remainders," the first remainder being the non-zero member of the first equation, the second remainder being the result of eliminating the highest power of  $x$  from the first two equations, the third remainder the result of eliminating the two highest powers of  $x$  from the

\* See Art. 5 of "On a theory of the syzygetic relations . . ."

first three equations, and so on. In other words, if the set of equations derived from  $A=0, B=0$  by Bezout's method be

$$\left. \begin{array}{l} c_{11}x^{m-1} + c_{12}x^{m-2} + \dots + c_{1m} = 0 \\ c_{21}x^{m-1} + c_{22}x^{m-2} + \dots + c_{2m} = 0 \\ \dots \dots \dots \dots \end{array} \right\},$$

then the second, third, . . . “simplified remainders” of A and B are

$$\left| \begin{array}{cccccc} c_{11} & c_{12}x^{m-2} & c_{13}x^{m-3} & \dots & c_{1m} \\ c_{21} & c_{22}x^{m-2} & c_{23}x^{m-3} & \dots & c_{2m} \end{array} \right|,$$

$$\left| \begin{array}{cccccc} c_{11} & c_{12} & c_{13}x^{m-3} & \dots & c_{1m} \\ c_{21} & c_{22} & c_{23}x^{m-3} & \dots & c_{2m} \\ c_{31} & c_{32} & c_{33}x^{m-3} & \dots & c_{3m} \end{array} \right|,$$

$$\dots \dots \dots \dots \dots \dots \dots$$

Proceeding from this, Trudi then says that if the non-zero members of the said derived equations be denoted by  $Y_1, Y_2 \dots$ , the “simplified remainders” can clearly be put in the form

$$Y_1, \quad \left| \begin{array}{ccc} c_{11} & Y_1 \\ c_{21} & Y_2 \end{array} \right|, \quad \left| \begin{array}{ccc} c_{11} & c_{12} & Y_1 \\ c_{21} & c_{22} & Y_2 \\ c_{31} & c_{32} & Y_3 \end{array} \right|, \quad \dots \dots \dots;$$

and, as by definition

$$Y_1 = a_0B - b_0A,$$

$$Y_2 = (a_0x + a_1)B - (b_0x + b_1)A,$$

$$Y_3 = (a_0x^2 + a_1x + a_2)B - (b_0x^2 + b_1x + b_2)A,$$

$$\dots \dots \dots \dots \dots \dots \dots$$

it follows that the said remainders have still another form, namely,

$$a_0B - b_0A,$$

$$\left| \begin{array}{cc} c_{11} & a_0 \\ c_{21} & a_0x + a_1 \end{array} \right| B - \left| \begin{array}{cc} c_{11} & b_0 \\ c_{21} & b_0x + b_1 \end{array} \right| A,$$

$$\left| \begin{array}{ccc} c_{11} & c_{12} & a_0 \\ c_{21} & c_{22} & a_0x + a_1 \\ c_{31} & c_{32} & a_0x^2 + a_1x + a_2 \end{array} \right| B - \left| \begin{array}{ccc} c_{11} & c_{12} & b_0 \\ c_{21} & c_{22} & b_0x + b_1 \\ c_{31} & c_{32} & b_0x^2 + b_1x + b_2 \end{array} \right| A,$$

$$\dots \dots \dots \dots \dots \dots \dots$$

—a result easily shown, by the use of the condensation-theorem, to be in agreement with a previous one in which the determinant coefficients of A and B are bigradients. He is also careful to note that although here, as usual,  $n$  is taken equal to  $m$ , no real restriction is thereby made, the case where  $m > n$  being viewable as a case in which the coefficients of  $x^{n+1}, x^{n+2}, \dots, x^m$  in B are equal to 0. For example, if the given equations be

$$\left. \begin{array}{l} ax^4 + bx^3 + cx^2 + dx + e = 0 \\ qx^2 + rx + s = 0 \end{array} \right\},$$

Bezout's derived equations (although not in Bezout's nor Trudi's notation) are

$$\left| \begin{array}{cc|c} a & bx^3 + cx^2 + dx + e & = 0 \\ . & qx^2 + rx + s & \\ \end{array} \right. \\ \left| \begin{array}{cc|c} ax + b & cx^2 + dx + e & = 0 \\ . & qx^2 + rx + s & \\ \end{array} \right. \\ \left| \begin{array}{cc|c} ax^2 + bx + c & dx + e & = 0 \\ q & rx + s & \\ \end{array} \right. \\ \left| \begin{array}{cc|c} ax^3 + bx^2 + cx + d & e & = 0 \\ qx + r & s & \\ \end{array} \right. \right\}$$

or, in their usual form,

$$\left. \begin{array}{cccc} aq \cdot x^2 + & ar \cdot x + & as & = 0 \\ aqx^3 + (ar + bq)x^2 + & (as + br)x + & bs & = 0 \\ arx^3 + (as + br)x^2 + (bs + cr - dq)x + (cs - eq) & = 0 \\ asx^3 + & bsx^2 + & (cs - eq)x + (ds - er) & = 0 \end{array} \right\}.$$

We then have for the simplified remainders of the 1<sup>st</sup> and 0<sup>th</sup> degrees the determinants

$$\left| \begin{array}{cccc} . & aq & ar \cdot x + as & \\ aq & ar + bq & (as + br) \cdot x + bs & \\ ar & as + br & (bs + cr - dq) \cdot x + (cs - eq) & \end{array} \right|, \\ \left| \begin{array}{cccc} . & aq & ar & as \\ aq & ar + bq & as + br & bs \\ ar & as + br & bs + cr - dq & cs - eq \\ as & bs & cs - eq & ds - er \end{array} \right|,$$

being only careful to note that both of these contain the irrelevant factor  $a^2$ . Trudi, however, does not point out that this factor would not have troubled us if we had noted at the outset that for the first two derived equations we might have substituted

$$\begin{aligned} qx^2 + rx + s &= 0 \\ qx^3 + rx^2 + sx &= 0, \end{aligned}$$

thus using Sylvester's method of derivation for the first  $m - n$  equations and Bezout's for the remaining  $n$ , as Rosenhain had shown in 1844.\*

The case where B is the derivate of A receives special attention (pp. 152–160), the object of course being to show that the quantities  $D_r, U_r, V_r$  are then expressible in terms of sums of like powers of the roots of the equation  $A = 0$ . The reason for the possibility of this transformation lies in the fact that the coefficients

$$b_0, b_1, b_2, \dots, b_{n-1}$$

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\* *Crelle's Journ.*, xxviii. p. 269.

are then equal to

$$a_0 s_0, \quad a_0 s_1 + a_1 s_0, \quad a_0 s_2 + a_1 s_1 + a_2 s_0, \quad \dots, \quad a_0 s_{n-1} + \dots + a_{n-1} s_0,$$

and that in addition we have

$$\begin{aligned} (a_0, a_1, \dots, a_n \langle s_n, s_{n-1}, \dots, s_0 \rangle) &= 0, \\ (a_0, a_1, \dots, a_n \langle s_{n+1}, s_n, \dots, s_1 \rangle) &= 0, \\ (a_0, a_1, \dots, a_n \langle s_{n+2}, s_{n+1}, \dots, s_2 \rangle) &= 0, \\ \dots &\dots \dots \dots \dots \dots \dots \dots \dots \end{aligned}$$

The results arrived at are

$$D_r = a_0^{2r+1} \left| \begin{array}{cccccc} s_0 & s_1 & s_2 & \dots & s_{r-1} & s_r \\ s_1 & s_2 & s_3 & \dots & s_r & s_{r+1} \\ s_2 & s_3 & s_4 & \dots & s_{r+1} & s_{r+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ & & & & & r+1 \end{array} \right|$$

$$= 0 \text{ when } r > n - 1$$

$$V_r = a_0^{2r} \left| \begin{array}{cccccc} s_0 & s_1 & s_2 & \dots & s_{r-1} & 1 \\ s_1 & s_2 & s_3 & \dots & s_r & x \\ s_2 & s_3 & s_4 & \dots & s_{r+1} & x^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ & & & & & r+1 \end{array} \right|$$

$$U_r = -a_0^{2r} \left| \begin{array}{cccccc} s_0 & s_1 & s_2 & \dots & s_{r-1} & . \\ s_1 & s_2 & s_3 & \dots & s_r & s_0 \\ s_2 & s_3 & s_4 & \dots & s_{r+1} & s_0x + s_1 \\ s_3 & s_4 & s_5 & \dots & s_{r+2} & s_0x^2 + s_1x + s_2 \end{array} \right|_{r+1}$$

Here again, however, Trudi loses his opportunity from not being acquainted with Cayley's multiplication-theorem of 1845, the use of which enables us to transform not only  $D_r$ , but the whole bigradient array of which  $D_r$  is the first determinant. In fact, it gives us for the case under consideration another condensation-theorem. For example, when

$$\Lambda = a_0 x^5 + a_1 x^4 + \dots + a_5$$

and we consequently have to consider the four "simplified remainders"

$$\left\| \begin{array}{c} (a_0, \dots, a_5)_1 \\ (b_0, \dots, b_4)_2 \end{array} \right\| (x^3, x^2, x, 1), \quad \left\| \begin{array}{c} (a_0, \dots, a_5)_2 \\ (b_0, \dots, b_4)_3 \end{array} \right\| (x^2, x, 1), \quad \left\| \begin{array}{c} (a_0, \dots, a_5)_3 \\ (b_0, \dots, b_4)_4 \end{array} \right\| (x, 1), \quad \left\| \begin{array}{c} (a_0, \dots, a_5)_4 \\ (b_0, \dots, b_4)_5 \end{array} \right\|$$

we find the condensation-results

$$\left\| \begin{array}{c} (a_0, \dots, a_5)_1 \\ (b_0, \dots, b_4)_2 \end{array} \right\| = a_0^2 \left\| \begin{array}{cccccc} s_0 & a_0 s_1 & a_0 s_2 + a_1 s_1 & a_0 s_3 + \dots & a_0 s_4 + \dots \\ s_1 & a_0 s_2 & a_0 s_3 + a_1 s_2 & a_0 s_4 + \dots & a_0 s_5 + \dots \end{array} \right\|,$$

$$\left\| \begin{array}{c} (a_0, \dots, a_5)_2 \\ (b_0, \dots, b_4)_3 \end{array} \right\| = a_0^4 \left\| \begin{array}{cccccc} s_0 & s_1 & a_0 s_2 & a_0 s_3 + a_1 s_2 & a_0 s_4 + a_1 s_3 + a_2 s_2 \\ s_1 & s_2 & a_0 s_3 & a_0 s_4 + a_1 s_3 & a_0 s_5 + a_1 s_4 + a_2 s_3 \\ s_2 & s_3 & a_0 s_4 & a_0 s_5 + a_1 s_4 & a_0 s_6 + a_1 s_5 + a_2 s_4 \end{array} \right\|,$$

$$\begin{vmatrix} (a_0, \dots, a_5)_3 \\ (b_0, \dots, b_4)_4 \end{vmatrix} = a_0^6 \begin{vmatrix} s_0 & s_1 & s_2 & a_0s_3 & a_0s_4 + a_1s_3 \\ s_1 & s_2 & s_3 & a_0s_4 & a_0s_5 + a_1s_4 \\ s_2 & s_3 & s_4 & a_0s_5 & a_0s_6 + a_1s_5 \\ s_3 & s_4 & s_5 & a_0s_6 & a_0s_7 + a_1s_6 \end{vmatrix},$$

$$\begin{vmatrix} (a_0, \dots, a_5)_4 \\ (b_0, \dots, b_4)_5 \end{vmatrix} = a_0^9 \begin{vmatrix} s_0 & s_1 & s_2 & s_3 & s_4 \\ s_1 & s_2 & s_3 & s_4 & s_5 \\ s_2 & s_3 & s_4 & s_5 & s_6 \\ s_3 & s_4 & s_5 & s_6 & s_7 \\ s_4 & s_5 & s_6 & s_7 & s_8 \end{vmatrix},$$

all in agreement with Cayley's original result of 1846 (*Hist.*, ii. pp. 162-164). Taking the second of these for proof, we multiply unity columnwise by the given bigradient array, obtaining

$$\begin{aligned} & \begin{vmatrix} 1 & . & . & . & . & -s_0 \\ . & 1 & . & -s_0 & -s_1 & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & \\ . & . & . & . & 1 & \end{vmatrix} \cdot \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & . \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & \\ . & b_0 & b_1 & b_2 & b_3 & b_4 & \\ b_0 & b_1 & b_2 & b_3 & b_4 & . \\ b_0 & b_1 & b_2 & b_3 & b_4 & . \end{vmatrix} \\ = & \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & . \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & \\ b_0 & b_1 & b_2 & b_3 & b_4 & b_4 & \\ a_0s_1 & a_0s_2 + a_1s_1 & a_0s_3 + a_1s_2 + a_2s_1 & a_0s_4 + \dots & a_0s_5 + \dots & a_0s_6 + \dots \\ a_0s_2 & a_0s_3 + a_1s_2 & a_0s_4 + a_1s_3 + a_2s_2 & a_0s_5 + \dots & a_0s_6 + \dots \end{vmatrix} \\ = & a_0^2 \begin{vmatrix} a_0s_0 & a_0s_1 + a_1s_0 & a_0s_2 + a_1s_1 + a_2s_0 & a_0s_3 + \dots & a_0s_4 + \dots \\ a_0s_1 & a_0s_2 + a_1s_1 & a_0s_3 + a_1s_2 + a_2s_1 & a_0s_4 + \dots & a_0s_5 + \dots \\ a_0s_2 & a_0s_3 + a_1s_2 & a_0s_4 + a_1s_3 + a_2s_2 & a_0s_5 + \dots & a_0s_6 + \dots \end{vmatrix} \end{aligned}$$

and thence the final form desired.

The question of the existence of multiple or repeated roots in an equation is next taken up (pp. 178-196), the main result being: *The equation A=0 will admit of only k distinct roots of the first determinant of each of the last n-k bigradient arrays arising from A, and its derivative vanishes: and this condition being fulfilled, the equation of the said roots will be got by equating to 0 the determinant formed by replacing the last column of D\_k by k zeros and the 0<sup>th</sup>, 1<sup>st</sup>, 2<sup>nd</sup>, ..., k<sup>th</sup> powers of x.* For example, A and its derivate being

$$\begin{aligned} & x^5 + x^4 - 5x^3 - x^2 + 8x + 4, \\ & 5x^4 + 4x^3 - 15x^2 - 2x + 8, \end{aligned}$$

and it having been found that the first determinants of the arrays

$$\left\| \begin{array}{c} (1, \dots, 4)_1 \\ (5, \dots, 8)_2 \end{array} \right\|, \quad \left\| \begin{array}{c} (1, \dots, 4)_2 \\ (5, \dots, 8)_3 \end{array} \right\|, \quad \left\| \begin{array}{c} (1, \dots, 4)_3 \\ (5, \dots, 8)_4 \end{array} \right\|, \quad \left\| \begin{array}{c} (1, \dots, 4)_4 \\ (5, \dots, 8)_5 \end{array} \right\|$$

have the values

$$54, \quad 0, \quad 0,$$

the proposition states that the number of distinct roots of the equation  $A=0$  is 2, i.e. 5–3, and that the equation of these two roots is

$$\left| \begin{array}{ccccc} 1 & 1 & -5 & -5 & . \\ . & 1 & 1 & -5 & . \\ . & . & 5 & 4 & 1 \\ . & 5 & 4 & -15 & x \\ 5 & 4 & -15 & -2 & x^2 \end{array} \right| = 0.$$

As a matter of fact, this equation is  $54(x+2)(x-1)=0$ , and  $A\equiv(x+2)^2(x-1)^3$ .

Lastly, Trudi takes up Sturm's theorem for determining the number of roots of an equation which lie between two given values. In dealing with it he brings forward a new series of functions as a substitute for Sturm's series, namely,

$$1, \quad \left| \begin{array}{ccccc} a_0 & a_1 & . & . & , \\ . & b_0 & 1 & . & , \\ b_0 & b_1 & x & . & , \\ & & & b_0 & 1 \\ & & & . & b_1 \\ & & & b_0 & b_2 \\ & & & b_1 & x \\ b_0 & b_1 & b_2 & b_3 & x^2 \end{array} \right|, \dots$$

Further, he points out that the individual members of this series can be lowered in grade by the use of his condensation-theorem, thus providing a variant of the series. He also notes that by means of the theorem which we have extended above into another condensation-theorem they can be transformed into

$$1, \quad \left| \begin{array}{ccccc} a_0^2 & s_0 & 1 & . & , \\ s_1 & x & . & , & , \\ s_1 & s_2 & x & . & , \\ s_2 & s_3 & x^2 & . & , \end{array} \right|, \dots$$

and so he arrives by a different route at Joachimsthal's series of 1854 (*Hist.*, ii. p. 171).

Trudi's work on bigradients, extending to 94 pages if both *Teoria* and *Applicazioni* be included, has suffered undeserved neglect. Why this should have been the case it is a little difficult to understand, its only demerits being an occasional wordiness, a not very acceptable notation, and a paucity of concrete examples. In his preface (p. vii) he tells us

that it was first communicated in a number of papers to the Naples Academy of Sciences in the year 1857. This being so, it was two years in advance of Zeipel's memoir on the same subject (*Hist.*, ii. pp. 370–372) and Bruno's text-book, a fact which it is important for the reader to recall if any small point of similarity between two modes of treatment should attract attention.

SALMON, G. (1866).

[LESSONS INTRODUCTORY TO THE MODERN HIGHER ALGEBRA. 2nd ed.  
viii + 296. Dublin.]

In a table of resultants (pp. 283–285) the final expansion of  $R_{25}$  is given, and the discriminant of  $(a, b, c, d, e \propto x^4, x^3y, \dots, y^4)$ .

SARDI, C. (1866): RAJOLA, L. (1866): TORELLI, G. (1866).

[Questione 47. *Giornale di Mat.*, iv. pp. 239–240: solution by L. Rajola,  
iv. p. 297.]

[Teorema sui determinanti a due scale, e soluzione della questione 47.  
*Giornale di Mat.*, iv. pp. 294–296.]

We have already seen how, from equating two forms of the resultant of a pair of rational integral equations, interesting identities may be obtained (*Hist.*, i. p. 487 at bottom: ii. pp. 369–370, 374–375). Another instance is here reached, the forms of eliminant used being Sylvester's bigradient and the eliminant which arises from successively substituting the roots of one of the equations in the non-zero member of the other equation and taking the product of the resulting expressions. If in connection with the latter we make use of Spottiswoode's determinant expression (*Hist.*, ii. p. 111) for such a non-zero number, the identity evolved will be purely and almost alarmingly determinantal.

BALTZER, R. (1864, 1870, 1875).

[THEORIE UND ANWENDUNG DER DETERMINANTEN, . . . 2<sup>te</sup> Aufl. 3<sup>te</sup> Aufl.  
4<sup>te</sup> Aufl. Leipzig.]

Putting (§ 11, 4)

$$\begin{aligned} A(x) &\equiv a_0x^m + a_1x^{m-1} + \dots + a_m \equiv a_0(x - a_1)(x - a_2) \dots (x - a_m) \\ B(x) &\equiv b_0x^n + b_1x^{n-1} + \dots + b_n \equiv b_0(x - \beta_1)(x - \beta_2) \dots (x - \beta_n) \end{aligned} \quad \left\{ \begin{matrix} m = n \\ m > n \end{matrix} \right.$$

and supposing  $x$  to be one of the roots of the equation  $B(x)=0$ , Baltzer predicates the  $n$  equations

$$\begin{aligned} 0 &= \{a_m - A(x)\} + a_{m-1}x + a_{m-2}x^2 + \dots \\ 0 &= \{a_m - A(x)\}x + a_{m-1}x^2 + \dots \\ 0 &= \{a_m - A(x)\}x^2 + \dots \\ &\dots \dots \dots \dots \dots \end{aligned}$$

and the  $m$  equations

$$\begin{aligned} 0 &= b_n + b_{n-1}x + b_{n-2}x^2 + \dots \\ 0 &= b_nx + b_{n-1}x^2 + \dots \\ 0 &= b_nx^2 + \dots \\ &\dots \dots \dots \end{aligned}$$

and so deduces

$$\left| \begin{array}{cccccc} a_m - A(x) & a_{m-1} & a_{m-2} & \dots \\ \cdot & a_m - A(x) & a_{m-1} & \dots \\ \cdot & \cdot & a_m - A(x) & \dots \\ \cdot & \cdot & \cdot & \ddots \\ b_n & b_{n-1} & b_{n-1} & \dots \\ \cdot & b_n & b_{n-1} & \dots \\ \cdot & \cdot & b_n & \dots \\ \cdot & \cdot & \cdot & \ddots \end{array} \right|_{n+m} = 0,$$

which must thus be the equation in  $A(x)$  whose roots are  $A(\beta_1), A(\beta_2), \dots, A(\beta_n)$ . Since the coefficient of the highest power of  $A(x)$  in it is  $(-1)^n b_0^m$ , it follows that

$$(-1)^n b_0^m \cdot A(\beta_1)A(\beta_2) \dots A(\beta_n) = \left| \begin{array}{cccccc} a_m & a_{m-1} & a_{m-2} & \dots \\ \cdot & a_m & a_{m-1} & \dots \\ \cdot & \cdot & a & \dots \\ \cdot & \cdot & \cdot & \ddots \\ b_n & b_{n-1} & b_{n-2} & \dots \\ \cdot & b_n & b_{n-1} & \dots \\ \cdot & \cdot & b_n & \dots \\ \cdot & \cdot & \cdot & \ddots \end{array} \right|_{n+m},$$

as Hesse in 1858 had shown by direct transformation.

The bigradient form of resultant is also used (§ 11, 7) to show that when  $A$  and  $B$  are of the same degree

$$\text{resultant}(A, B + \lambda A) = \text{resultant}(A, B).$$

A fresh proof is given of Jacobi's theorem \* that if  $\phi$  be a given function of the  $(m+n-1)^{\text{th}}$  degree in  $x$ , it is possible to determine two functions  $u, v$  of the  $(n-1)^{\text{th}}, (m-1)^{\text{th}}$  degrees so as to have

$$uA + vB = S\phi,$$

---

\* Crelle's Journ., xv. (1835) p. 108, where however  $m=n$ .

52 Proceedings of the Royal Society of Edinburgh. [Sess.  
where S is Sylvester's bigradient. This consists simply in taking the  
 $1+n+m$  equations

$$\left. \begin{array}{l} \phi = c_{m+n-1} + c_{m+n-2}x + c_{m+n-3}x^2 + \dots \\ A = a_m + a_{m-1}x + a_{m-2}x^2 + \dots \\ xA = a_mx + a_{m-1}x^2 + \dots \\ x^2A = a_mx^2 + \dots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ B = b_n + b_{n-1}x + b_{n-2}x^2 + \dots \\ xB = b_nx + b_{n-1}x^2 + \dots \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{array} \right\}$$

and deducing

$$\left| \begin{array}{cccccc} \phi & c_{m+n-1} & c_{m+n-2} & c_{m+n-3} & \dots \\ A & a_m & a_{m-1} & a_{m-2} & \dots \\ xA & . & a_m & a_{m-1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B & b_n & b_{n-1} & b_{n-2} & \dots \\ xB & . & b_n & b_{n-1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \right|_{+n+m} = 0.$$

Jacobi's theorem of 1835 regarding Bezout's condensed eliminant suggests the similar theorem regarding the bigradient eliminant,\* namely, if w be a common root of the equations

$$a_0x^m + a_1x^{m-1} + \dots = 0, \quad b_0x^n + b_1x^{n-1} + \dots = 0,$$

then the signed primary minors associated with any row of

$$\left| \begin{array}{c} (a_0, \dots, a_m)_n \\ (b_0, \dots, b_n)_m \end{array} \right|$$

are proportional to

$$w^{m+n-1}, w^{m+n-2}, \dots, w, 1.$$

In dealing with the highest-common-factor of A and B and with the subject of elimination Baltzer profits far less than he ought to have done from the work of Trudi, whom indeed he does not mention.

ISÈ, E. (1873); JANNI, V. (1874).

[Sul grado della risultante. *Giornale di Mat.*, xi. p. 253.]

[Sul grado dell' eliminante del sistema di due equazioni. *Giornale di Mat.*, xii. p. 27.]

\* Gordan (1870) in quoting the two from Baltzer says that  $mn$  of the primary minors of the former eliminant are secondary minors of the latter. (*Math. Annalen*, iii. p. 356.)

The bigradient form of eliminant is here used in the establishing of the proposition that if the coefficients  $a_r, b_r$ , be functions of the  $r^{\text{th}}$  degree in one and the same variable  $y$ , the eliminant is of the  $(mn)^{\text{th}}$  degree in the same variable. Janni's proof, though not quite so good as it might have been, is the more interesting. The eliminant being

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & \cdot & \cdot \\ \cdot & b_0 & b_1 & b_2 & \cdot \\ \cdot & \cdot & b_0 & b_1 & b_2 \end{vmatrix},$$

he, in effect, multiplies the columns in reverse order by  $y^0, y^1, y^2, y^3, y^4$  respectively, and then divides the rows in order by  $y^4, y^3, y^2, y^1, y^0$  respectively, thus obtaining

$$\begin{vmatrix} a_0 & a_1y^{-1} & a_2y^{-2} & a_3y^{-3} & \cdot \\ \cdot & a_0 & a_1y^{-1} & a_2y^{-2} & a_3y^{-3} \\ b_0y^2 & b_1y & b_2 & \cdot & \cdot \\ \cdot & b_0y^2 & b_1y & b_2 & \cdot \\ \cdot & \cdot & b_0y^2 & b_1y & b_2 \end{vmatrix}.$$

In this equivalent form the elements of the first two rows are all now of the degree 0 in  $y$ , and those of the last three rows are all of the degree 2, whence comes at once the desired result.

It should be noted that the procedure shows each term to be of the  $(mn)^{\text{th}}$  degree in  $y$ ; in other words, that the eliminant is homogeneous. Also, dispensing in the end with  $y$ , we may deduce the isobarism of the eliminant, its weight being  $mn$ .

ZEUTHEN, H. G. (1874): MADSEN, V. H. O. (1875).

[En Bemærkning om Beviserne for Hovedscetningen om Elimination mellem to algebraiske Ligninger. *Tidsskrift for Math.* (3), iv. pp. 165–171.]

[En Bemærkning om Sylvesters dialytiske Eliminationsmethode. *Tidsskrift for Math.* (3), v. pp. 144–145.]

Zeuthen repeats Salmon's mode of 1859 (*Hist.*, ii. pp. 373–374) of using Euler's treatment of two integral equations in  $x$  which have more than one common root: he is, however, more detailed, and takes the number of roots to be  $p$ .

LEMONNIER, H. (1875, 1878).

[Théorèmes concernant les équations qui ont des racines communes.

*Comptes-Rendus . . . Acad. des Sci. (Paris)*, lxxx. pp. 111–112, 252–255.]

[Mémoire sur l'élimination. *Annales de l'École Norm. Sup.* (2), vii. pp. 77–96, 151–214.]

Lemonnier's condition for the equations

$$a_0x^m + \dots + a_m = 0, \quad b_0x^n + \dots + b_n = 0$$

having  $k$  common roots is different from Trudi's, but fortunately for comparison is very easily expressed in Trudi's notation. It is\* that the first  $k$  determinants of

$$\begin{vmatrix} (a_0, \dots, a_m)_{n-k+1} \\ (b_0, \dots, b_n)_{m-k+1} \end{vmatrix}$$

shall vanish, and the first determinant of

$$\begin{vmatrix} (a_0, \dots, a_m)_{n-k} \\ (b_0, \dots, b_n)_{m-k} \end{vmatrix}$$

shall not vanish. The former part of the condition recalls Zeipel's of 1859 : the latter is an important necessary adjunct. When, however, the equation of the common roots

$$\begin{vmatrix} (a_0, \dots, a_m)_{n-k} & |(x^k, x^{k-1}, \dots, x^0) = 0 \\ (b_0, \dots, b_n)_{m-k} & \end{vmatrix}$$

happens to be given along with the condition, it is less necessary to mention the latter part, as the determinant involved is the coefficient of  $x^k$  in the said equation.

MUIR, T. (1876).

[New general formulae for the transformation of infinite series into continued fractions. *Trans. Roy. Soc. Edin.*, xxvii. pp. 467–471.]

[On the transformation of Gauss' hypergeometric series into a continued fraction. *Proc. London Math. Soc.*, vii. 112–118.]

The fundamental theorem, which is established in two different ways, is not essentially different from Heilermann's of 1845 (*Hist.*, ii. p. 361). The second of the two ways is the more interesting. Beginning with the series

$$\begin{aligned} a_0 + a_1x + a_2x^2 + a_3x^3 + \dots, & \quad \text{or } f_0, \\ b_0 + b_1x + b_2x^2 + b_3x^3 + \dots, & \quad \text{or } f_1, \end{aligned}$$

---

\* This is in accordance with the statement in § 13 of the complete memoir, and is somewhat different from that first published.

and subtracting  $b_0$  times the first from  $a_0$  times the second, and dividing the result by  $x$ , we obtain

$$\left| \begin{array}{cc} a_0 & a_1 \\ b_0 & b_1 \end{array} \right| + \left| \begin{array}{cc} a_0 & a_2 \\ b_0 & b_2 \end{array} \right| x + \left| \begin{array}{cc} a_0 & a_3 \\ b_0 & b_3 \end{array} \right| x^2 + \dots, \text{ or } f_2 \text{ say};$$

and by subtracting  $|a_0 b_1|$  times the second from  $b_0$  times this third series and dividing by  $x$  there results

$$\left| \begin{array}{ccc} a_0 & a_1 & a_2 \\ . & b_0 & b_1 \\ b_0 & b_1 & b_2 \end{array} \right| + \left| \begin{array}{ccc} a_0 & a_1 & a_3 \\ . & b_1 & b_2 \\ b_0 & b_1 & b_3 \end{array} \right| x + \left| \begin{array}{ccc} a_0 & a_1 & a_4 \\ . & b_1 & b_3 \\ b_0 & b_1 & b_4 \end{array} \right| x^2 + \dots, \text{ or } f_3 \text{ say};$$

and so on. The outcome is

$$\frac{a_0 + a_1 x + a_2 x^2 + \dots}{b_0 + b_1 x + b_2 x^2 + \dots} = \frac{\theta_1}{\theta_0} - \frac{\theta_2 x}{\theta_1} - \frac{\theta_3 x}{\theta_2} - \frac{\theta_1 \theta_4 x}{\theta_3} - \dots.$$

where  $\theta_0, \theta_1, \theta_2, \dots$  are the first terms of  $f_0, f_1, f_2, \dots$  respectively.

#### VENTÉJOLS, . (1877).

[Sur un problème comprenant la théorie de l'élimination. *Comptes-Rendus . . . Acad. des Sci. (Paris)*, lxxxiv. pp. 546–549.]

Ventéjols' subject would have been much better described by Lemonnier's title of 1875. In substance nothing fresh is brought forward.

#### DICKSON, J. D. H. (1877).

[A class of determinants. *Trans. Roy. Soc. Edin.*, xxviii. pp. 625–631.]

[The numerical calculation of a class of determinants, and a continued fraction. *Proc. London Math. Soc.*, x. pp. 226–228.]

The determinants here considered are the bigradients dealt with by Heilermann (1845) and Muir (1876). They also arise in the same connection.

#### MANSION, P. (1878).

[Sur l'élimination. *Bulletin . . . Acad. . . . de Belgique*, xlvi. pp. 899–903.]

What is interesting here is Mansion's mode of obtaining the evanescent bigradient array that results from the existence of common roots. The equations being

$$A(x) \equiv a_0 x^5 + \dots + a_5 = 0, \quad B(x) \equiv b_0 x^4 + \dots + b_4 = 0$$

and the common roots  $\alpha, \beta, \gamma$ , it follows that

$$\begin{vmatrix} \alpha^m & \alpha^n & A(\alpha) \\ \beta^m & \beta^n & A(\beta) \\ \gamma^m & \gamma^n & A(\gamma) \end{vmatrix}, \begin{vmatrix} \alpha^m & \alpha^n & \alpha A(\alpha) \\ \beta^m & \beta^n & \beta A(\beta) \\ \gamma^m & \gamma^n & \gamma A(\gamma) \end{vmatrix}, \begin{vmatrix} \alpha^m & \alpha^n & B(\alpha) \\ \beta^m & \beta^n & B(\beta) \\ \gamma^m & \gamma^n & B(\gamma) \end{vmatrix}, \begin{vmatrix} \alpha^m & \alpha^n & \alpha B(\alpha) \\ \beta^m & \beta^n & \beta B(\beta) \\ \gamma^m & \gamma^n & \gamma B(\gamma) \end{vmatrix}, \begin{vmatrix} \alpha^m & \alpha^n & \alpha^2 B(\alpha) \\ \beta^m & \beta^n & \beta^2 B(\beta) \\ \gamma^m & \gamma^n & \gamma^2 B(\gamma) \end{vmatrix}$$

are all equal to 0; so that, if we temporarily write

$$(m, n, p) \text{ for the alternant } |\alpha^m \beta^n \gamma^p|,$$

we have

$$\begin{aligned} (mn5)a_0 + (mn4)a_1 + (mn3)a_2 + (mn2)a_3 + (mn1)a_4 + (mn0)a_5 &= 0 \\ (mn6)a_0 + (mn5)a_1 + (mn4)a_2 + (mn3)a_3 + (mn2)a_4 + (mn1)a_5 &= 0 \\ (mn4)b_0 + (mn3)b_1 + (mn2)b_2 + (mn1)b_3 + (mn0)b_4 &= 0 \\ (mn5)b_0 + (mn4)b_1 + (mn3)b_2 + (mn2)b_3 + (mn1)b_4 &= 0 \\ (mn6)b_0 + (mn5)b_1 + (mn4)b_2 + (mn3)b_3 + (mn2)b_4 &= 0. \end{aligned}$$

Here, however, by taking any two of the numbers 0, 1, 2, 3, 4, 5, 6 as values for  $m, n$  two of the alternants will disappear, and we shall be able to eliminate the five others, the final and complete result thus being in Cayley's notation

$$\begin{vmatrix} . & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 \\ a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & . \\ . & . & b_0 & b_1 & b_2 & b_3 & b_4 \\ . & b_0 & b_1 & b_2 & b_3 & b_4 & . \\ b_0 & b_1 & b_2 & b_3 & b_4 & . & . \end{vmatrix} = 0.$$

For example, putting  $m, n$  equal to 0, 1, then equal to 0, 2, and finally equal to 1, 2, we should have the particular three results which in accordance with our usage under Trudi we might write

$$\begin{vmatrix} (a_0, \dots, a_5)_2 \\ (b_0, \dots, b_4)_3 \end{vmatrix} = 0.$$

All this, however, is considerably modified from Mansion's exposition.

GÜNTHER, S. (1879).

[Eine Relation zwischen Potenzen und Determinanten. *Zeitschrift f. Math. u. Phys.*, xxiv. pp. 244–248.]

The subject here is simply the evaluation of the bigradient which is the discriminant of  $(x^{m+2}-1)/(x-1)$ , the result being

$$(m+2)^m.$$

For example, when  $m$  is 2,

$$\begin{vmatrix} 1 & 1 & 1 & 1 & . \\ . & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & . & . \\ . & 1 & 2 & 3 & . \\ . & . & 1 & 2 & 3 \end{vmatrix} = 4^2.$$

Gunther's proof is unnecessarily lengthy. The determinant can be readily transformed into one whose diagonal has for its elements 1 repeated  $m+1$  times and  $m+2$  repeated  $m$  times, and whose other terms all vanish. For example, when  $m$  is 2, the requisite operations are

$$\begin{aligned} \text{row}_5 &= \text{row}_4 + \text{row}_2, \\ \text{row}_4 &= \text{row}_3 + \text{row}_1, \\ \text{row}_3 &= \text{row}_2 - \text{row}_1. \end{aligned}$$

MANSION, P. (1879).

[On the equality of Sylvester's and Cauchy's eliminants. *Messenger of Math.*, ix. pp. 60-63.]

Mansion's proof is not essentially different from the process of applying Trudi's condensation-theorem to Sylvester's bigradient. The additional fact, to which Mansion draws attention, namely, that many minors of the one eliminant have equivalents among the minors of the other, is also virtually included in Trudi. Thus, the four identities which Mansion indicates in the form (see his fig. 11)

$$\begin{array}{ccccccccc}
 a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & \cdot & \cdot \\
 \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & \cdot \\
 \cdot & \cdot & a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\
 \cdot & \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & \cdot \\
 \cdot & \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & \cdot \\
 \cdot & \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & \cdot & \cdot \\
 \cdot & \cdot & b_0 & b_1 & b_2 & b_3 & \cdot & \cdot & \cdot \\
 \cdot & b_0 & b_1 & b_2 & b_3 & \cdot & \cdot & \cdot & \cdot \\
 b_0 & b_1 & b_2 & b_3 & \cdot & \cdot & \cdot & \cdot & \cdot
 \end{array} = \begin{array}{cccccc}
 \cdot & \cdot & b_0 & b_1 & b_2 & b_3 \\
 \cdot & b_0 & b_1 & b_2 & b_3 & \cdot \\
 b_0 & b_1 & b_2 & b_3 & \cdot & \cdot \\
 \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\
 \mu_1 & \mu_2 & \mu_3 & \mu_4 & \mu_5 & \mu_6 \\
 \nu_1 & \nu_2 & \nu_3 & \nu_4 & \nu_5 & \nu_6
 \end{array}$$

where the  $\lambda$ 's,  $\mu$ 's,  $\nu$ 's stand for

$$\begin{array}{cccccc}
a_0 b_1 & a_0 b_2 + a_1 b_1 & a_0 b_3 + a_1 b_2 + a_2 b_1 & a_1 b_3 + a_2 b_2 + a_3 b_1 - a_4 b_0 & a_2 b_3 + a_3 b_2 - a_5 b_0 & a_3 b_3 - a_6 b_0 \\
a_0 b_2 & a_0 b_3 + a_1 b_2 & a_1 b_3 + a_2 b_2 & a_2 b_3 + a_3 b_2 + a_5 b_0 & a_3 b_3 - a_6 b_0 + a_4 b_2 - a_5 b_1 & a_4 b_3 - a_6 b_1 \\
a_0 b_3 & a_1 b_3 & a_2 b_3 & a_3 b_3 - a_6 b_0 & a_4 b_3 - a_5 b_1 & a_5 b_3 - a_6 b_2
\end{array}$$

are only four of the ten noted by Trudi, the others being excluded, so to speak, by drawing three vertical dotted lines on the right of each determin-

nant instead of continuing the horizontal dotted lines all the way towards the right.

From the foregoing there is probably no serious omission of papers dealing directly with bigradients and in particular with the bigradient eliminant. But, as it is possible to study the subject of the common roots of two integral equations without direct reference to bigradients, and as the other determinants that may then be used can generally be transformed into bigradients, it will doubtless be of service to the student of determinants to give the following list of titles of papers on elimination. When taken together with the preceding papers on the same subject they will also be helpful to the student of the theory of equations:—

- 1870. GORDAN, P. Ueber die Bildung der Resultante zweier Gleichungen. *Math. Annalen*, iii. pp. 355–414.
- 1872. NAEGELSBACH, H. Ueber die Resultante zweier ganzen Functionen. *Zeitschrift f. Math. u. Phys.*, xvii. pp. 333–346.
- 1876. DARBOUX, G. Sur la théorie de l'élimination entre deux équations à une variable. *Bull. des Sci. Math.*, x. pp. 56–64: (2) i. pp. 54–64.
- 1877. ROUCHÉ, E. Sur l'élimination. *Nouv. Annales de Math.* (2), xvi. pp. 105–113.
- 1877. IGEL, B. Einige Sätze und Beweise zur Theorie der Resultante. *Sitzungsb. . . . Akad. d. Wiss. (Wien)*, lxxvi. pp. 145–168.
- 1877. FORESTIER, C. Exposition succincte de quelques méthodes d'élimination entre deux équations. *Mém. de l'Acad. des Sci. (Toulouse)* (7), ix. pp. 142–163.
- 1879. BIEHLER, C. Sur la théorie des équations. *Dissert.* 60 pp. Paris.
- 1879. FALK, M. Sur la méthode de l'élimination de Bezout et Cauchy. *Nova Acta Reg. Soc. (Upsala)*, x. No. 15, 36 pp.
- 1879. HIOUX, V. Note sur la méthode d'élimination Bezout-Cauchy. *Nouv. Annales de Math.* (2) xviii. pp. 289–295.
- 1879. MANSION, P. Sur l'élimination. *Bull. . . . Acad. . . . Belgique* (2), xlvi. pp. 899–903: xlvii. pp. 532–541: xlviii. pp. 463–472, 473–490, 491–526.

The papers of Falk and Manson devote some little space to reviewing the work of their predecessors, and are therefore additionally helpful. They do not, however, mention Trudi, nor indeed does any one of the other writers of the period.

It may be noted as a significant fact in connection with the history of the subject that in 1876 the editors of the *Nouvelles Annales* found themselves called on to republish Cauchy's important paper of 1840 (see *Hist.*, i. pp. 240-243). Its reprint occupies pp. 385-416, 433-451 of vol. xv. of the second series. On this account, save for junior readers, the "exposition succincte" above noted was quite unnecessary.

#### LIST OF AUTHORS

whose writings are herein dealt with.

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1864. BALTZER, R. . . . .	50	1877. VENTÉJOLS . . . . .	55
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1875. BALTZER, R. . . . .	50	1878. MANSION, P. . . . .	55
1873. ISÈ, E. . . . .	52	1879. GÜNTHER, S. . . . .	56
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1874. ZEUTHEN, H. G. . . . .	53		

*(Issued separately February 19, 1914.)*

VI.—The Kinetic Energy of Viscous Flow through a Circular Tube. By Professor A. H. Gibson, D.Sc., University College, Dundee.

(MS. received October 11, 1913. Read December 15, 1913.)

IN the stream-line flow of a viscous fluid through a circular pipe of radius  $a$ , the velocity of flow at any radius  $x$  is given by

$$v = k(a^2 - x^2),$$

where

$$k = \frac{1}{4\mu} \cdot \frac{dp}{dl}.$$

From this it follows that the kinetic energy of the moving column per unit volume is given by

$$\begin{aligned} & \frac{w}{2g} \int_0^a 2\pi x v^3 dx \\ &= \frac{w}{2g} \cdot 2\pi k^3 \int_0^a (a^6 x - 3a^4 x^3 + 3a^2 x^5 - x^7) dx \\ &= \frac{w}{2g} \cdot \frac{1}{4}\pi k^3 a^8. \end{aligned} \quad (1)$$

The mean velocity  $\bar{v}$  of flow through such a tube is given by

$$\bar{v} = \frac{1}{2}ka^2,$$

so that the apparent kinetic energy, or the product of the mass flow and one-half the square of its mean velocity, is

$$\begin{aligned} & \frac{w}{2g} \cdot \pi a^2 [\frac{1}{2}ka^2]^3 \\ &= \frac{w}{2g} \cdot \frac{1}{8}\pi k^3 a^8. \end{aligned} \quad (2)$$

The true kinetic energy is therefore twice that calculated as in (2) from the mean velocity.

In the majority of experiments carried out to determine the coefficient of viscosity of a fluid, the head necessary to maintain a measured velocity of flow through a tube of known diameter and length

is measured, and the coefficient  $\mu$  is then determined from Poiseuille's equation,

$$h = \frac{32\mu l \bar{v}}{d^2} . . . . . \quad (3)$$

Where  $h$  is measured by the difference of pressures at piezometer orifices at two points in the wall of the tube, this equation is rigorously true. Where, however, as is more often the case, the upper end of the tube projects into a reservoir of the fluid, while its lower end discharges freely, the head being measured from the free surface in the reservoir to the centre of the discharging end of the tube, the true equation becomes

$$h = \frac{32\mu l \bar{v}}{d^2} + \text{K.E. of discharge} + \text{head loss at entrance to tube}. \quad . . . (4)$$

The two last terms become decreasingly important as the length of the tube increases, but, with a fairly short tube, account for a very appreciable portion of the whole head. In many cases in which the details of viscosity experiments have been published, the kinetic energy of discharge has been calculated erroneously from the mean velocity as in formula (2), while various allowances, varying from zero to  $\frac{\bar{v}^2}{2g}$  have been made for the head loss at entrance to the tube. In a thin-walled tube projecting into a reservoir, the loss at entrance, with an inviscid fluid, may readily be shown to be equal to  $\frac{\bar{v}^2}{2g}$ , while the effect of viscosity is to reduce this loss somewhat. As the walls of the tube become thicker the conditions approximate more nearly to that of a tube whose end opens flush with the sides of the reservoir, in which case, in large tubes conveying water, the loss is approximately  $\cdot 5 \frac{\bar{v}^2}{2g}$ . The true value of this loss for any actual projecting

tube may therefore be expected to be given by  $c \frac{\bar{v}^2}{2g}$ , where  $c$  increases with the relative thickness of the walls, and, for such a fluid as water, has a value somewhere between  $\cdot 5$  and  $1 \cdot 0$ .

The following experiments have been carried out with a view of checking the accuracy of the deductions leading to formula (1), and of determining the value of  $c$  for tubes of small bore. In each case the tube used projected for some distance into the upper reservoir and discharged freely at its lower end. The head from the free surface in the reservoir to the centre of the outlet was measured, and the discharge was collected and measured. The fluid was water,

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and the purely viscous loss was calculated from Poiseuille's value of  $\mu$ , viz.,

$$\mu = \frac{0.0000181}{1 + \cdot03368t + \cdot000221t^2}.$$

Three tubes were used, of the following dimensions:—

Tube.	Diameter in cm.		Length. Cm.	Ratio external internal diameter = $m$ .
	Internal.	External.		
A	.0715	.1073	4.29	1.50
B	.1470	.1960	6.66	1.33
C	.2580	.9000	38.20	3.50

For tubes A and B,  $t = 14.2^\circ$  C., making  $\mu = .0000119$ .

For tube C,  $t = 12.9^\circ$  C.,  $\mu = .0000123$ .

Since the pressure in the interior of the jet at the point of discharge is greater than atmospheric by an amount  $\frac{T}{r}$ , where T is the surface tension and r the radius of the jet, the effective head is less by this amount than the measured head.

Thus, for tube A,  $T = .073$  grms. per cm. ;  $\frac{T}{r} = 2.028$  cm.

,, B,  $T = .073$  ,;  $\frac{T}{r} = 1.00$  ,,

,, C,  $T = .075$  ,;  $\frac{T}{r} = 0.57$  ,,

This correction is applied in the following tables, which give the experimental results.

TUBE A.

Experiment.	Loss of head in cm.				$\bar{v}$ c. per sec.	$\frac{\bar{v}^2}{2g}$ .	k.
	Total.	In viscous resistance.	Due to T at outlet.	Residue $(= k \frac{\bar{v}^2}{2g})$ .			
1	16.50	12.400	2.028	2.072	39.00	.775	2.68
2	16.22	12.200	"	2.002	38.40	.750	2.68
3	8.48	5.980	"	0.475	18.82	.1805	2.63
4	8.35	5.870	"	0.452	18.48	.1740	2.60
					Mean =		2.65

TUBE B.

Experiment.	Loss of head in cm.				$\bar{v}$ c. per sec.	$\frac{\bar{v}^2}{2g}$	$k$ .
	Total.	In viscous resistance.	Due to T at outlet.	Residue $(=k\frac{\bar{v}^2}{2g})$ .			
1	16.02	8.155	1.000	6.865	69.60	2.470	2.78
2	16.64	8.390	"	7.250	71.30	2.595	2.80
3	12.56	6.905	"	4.655	58.75	1.760	2.65
4	11.81	6.550	"	4.260	55.70	1.581	2.69
5	7.50	4.501	"	1.999	38.23	0.745	2.68
6	7.46	4.490	"	1.970	38.20	0.744	2.65
					Mean =		2.71

TUBE C.

1	16.90	11.98	0.57	4.35	57.90	1.705	2.55
2	16.60	11.85	"	4.18	56.90	1.646	2.54
3	12.43	9.30	"	2.56	44.65	1.015	2.53
4	12.55	9.36	"	2.62	44.95	1.029	2.54
	Mean =						
							2.54

From these results it appears that the sum of the residual losses is between 2.5 and 3.0 times  $\frac{\bar{v}^2}{2g}$ . Of this, the kinetic energy of discharge accounts for  $2\frac{\bar{v}^2}{2g}$ . The remainder, incurred at entry to the tube, is equal to  $c\frac{\bar{v}^2}{2g}$ , where  $c$  has mean values as follow:—

Ratio outer diameter inner $(=m)$	1.33	1.50	3.50
Mean value of $c$	.71	.65	.54
$\frac{1}{2 - \frac{1}{m^2}}$	.70	.64	.52

The value of  $c$  appears to increase slightly with an increase in speed. Its mean values over the range of values of  $m$  occurring in the experiments are given with fair accuracy by the relationship

$$c = \frac{1}{2 - \frac{1}{m^2}},$$

and as this also satisfies the two extreme conditions, *i.e.* makes  $c=1$  when  $m=1$ , as in a thin-walled tube, and makes  $c=.5$  when  $m$  is very large, values calculated by this relationship are probably fairly accurate for all intermediate values of  $m$ .

VII.—The Axial Inclination of Curves of Thermoelectric Force: a Case from the Thermoelectrics of Strained Wires. By John M'Whan, M.A., Ph.D., Lecturer in Mathematics in the University of Glasgow. *Communicated by Professor ANDREW GRAY, LL.D., F.R.S.*

(MS. received October 17, 1913. Read February 16, 1914.)

IN a communication to the Royal Society of Edinburgh,\* Mr J. D. Hamilton Dickson has examined with great care the valuable results of Professors Dewar and Fleming on the thermoelectromotive forces of various couples, and has come to the conclusion that the curve representing the thermo-E.M.F. is in every case a parabola whose axis is, not vertical as had always been assumed, but inclined a definite though very small angle to the E.M.F.-axis.

This remarkable result has led me to go back to some experiments which I made a few years ago on the thermoelectric properties of longitudinally strained metal wires, to see if by any chance the same phenomenon might be detected there, and in one instance (only) I have been able to establish its existence unmistakably. The experiments in question, which I have described elsewhere,† were made on couples consisting each entirely of one and the same pure metal; but one wire of the couple might be subjected to any desired longitudinal tension while the other remained unstrained. The temperatures of the junctions were the same in all the experiments, one junction being steam-heated, the other water-cooled.

On reference to the curves showing the relation of the tension in the strained wire to the thermo-E.M.F. of the couple, only one was found to be of parabolic shape, namely, that for nickel. The E.M.F.'s in non-magnetic metals nearly all obey a straight-line law up to the point where overstrain sets in: the only other magnetic metal tested, bismuth, gave no simple relation between E.M.F. and strain. A closer examination of the nickel-curve showed that, while it afforded good grounds for suspecting an inclination of the axis, the observations had been neither numerous enough nor of the high order of accuracy necessary for certainty, and it was accordingly decided to repeat the experiment. This repetition proved extremely laborious, and only after some five or six attempts (each necessitat-

\* *Trans. R.S.E.*, xlvi. 737–791, 1910–11.

† *Diss.*, Göttingen, 1911.

ing the mounting of a fresh and freshly annealed wire) was the accompanying curve, the figures for which appear in Table I., obtained, in which only one of the thirty plotted points lies appreciably wide.

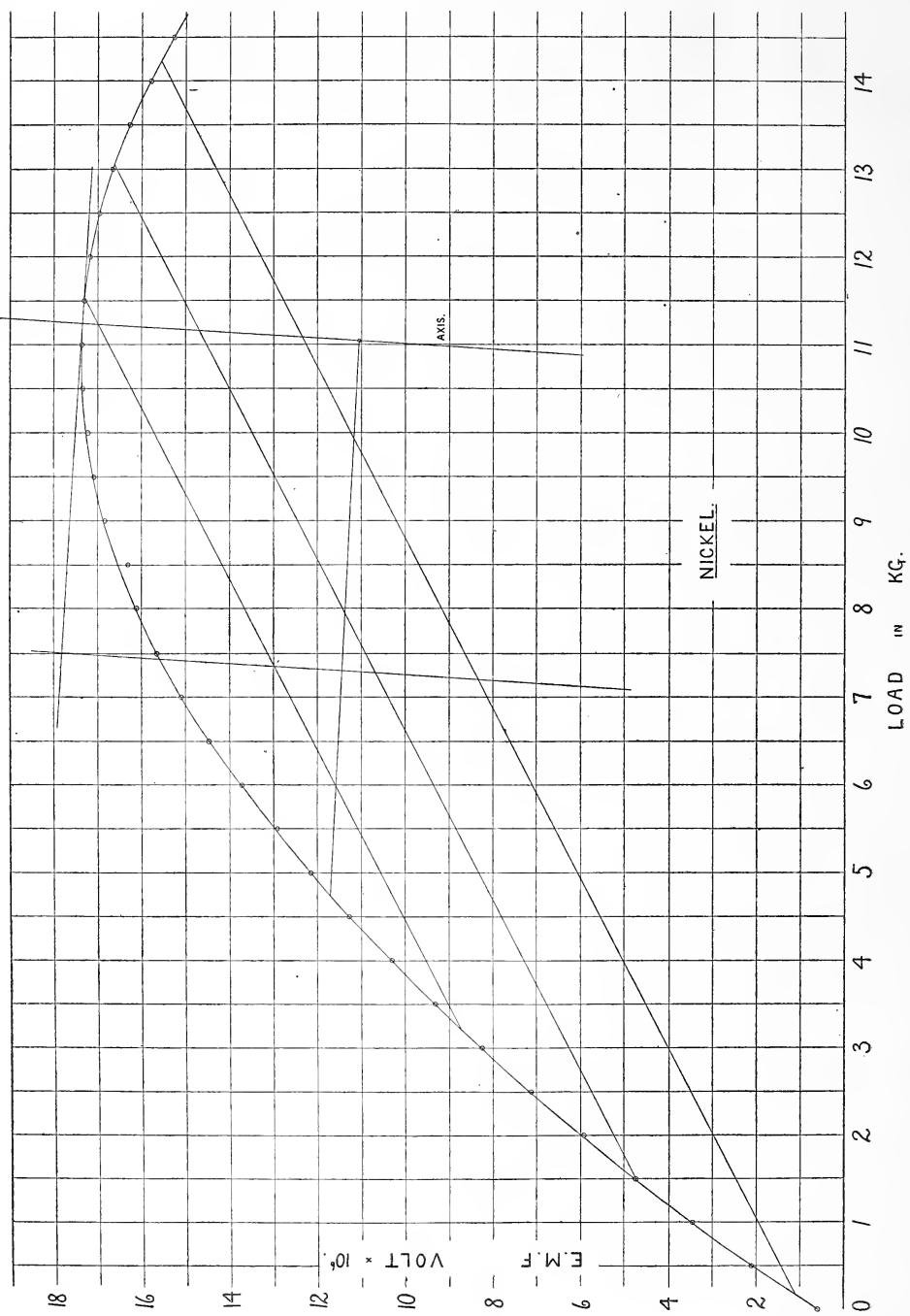
TABLE I.—TABLE SHOWING LOADS CARRIED BY STRAINED WIRE (MEAN DIAMETER 0·682 MM.) AND CORRESPONDING E.M.F.'S, REDUCED TO MEAN TEMPERATURE-RANGE OF 75° C. June 4, 1913.

Load (kg.) . . .	0	0·5	1	1·5	2	2·5
E.M.F. (volt $\times 10^6$ ) . . .	0·600	2·101	3·459	4·732	5·932	7·120
3	3·5	4	4·5	5	5·5	
8·261	9·318	10·300	11·289	12·141	12·920	
6	6·5	7	7·5	8	8·5	
13·718	14·471	15·092	15·649	16·101	16·299	
9	9·5	10	10·5	11	11·5	
16·833	17·080	17·211	17·330	17·363	17·302	
12	12·5	13	13·5	14	14·5	
17·178	16·970	16·661	16·272	15·800	15·260	

It may be remarked, with reference to this curve and the table of values, that the loads given as abscissæ do not include the weight (175 gm.) of the weight-pan. This accounts largely, though not entirely, for the fact that "zero" load on the curve shows an E.M.F. of  $0\cdot60 \times 10^{-6}$  volt; the unavoidable manipulation of the annealed wire in mounting it (and consequent slight strain) gave rise to an observed E.M.F. at (true) zero load of about  $0\cdot066 \times 10^{-6}$  volt, which accounts for the remainder of the discrepancy. During the experiment—which lasted some eight consecutive hours—it was, of course, impossible to secure that the temperatures inside the steam jacket and cold-water jacket which surrounded the two junctions of the couple should remain constant. Besides the unavoidable small variations due to the replenishment of the boiler and other incidental parts of the experiment, there comes into play a very considerable thermoelastic cooling effect\* at the junctions as the load is increased. Thus, though these junction-temperatures, which were read between every two galvanometer readings, proved fairly steady in the neighbourhood of 97° C. and 21° C. respectively, varying at most by about half a degree, for greater accuracy all the observations of E.M.F. used in plotting the curve were reduced by interpolation to a mean temperature-difference of 75° C. The reduction was easily performed from a series of temperature-E.M.F. curves for different constant loads, prepared in connection with the original experiments.

*Examination of the Curve.*—The curve, when drawn to a large scale

\* *Loc. cit.*, pp. 36-43.



(1 inch = 0.5 kg. for abscissæ, =  $10^{-6}$  volt for ordinates), was examined to determine (i.) if it was a parabola, and (ii.) if so, if its axis was vertical or inclined. The method of examination differed from that employed by Dickson, in that it was first tacitly assumed that the curve was a parabola, the fact that the midpoints of several parallel chords were found to lie on one straight line giving support to this assumption, while not completely justifying it. The straight line in question then gave the direction of the axis. To determine the vertex of the curve, the tangent to it at right angles to the axial direction was drawn: this is easily performed with considerable accuracy by the aid of a long slip of glass having one straight line ruled on it all its length, and a shorter one at right angles across the slip. The shorter one is made to cover the determined axial direction, and then slid along it until the longer one just touches the curve. (The glass is, of course, turned with the lines next the paper.) The axis was then drawn at right angles to the tangent at the vertex, and the focus determined by trial, again with the glass slip, by finding the point on the axis whose distance from the vertex was half its perpendicular distance from the curve. This determined the latus rectum. Lastly, the inclination of the curve-axis to the E.M.F.-axis was measured roughly by protractor and (much more accurately) by square-counting, to find its tangent. The results of these various measurements were:

Co-ordinates of vertex	.	.	(22°48", 17°34")
Length of latus rectum	.	.	25.2"
Axial inclination,			
(i.) by protractor (mean of 8 readings)	.	3° 54'	(about)
(ii.) by square-counting ( $\tan \omega = \frac{8}{120.8}$ )	.	3° 48'	

From these data it is now possible to calculate the equation of the parabola, and the final stage of the work is then the verification that the values given in Table I. satisfy this equation to a sufficient degree of accuracy. The equation, as calculated to seven significant figures for each coefficient, was

$$9956077x^2 - 1322564xy + 43922y^2 - 407990900x + 279653900y - 206538300 = 0.$$

For any assigned value of  $x$  the equation gives two values of  $y$ , one of which, since the coefficient of  $y^2$  is very small compared with the other coefficients, will be practically infinite (and negative). Neglecting these infinite solutions as foreign to the problem, Table II. gives a comparison of the E.M.F.'s ( $y$ ) corresponding to various loads  $\left(\frac{x}{2}\right)$  as calculated from this equation,

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and the E.M.F.'s actually observed at the same loads. The agreement is at once sufficiently obvious: from the arrangement of the signs in the column

TABLE II.

Load (kg.).	Calculated value of E.M.F. (Volt $\times 10^6$ ).	Observed value of E.M.F.	Difference.
0	0·641	0·600	+0·041
1	3·547	3·459	+0·088
2·5	7·307	7·120	+0·187
4	10·511	10·300	+0·211
5	12·326	12·141	+0·185
6	13·877	13·718	+0·159
7·5	15·685	15·649	+0·036
9	16·855	16·833	+0·022
10	17·262	17·211	+0·051
11	17·362	17·363	-0·001
12·5	16·915	16·970	-0·055
14·5	15·148	15·260	-0·112

of differences it seems probable that the determination of the axial inclination has been slightly at fault (rather too large), and that a more accurate determination would make the agreement still more striking. As it is, the initial assumption that the curve is a parabola appears sufficiently justified.

(Issued separately March 20, 1914.)

VIII.—The Path of a Ray of Light in a Rotating Homogeneous and Isotropic Solid. By E. M. Anderson, M.A., B.Sc. *Communicated by THE GENERAL SECRETARY.*

(MS. received November 3, 1913. Read January 19, 1914.)

THE path of a ray of light in irrotationally moving media was first investigated mathematically by Fresnel, who showed that to account for the observed phenomena it is necessary to suppose that, the aether being fixed, the medium imparts  $\frac{\mu^2 - 1}{\mu^2}$  of the amount of its own motion, resolved in the line of the refracted ray, to the advancing disturbance. This conclusion is also a necessary consequence of the modern Theory of Relativity, which, while holding that, with reference to axes with regard to which the medium is stationary at any point, the speed of light depends only on the refractive index, for any other system leads to the above result.

Using this formula, it is possible to calculate the path of a ray of light in a rotating homogeneous and isotropic solid, when the velocity produced by rotation is small compared to that of light itself. We shall first consider the case of a body rotating about an axis fixed in space, and at

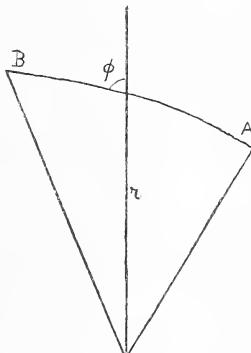


FIG. 1.

right angles to the line joining two points A and B, through which we will suppose the path to pass. If  $r$  be the distance of the disturbance at any moment from this axis, and  $\phi$  the angle made by the ray with the direction of  $r$  produced; if further  $\omega$  be the velocity of rotation, and  $c$  the speed of light; then the total velocity of the ray is

$$\frac{c}{\mu} + \frac{\omega r \sin \phi (\mu^2 - 1)}{\mu^2},$$

or

$$D + Er \sin \phi,$$

where

$$D = \frac{c}{\mu} \quad \text{and} \quad E = \omega \frac{\mu^2 - 1}{\mu^2}.$$

Then the condition for a brachistochrone is first evidently that the path shall lie entirely in the plane of rotation, and further

$$\delta \int \frac{ds}{D + Er \sin \phi} = 0,$$

or, neglecting the second order of small quantities

$$\delta \int \frac{ds}{D} - \delta \int \frac{ds Er \sin \phi}{D^2} = 0.$$

If we consider the part of the brachistochrone intercepted between A and B, then  $\int ds$  is the total length of the curve AB, while  $\int ds r \sin \phi$  is twice the area traced out by the radius vector.

It is easy to see that, while an increase of the length of the path leads to an increase in the time taken, an increase in the area covered by the radius vector will have the opposite effect, if area be reckoned positive when swept out in the direction in which the solid is rotating. The last equation may be interpreted to mean that for any slight deviation from a brachistochrone these two causes of variation must exactly balance, and that they will do so if an increase in length is accompanied by  $D/2E$  times the same increase in area. Substituting we find

$$\frac{D}{2E} = \frac{\mu c}{2\omega(\mu^2 - 1)}.$$

It is easy to show that a curved path, with a certain radius of curvature,

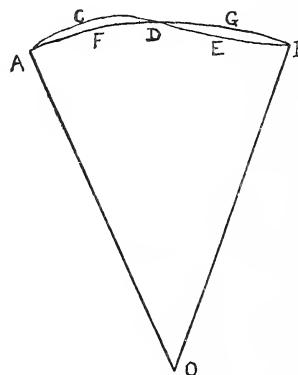


FIG. 2.

constant within the limits already assigned, will satisfy the above condition. In the first place, it is a well-known theorem, and may be proved by the

calculus of variations, that for any given length of line joining two points A and B (fig. 2), that form of curve which subtends a maximum area with a third point O in the same plane is an arc of a circle. We shall not, however, use this result, but another which follows from it, or may be regarded as the last step in its demonstration, namely, that if AFDGB be the circular arc of required length, and ACDEB a slight variation from it having exactly the same length, then the areas OACDEB and OAFDGB are equal to the first order of small quantities.

If now (fig. 3) we consider ALB as a possible circular path for light in the rotating solid, and let APB be any slight deviation from it, in the same plane, but not necessarily of the same length as ALB; then if the lengths of the two curves be not the same, let AMB be the circular

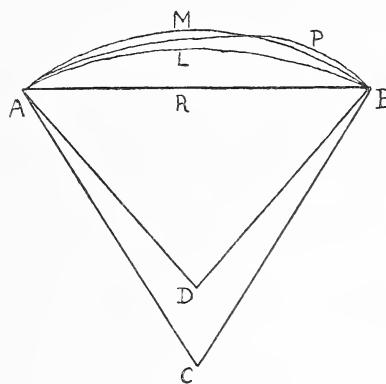


FIG. 3.

arc joining A and B which has the same length as APB. Then by the theorem just stated AMB and APB subtend the same area at any point in their plane to the first order of small quantities. Thus the time taken by an ethereal disturbance to traverse the paths AMB and APB will be equal with the same degree of accuracy.

Let us next consider the difference of length, and of area subtended by the two circular arcs ALB and AMB. Let R be the centre of the chord AB, and C and D the respective centres of the two circles. If then

$$AR = \alpha$$

$$AD = \rho$$

$$AC = \rho + \epsilon,$$

then the area enclosed by the two arcs is

$$\epsilon \frac{d}{d\rho} \left( \rho^2 \sin^{-1} \frac{\alpha}{\rho} - \alpha \sqrt{\rho^2 - \alpha^2} \right),$$

which equals

$$2\epsilon \left( \rho \sin^{-1} \frac{\alpha}{\rho} - \frac{\alpha \rho}{\sqrt{\rho^2 - \alpha^2}} \right).$$

The difference in length of the two arcs is

$$\epsilon \frac{d}{d\rho} \left[ 2\rho \sin^{-1} \frac{\alpha}{\rho} \right] = 2\epsilon \left[ \sin^{-1} \frac{\alpha}{\rho} - \frac{\alpha}{\sqrt{\rho^2 - \alpha^2}} \right],$$

the former result divided by  $\rho$ , where  $\rho$  is the radius of curvature.

Thus the area enclosed between two nearly coincident circular arcs, ending at the same points, is  $\rho$  times their difference of length. Also, from what we have already seen, the difference in the area subtended at any point in its plane by a circular arc and any slight deviation from it will be  $\rho$  times the difference in length of the two curves. This assumes that the point of subtension is so situated that the area swept out by the radius vector is wholly positive or wholly negative. Obviously, for a point on the concave side the difference of area will be of the same sign as the difference in length, while for a point on the convex side the sign will be opposite.

Now, we have seen that the condition for a brachistochrone in the case we are considering is that an increase of length in the curve between any two points shall be accompanied by an increase of area subtended at the centre of rotation of  $\frac{\mu c}{2\omega(\mu^2-1)}$  times the amount, area being counted positive when traced out in the same direction as the body is rotating. This condition is obviously satisfied by a circular arc whose radius of curvature

$$\rho = \frac{c}{\omega} \times \frac{\mu}{2(\mu^2-1)},$$

and which is concave or convex to the centre of rotation according as the wave motion with regard to that centre is in the same or in the opposite direction to the rotation.

We will next consider the case where the medium, in addition to its rotatory motion, has an uniform translatory motion  $v$ , as before, small compared to the velocity of light. If  $\theta$  be the angle between the direction of the ray and that of  $v$ , the total velocity is

$$\frac{c}{\mu} + \frac{\omega r \sin \phi (\mu^2 - 1)}{\mu^2} + \frac{v \cos \theta (\mu^2 - 1)}{\mu^2},$$

or

$$D + Er \sin \phi + F \cos \theta,$$

where

$$F = \frac{v(\mu^2 - 1)}{\mu^2}.$$

The condition for a brachistochrone is

$$\delta \int \frac{ds}{D + Er \sin \phi + F \cos \theta} = 0$$

or to the first order of small quantities

$$\delta \int \frac{ds}{D} - \delta \int \frac{ds E r \sin \phi}{D^2} - \delta \int \frac{ds F \cos \theta}{D^2} = 0.$$

Now, as the projection of the path AB on a line drawn parallel to  $v$  is a constant, the last term vanishes, and we therefore arrive at our previous result.

In both this and the previous case the curves calculated are those followed in space with regard to the assumed axes of no velocity. The paths traced out in the solid itself can be deduced as follows. It is easy to show that a ray which penetrates the rotating body in a straight line, with velocity  $\frac{c}{\mu}$ , will leave a trace in the solid of curvature  $\frac{2\omega\mu}{c}$ , which will be convex to the centre of rotation if the ray be moving with regard to the centre in the direction of rotation. The actual curvature of the path of light is  $\frac{2\omega(\mu^2-1)}{c\mu}$  in the opposite direction. By subtraction we get  $\frac{2\omega}{c\mu}$  as the curvature of the path traced out in the solid; the radius of curvature is  $\frac{c\mu}{2\omega}$ , and the curve will be convex or concave to the centre of rotation according as the radius vector of the disturbance moves along with or against the rotation. This result will apply to both the cases so far considered.

We have next to deal with the case in which the path of the ray is not in the plane of rotation. Let us consider what path will be followed between

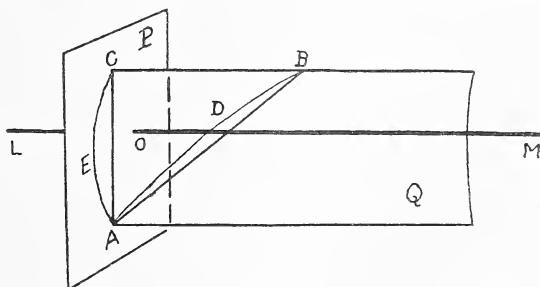


FIG. 4.

two points A and B in a medium rotating about an axis LM (fig. 4). Let P be the plane passing through A and perpendicular to LM; O the point

of intersection of that axis; C the projection of B on P. Join CA and AB, and let  $\psi_0$  be the angle CAB. Let ADB be a possible path for light between A and B, and let AEC be its orthogonal projection on P. Let  $\psi$  be the angle between any element of the curve and the corresponding element of the projection;  $r$  the distance of the latter element from the point O, and  $\phi$  the angle it makes with the direction of  $r$  produced. Then the speed of light at any point on ADB is

$$\frac{c}{\mu} + \omega r \sin \phi \cos \psi \frac{\mu^2 - 1}{\mu^2},$$

or

$$D + Er \sin \phi \cos \psi,$$

where D and E have the same meanings as before. The condition for a brachistochrone is therefore

$$\delta \int \frac{ds}{D} - \delta \int \frac{Er \sin \phi \cos \psi ds}{D^2} = 0.$$

If now  $ds'$  be the element of the projection corresponding to any element  $ds$ ,  $ds' = ds \cos \psi$ , and our condition may be written

$$\delta \int \frac{ds}{D} - \delta \int \frac{Er \sin \phi ds'}{D^2} = 0.$$

Let Q be the cylindrical surface whose generators are parallel to LM, and which passes through the curve ADB and its projection. Then it is evident that for any curve on Q joining A and B the latter half of the expression vanishes. Hence the curve of this class which most nearly satisfies the brachistochrone condition is the shortest in length, or otherwise the curve for which  $\psi$  is constant.

We may therefore confine ourselves to curves for which this condition is fulfilled. Denoting the length of the arc AEC by  $l$ , and that of CB by  $a$ ,

$$\tan \psi = \frac{a}{l}, \quad \text{and} \quad \cos \psi = \frac{l}{\sqrt{l^2 + a^2}}.$$

The condition may therefore be written

$$\delta \int \frac{ds' \sqrt{l^2 + a^2}}{Dl} - \delta \int \frac{Er \sin \phi ds'}{D^2} = 0.$$

As  $\int \frac{ds'}{l} = 1$ , we get

$$\frac{l \delta l}{D \sqrt{l^2 + a^2}} - \delta \int \frac{Er \sin \phi ds'}{D^2} = 0,$$

or in other words,  $\psi_0$  being the rectilineal angle CAB, the increase of area subtended at O by the curve AEC must be  $\frac{D \cos \psi_0}{2E}$  times its increase of length, to the first order of small quantities. From this it follows that

the radius of curvature of the projected curve is  $\frac{c \cos \psi_0 \mu}{2\omega(\mu^2 - 1)}$ . The curve itself follows from this condition and the fact that  $\psi$  is constant. Within the limits considered it is part of a circular helix. Its radius of curvature is  $\frac{c\mu}{2\omega \cos \psi_0 (\mu^2 - 1)}$  in a direction perpendicular to the axis of rotation. This conclusion holds good whether or not we consider our rotating body to have also a translatory motion with regard to the axes of reference.

NOTE ON MR ANDERSON'S PAPER. By Sir JOSEPH LARMOR,  
M.P., F.R.S.

Mr E. M. Anderson's elegant argument may be paraphrased as follows:—

Let  $v$  be any coplanar velocity of the medium, and set  $E = v(\mu^2 - 1)/\mu^2$  and  $D = c/\mu$ ; then the time of passage of a ray restricted to any artificial path is

$$\int \frac{ds}{D + E \cos \phi}, \quad \text{approximately} \quad \frac{1}{D} \int ds - \frac{1}{D^2} \int E \cos \phi \cdot ds.$$

Now

$$\int E \cos \phi ds = \frac{\mu^2 - 1}{\mu^2} \int v \cos \phi ds,$$

where the integral expresses, for a complete circuit, the circulation of the medium in the sense introduced by Lord Kelvin into Hydrodynamics. Here a circuit can be completed by any unvaried return path. Now, in coplanar Kinematics, the circulation round the contour of any area is  $[2\omega d(\text{area})]$ , where  $\omega$  is the velocity of differential rotation or the vorticity. When  $\omega$  is uniform, the time of passage of the ray is, for a ray in the plane of motion,

$$\delta\tau = \frac{1}{D} (\text{length}) - \frac{2\omega}{D^2} \frac{\mu^2 - 1}{\mu^2} (\text{area}).$$

Now, when the length is maintained constant,  $\delta(\text{area}) = 0$  for all possible variations when the curve is a circular arc. Therefore, as Mr Anderson reasons, when the length also is allowed to vary

$$\delta(\text{area}) = A\delta(\text{length}),$$

where the value of  $A$  can be calculated from the circular form. A particular circular arc can then be selected which will make  $\delta\tau$  vanish for all small variations of its form without restriction to constant length; and this will be the path of an unconstrained ray.

If an additional uniform velocity is imposed at right angles to this coplanar (or rather uniform laminar) motion, the vorticity will remain unaltered; thus, in the expression for  $\delta\tau$  (the ray being now free of any restriction to a plane) the area that occurs will be that of the projection on the plane of the laminar motion. Now, even in the most general coplanar motion, when the vorticity is not uniform,  $\int \omega d(\text{area})$  will be stationary for small variations which leave the length unvaried, only when the curve is a geodesic on the cylinder, perpendicular to the laminar motion, on which it lies; for otherwise a displacement of the curve, considered as a thread, on the cylinder will make it slack, and the cylinder can be expanded to take in more area. This geodesic, as it would unroll into a straight line, will have the length of its projection on the plane of the laminar motion unvaried while the shape of the cylinder thus varies. The shape of the cross-section of the cylinder on which the ray lies will therefore be such that for given length of its arc  $\int \omega d(\text{area})$  is stationary. In particular, if  $\omega$  is uniform, it will be an arc of a circle.\* This brings us to Mr Anderson's result, in a somewhat extended form.

If a uniform material medium is in motion through the æther with vorticity  $\omega$  restricted to be constant in magnitude and direction, all rays of light travel in it along helices traced on cylinders of constant radius  $\frac{c}{2\omega} \cdot \frac{\mu}{\mu^2 - 1} \cos \psi_0$ , having their axes in the direction of the constant vorticity.

\* The same argument establishes that a flexible conductor carrying an electric current in a uniform magnetic field will when free assume the form of a circular helix; cf. *Proc. Lond. Math. Soc.*, vol. xvi., 1884, p. 169.

IX.—*Principia Atmospherica: a Study of the Circulation of the Atmosphere.* An Address delivered at the request of the Council before the Royal Society of Edinburgh, on 1st December 1913. By W. N. Shaw, LL.D., Sc.D., F.R.S., Director of the Meteorological Office, Reader in Meteorology in the University of London.

(Read December 1, 1913. MS. received December 12, 1913.)

#### INTRODUCTION.

EVERY science has two aspects or two stages in its development. In the first, the inductive stage, observations are made and compiled, and axioms or laws are laid down. In the second or deductive stage the laws are applied by syllogistic reasoning, mathematical or otherwise, to elicit conclusions which either disclose new facts or show the inevitable connection between facts already known, and, in either case, complete the claim of the study to the rank of a science.

The different sciences vary greatly in the stage of development which they present. The science of geometry has almost forgotten the origin of its own laws and axioms, and occupies itself with the most complicated deductive propositions, the forms of which are used to guide the deductions of other sciences. Biology is still in the inductive stage: no one ventures yet to predict in what form the horse will be found a million or even a thousand years hence.

These different aspects of science appeal with different force to different types of human mind. Observers are comparatively rare; true inducers, those who have the patience and the insight to arrange the facts and formulate the underlying laws, are extremely rare; deducers, those who draw conclusions, not always mathematical or strictly logical, make up the balance of the human race.

Many years ago, in 1862, Dr Alexander Buchan, in a contribution to this Society which was subsequently elaborated in a volume of the results of the *Challenger* Expedition, laid the foundations of our inductive knowledge of the atmospheric circulation by a series of maps of the distribution of pressure over the surface of the globe. With great pleasure I take the opportunity afforded to me by your invitation to address you on recent developments of the science of meteorology, particularly in the investigation of the upper air, to put before you a representation of the knowledge of

the atmospheric circulation as it presents itself to my mind, arranged in the normal scientific form, with axioms which represent inductive laws, with postulates or lemmas which represent groups of observed facts, and with propositions leading to conclusions which are susceptible of verification.

### Synopsis.

#### SECTION I.—AXIOMS OR LAWS OF ATMOSPHERIC MOTION.

##### 1. *The Law of the Relation of Motion to Pressure.*

In the upper layers of the atmosphere, the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surfaces at that level, and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.

##### 2. *The Law of the Computation of Pressure and of the Application of the Gaseous Laws.*

The pressure at any point in the atmosphere and at any instant is the weight of the column of air which stands upon one unit of horizontal area containing the point. The numerical values of pressure, temperature, and density at any point of the atmosphere are therefore related by the usual formulae for the gaseous laws.

##### 3. *The Law of Convection.*

Convection in the atmosphere is the descent of colder air in contiguity with air relatively warmer.

##### 4. *The Law of the Limit of Convection.*

Convection in the atmosphere is limited to that portion of it, called the troposphere, in which there exists a sensible fall of temperature with height. The upper layer of the atmosphere, in which there is no sensible fall of temperature with height and therefore no convection, is called the stratosphere.

##### 5. *The Law of Saturation.*

The amount of water vapour contained in a given volume of air cannot exceed a certain limit, which depends upon the temperature and upon nothing else.

#### SECTION II.—LEMMAS OR POSTULATES.

*Lemma 1.*—In the stratosphere, from 11 kilometres upwards it is colder in the high pressure than in the low pressure at the same level; and in the troposphere, from 9 kilometres downwards to 1 kilometre, it is warmer in the high pressure than in the low pressure at the same level. [W. H. Dines, *M.O.*, 210b.]

*Lemma 2.*—The average horizontal circulation in the Northern hemisphere in January between 4 kilometres and 8 kilometres consists of a figure-of-eight orbit from west to east along isobars round the pole, with lobes over the continents and bights over the oceans.

The average circulation at the surface is the resultant of the circulation at 4 kilometres combined with a circulation in the opposite direction of similar shape due to the distribution of temperature near the surface. [L. Teisserenc de Bort, *Ann. du Bureau Central Météorologique*, 1887; and W. N. Shaw, *Proc. Roy. Soc.*, vol. lxxiv. p. 20, 1904.]

## SECTION III.—PROPOSITIONS.

*Proposition 1.*—To define the conditions for the persistence of the existing motion of the atmosphere.

*Proposition 2.*—To show that the rate of increase of pressure-difference per kilometre of height is  $34.2 \frac{p}{\theta} \left( \frac{\Delta\theta}{\theta} - \frac{\Delta p}{p} \right)$ ; and hence that the distribution of pressure in the stratosphere is the dominant factor in the circulation of the air at the surface; that the intermediate layers between 4 kilometres and 8 kilometres exert little influence upon the distribution of pressure.

*Proposition 3.*—To show that the wind velocity across the slope of pressure at any level is proportional to  $\theta \frac{\Delta p}{p}$ ; and thence to show how to utilise observations of the pressure and temperature to calculate the wind velocity at any level.

*Proposition 4.*—To show that the wind velocity generally increases with height until the substratosphere is reached, and falls off with increase in height in the stratosphere.

*Proposition 5.*—To show how the distribution of pressure and temperature in the upper air can be calculated from the observations of structure represented by a sounding with a pilot balloon, and thence to account for the local distribution of rainfall when an upper current from the north-west crosses a lower current from the south-west.

*Proposition 6.*—To account for the average general circulation over the Northern hemisphere in the four-kilometre level as set out in Lemma 2.

## SECTION I.—AXIOMS OR LAWS OF ATMOSPHERIC MOTION.

The time has arrived when it seems possible and desirable to formulate the laws and principles which can be effectively employed at the present day in the explanation of many of the recognised phenomena of the structure and circulation of the atmosphere, and to illustrate their application. These laws and principles are the result of observations sometimes suggested or controlled by theory. They are of the nature of axioms or inductions, about the validity of which a good deal of discussion is possible. Into that discussion I do not now propose to enter. The axioms really depend for their justification upon their effectiveness in explaining observed facts. They are set out as follows:—

1. *The Law of the Relation of Motion to Pressure.*

In the upper layers of the atmosphere, the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surfaces at that level, and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.

The line of argument in favour of this law, which cannot strictly speaking be either verified or contradicted by any available process of observation, is as follows: The condition specified in the law is the condition of

kinematic equilibrium towards which all atmospheric motions tend, and have tended either since the earth began to rotate as it does now, or the atmosphere was first formed, whichever of those events is the later in time. Any deviation from the equilibrium state is by infinitesimal steps during which readjustment to the equilibrium condition has been taking place automatically. Hence any finite difference from the equilibrium state can only occur in quite exceptional conditions. Consequently if there is an ascertained difference from the equilibrium condition it requires explanation just as the divergences from the uniformity contemplated by the First Law of Motion require explanation.

An allowance for "curvature of path" is one of the differences of which account may have to be taken. Its importance depends upon the latitude. For the half of the globe north of  $30^{\circ}$  N. and south of  $30^{\circ}$  S. it is generally negligible, but near the equator it becomes the paramount consideration in the question of the persistence of distribution. Thus rotary systems, small or large, are the only possible isobars for a synchronous chart of an equatorial region, if one were drawn. The long sweeps of "parallel isobars" with which we are concerned in this paper would be inadmissible there.

Near the surface there is always a component of motion along the gradient from high pressure to low pressure. In this region the friction due to obstacles and to the viscosity of the air prevents the steady state being reached, and in consequence the centrifugal force due to the velocity of motion is not adequate to balance the pressure.

This modification of the general principle in the case of surface air may be inferred from the fact that in all maps of the distribution of pressure and wind at the surface there is evidence of a flow across the isobars. The maps are not always conclusive, as they are for sea level and not station level; but no person of experience will doubt the general truth of the statement, which in books often takes the form of postulating convergence towards centres of low pressure and divergence from centres of high pressure.

## 2. *The Law of the Computation of Pressure and of the Application of the Gaseous Laws.*

The pressure at any point in the atmosphere and at any instant is the weight of the column of air which stands upon one unit of horizontal area containing the point.

This principle assumes that the motion of the air is so slow that the hydrostatical forces are not interfered with. Explosion or elastic wave-motion would invalidate the law. It therefore assumes that the

atmosphere is free from explosions and elastic wave-motions, or that their effect is so small that it does not enter into meteorological calculation.

It follows that the numerical values of pressure, temperature, and density at any point of the atmosphere are related by the usual formulæ for the gaseous laws. In other words, when due allowance is made for the difference of composition in consequence of the variation in the amount of water vapour or other possible causes, the relation  $p = R\theta\rho$  holds, where  $p$ ,  $\theta$ ,  $\rho$  are the pressure, temperature (on the absolute scale), and density of the air, and  $R$  is a "constant" which is altered by an alteration in the composition of the air, but not by other causes.

### 3. *The Law of Convection.*

**Convection in the atmosphere is the descent of colder air in contiguity with air relatively warmer.**

The law is advisedly stated in this form (although objections may be taken to it for want of strictness) because the driving power of the convective circulation comes from the excess of density of the descending portion, and the excess of density in atmospheric air is due in nearly all cases to low temperature. Differences of density might be caused by differences of pressure or by differences in the amount of moisture contained in equal volumes. But finite differences of pressure cannot persist in contiguous masses of air; the amount of water vapour in air at the ordinary temperatures with which a meteorologist has to deal is only a small fraction of the whole mass, and the colder the air is, the less water vapour is required to saturate it. Consequently, although it would be possible in a physical laboratory to display a sample of air which, though warmer, is yet denser than another cooler sample on account of the humidity of the latter, the conditions would not easily occur in nature, and the motive power for convection would be exceedingly small. Such cases may therefore be left out of account, and we may consider that, of two contiguous masses of air, the colder is the denser.

The law of convection is usually stated with regard to the warmer part of the convective circulation, and takes the briefer form that warm air rises. The general adoption of this briefer form is due to the fact that the warming of air at the surface is a matter of common knowledge, and it occurs in the daytime, when its effects in producing a local convective circulation are often quite distinctly visible. The form which is adopted here, however, is preferable, because in any case it is the cooler and heavier air in the neighbourhood which must be looked for if the true cause of the circulation is to be found; and, although on the smaller scale the

heavier air is not far to seek, it is not so easily identified on the scale of a meteorological chart.

**Convection in the atmosphere may also be due to the variation in the gravitational acceleration due to the motion of the air with reference to the earth.**

The gravitational acceleration depends partly on the statical attraction of the earth's mass and partly on the centrifugal action due to rotation. The ordinary values of the constant of gravitation assume the rotation to be that of the solid earth, and the acceleration of gravity upon air moving over the earth's surface is consequently different from that for calm air. Hence the air which forms part of a westerly wind is *specifically lighter* than air at the same temperature and pressure which is calm ; and, on the other hand, air which forms part of an easterly wind is *specifically heavier*. These variations in what, contrary to the usual convention, may rightly be called the "specific gravity of the air" have not yet been generally taken into account in meteorological practice, but they are of real significance, and are the subject of certain classical papers by von Helmholtz and Brouillin on the circulation of the atmosphere.

#### 4. *The Law of the Limit of Convection.*

**Convection in the atmosphere is limited to that portion of it in which there exists a sensible fall of temperature with height.**

This portion, which comprises about three-fourths of the atmosphere, is called the *troposphere*, and is a layer of air about 10 kilometres thick surrounding the whole earth. It is surrounded by an outer spheroid of air comprising the remaining fourth part of the atmosphere, which is called the *stratosphere*, in which there is no sensible fall of temperature with height. The boundary between these two layers is not at a fixed height ; it is apparently a flexible, and therefore deformable, surface, but it is not penetrable by air.

The height of the boundary differs in different latitudes, being highest over the equator and getting gradually lower towards the poles ; it differs also in different localities, being higher over an area of high pressure than over one of low pressure. The local differences are due to deformations of the boundary by the accumulation or withdrawal of air from underneath. At any place the boundary oscillates about a mean position which should be regarded as the height of the boundary of the stratosphere for the place. There is no physical reason why the boundary of the stratosphere should not be penetrated. All that is required to produce that effect is an accumulation of air warm enough to cause upward convection. All that

can be said is that there is no example of the approach to such an accumulation. There are a sufficient number of examples in which there is a reversal of fall of temperature just below the stratosphere, and these show that the stratosphere has, if anything, a little to spare in the way of resistance against penetration. Hence, from the point of view of meteorological theory we regard the stratosphere as impenetrable.

##### 5. *The Law of Saturation.*

**The amount of water vapour contained in a given volume of air cannot exceed a certain limit which depends upon the temperature and upon nothing else.**

This is really simply a statement of Dalton's law of the saturation of a gas with the vapour of a liquid, but it is quoted here partly because it refers to the only form of variation in chemical composition to which the meteorological atmosphere is subject, and also partly in order to avoid a misapprehension that is very widespread. It is a well-known physical principle that when a vapour is condensed the "latent heat of vaporisation," which, in the case of water vapour, is very large, is liberated. The statement of the principle is not complete; it should go on to say that the condensation cannot take place unless provision has been made for disposing of the heat which will be liberated. In the case of the atmosphere it is often assumed that no provision of the kind is required, and that the air will, in consequence, *be warmed* by the heat set free. Herein lies the misapprehension. Vapour of water in air will not condense unless the air is cooled, and the amount of condensation will be limited by the amount of the cooling.

It should, however, be noted that the wording of the law as here given, namely, that the limiting amount of water vapour depends upon the temperature *and upon nothing else*, implies a statement about the atmosphere about which it is necessary to be explicit. Since Dalton's law was enunciated, the researches of Aitken and others have shown that the cooling of a mass of air below the "saturation point" causes condensation only if there are nuclei upon which drops of water can form. In the absence of such nuclei, laboratory experiments have shown that condensation does not take place until the limits of saturation have been largely exceeded; "four-fold saturation" is necessary in such a case. Air without nuclei cooled below its "saturation point" is said to be supersaturated, and the statement of the law of saturation as set out implies that *supersaturation does not exist in the free air*. This is another case in which there is no physical reason to prevent anyone imagining circumstances in which supersaturation

might exist; all that can be said is that no such circumstances have been demonstrated, and the ready formation of clouds at all heights seems to indicate that such circumstances are quite unlikely. Hence the meteorologist is entitled to infer, as the result of a meteorological though not of a physical law, that *condensation* in the form of cloud, or if necessary of rain, *will always accompany the reduction of temperature of the air below the point of saturation*, and the amount of condensation will depend upon the reduction of temperature and upon nothing else.\*

These five laws express the special principles with which the meteorologist must approach the consideration of the circulation of the atmosphere, with all its complexities and its perplexities. The rest must depend upon the application of the ordinary principles of dynamics and physics to the results of observations which indicate the pressure, temperature, and density of the air in its actual condition when under consideration. It is my object in this paper not to discuss or to justify these principles, but to show how far they lead us in the explanation of some of the more general phenomena of the atmospheric circulation.

The form which has been adopted for this communication has been chosen for the purpose of drawing a distinction between the inductive, the observational, and the deductive aspects of the questions which are treated. Just as, in the cases of motion treated in text-books of dynamics, there is ample opportunity for discussion as to the form of words which shall be used for the laws of motion and the grounds for their acceptance or rejection, starting from the consideration that there never has been an actual example of a body free from the action of force, so, in the case of atmospheric motion, there is no lack of opportunity for the discussion of the laws as here set out, starting from the consideration that no actual case can be quoted in which we are certain that the laws are strictly obeyed. And further, just as in the case of the dynamics of the heavenly bodies the whole subject is reduced to a manageable form by setting out to explain the changes of motion and their causes instead of pondering over the ultimate origin and cause of the state of motion which exists at any particular epoch, so in the study of the circulation of the atmosphere we may profitably turn our attention to the changes in the motion related to the varying distributions of pressure, and leave for the time being the endeavour to give a short answer to the question, "What is the ultimate

\* The supersaturation of atmospheric air is discussed in Dr Alfred Wegener's *Thermodynamik der Atmosphäre*, Leipzig, J. A. Barth, 1911. Humidities, by the hair hygrometer, up to 107 per cent. are cited on p. 254 of that work.

cause of any given distribution of pressure, with its attendant atmospheric motion?"

We proceed, therefore, first to define in two lemmas the average condition of the atmosphere which we wish the reader to keep in mind, and secondly to apply the laws which have been already enunciated to make certain deductions or establish certain propositions with regard to the circulation of the atmosphere, which are set out in the synopsis.

## SECTION II.—LEMMAS OR POSTULATES.

### *Lemma 1.*

In the stratosphere from 11 kilometres upwards it is colder in the high pressure than in the low pressure at the same level; and in the troposphere, from 9 kilometres downwards to 1 kilometre, it is warmer in the high pressure than in the low pressure at the same level.

*Proof.*—Table of average values of pressure and temperature at different levels over high pressure (1031 mb.) and low pressure (984 mb.) at the surface; with pressure differences and temperature differences at each level. Compiled from the diagram and tables of W. H. Dines, F.R.S., in *Geophysical Memoirs*, No. 2, M.O. Publication, 210b.

TABLE I.

k.	Pressure.		$\Delta p$ mb.	$\Delta\theta$ $^{\circ}$ A.	Temperature.	
	Low mb.	High mb.			Low 984 mb. $^{\circ}$ A.	High 1031 mb. $^{\circ}$ A.
15	116	123	7			
14	135	146	11	- 9	224	215
13	157	171	14	- 11	226	215
12	183	201	18	- 8	225	217
11	212	235	23	- 4	225	221
10	247	273	26	+ 1	225	226
9	288	317	29	+ 7	226	233
8	335	366	31	+ 13	227	240
7	388	422	34	+ 15	232	247
6	449	483	34	+ 14	240	254
5	516	552	36	+ 13	248	261
4	591	628	37	+ 12	255	267
3	675	713	38	+ 9	263	272
2	767	807	40	+ 8	269	277
1	870	913	43	+ 4	275	279
0	984	1031	47	+ 3	279	282

Standard deviation of  $P_g$  13·8 mb.

Standard deviation of  $P_s$  14·1.

Correlation coefficient between the variations of  $P_g$  and  $P_s$  from the means for the month (English ascents) .80.

The table which is here given summarises the results of an important investigation by Mr Dines into the relation between the changes of pressure

at the 9-k. level and the corresponding changes at the surface. The changes which he dealt with were chronological, and I have extended the conclusion in applying it to topographical differences. This extension is justified if the places between which the differences are to be taken are sufficiently close together to be influenced by the same barometric system, and if the chronological sequence is followed in individual cases. That the latter condition is generally satisfied is shown by the high correlation coefficient between the variations at 9 k. and at the surface.

The conclusion as to the relation between temperature and pressure in the upper air which is drawn from this table is supported by the gradual evolution of meteorological ideas on the subject. Originally it was assumed that high pressure meant relatively dense air and low pressure relatively light air from the surface upwards. Sometimes temperature and sometimes moisture was held accountable for the levity; but the view first put forward by von Hann that, in ordinary circumstances, the air over high pressure is warmer than that over low pressure has gradually developed until it may now be regarded as an accepted principle in meteorology. It is borne out by the simultaneous soundings which have occasionally been obtained from places within the same barometric system; and apparently the disturbances in the specified order are mostly confined to the lowest reaches of the atmosphere. This last point also is well illustrated by the figures of the table, which show a gradual falling off, on the average, of the temperature differences in the lowest three kilometres.

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*Lemma 2.*

The average horizontal circulation in the Northern hemisphere in January between 4 kilometres and 8 kilometres consists of a figure-of-eight orbit from west to east along isobars round the pole, with lobes over the continents and bights over the oceans.

The average circulation at the surface is the resultant of the circulation at 4 kilometres combined with a circulation in the opposite direction of similar shape due to the distribution of temperature near the surface.

[L. Teisserenc de Bort, *Ann. du Bureau Central Météorologique*, 1887; and W. N. Shaw, *Proc. Roy. Soc.*, vol. lxxiv. p. 20, 1904.]

This lemma is introduced in order to supply the reader with a suitable general picture of the atmospheric circulation in the upper air, and the modification which it must undergo in the lowest layers in consequence of the distribution of temperature near the surface. As will be seen from Proposition 2, which follows, the similarity of pressure-distribution at all heights depends upon the equality of  $\Delta\theta/\theta$  and  $\Delta p/p$ . Consequently, a

circulation along parallels of latitude from west to east in which the air nearer the poles is the colder is a circulation which may remain practically identical at all heights, and is suggestive of durability and persistence.

The distribution of pressure at the 4-k. level given by M. Teisserenc de Bort suggests that the actual circulation in the upper air is not a circulation along parallels of latitude, but yet is an approximation thereto, being something intermediate between a circle and a figure-of-eight.

That the circulation at the 4-k. level remains of the same general character up to the 8-k. level is suggested by the fact that in those regions distribution of temperature is such as to cause very little change in pressure-differences, in accordance with the formula of Proposition 2.

It may be remarked that the distribution was calculated by M. L. Teisserenc de Bort from the distribution of pressure and temperature at the surface, and is subject to two uncertainties: first, the reduction of the original pressure readings to sea-level; and secondly, their further reduction to the 4-k. level. The uncertainties arise from the uncertainty in the values of the temperature of the air "below the ground" in the reduction to sea-level, and above the ground in the reduction to the 4-k. level. To a certain extent these two uncertainties compensate each other in the important features of the result, and the conclusion as to the circulation at which M. Teisserenc de Bort had arrived, is supported by the results of Hildebrandsson's discussion of the international cloud observations (see Hildebrandsson and Teisserenc de Bort, *Les Bases de la Météorologie dynamique*, vol. ii., Gauthier-Villars, Paris), and by other considerations of a more general character.

The statements of these two lemmas are based upon observation and are, therefore, liable to modification or correction in detail as the results of observation become more conclusive. They are, however, sufficiently well established to justify their use in the further consideration of meteorological problems.

### SECTION III.—PROPOSITIONS.

We now proceed to the consideration of the propositions which are set out in the Synopsis. I shall deal in detail with only three of the propositions, numbered 1, 5, and 6 respectively, because the remaining three, numbered 2, 3, and 4, have already been dealt with in a paper communicated to the Scottish Meteorological Society, with the title of "The Calculus of the Upper Air, and the Results of the British Soundings in the International Week of May 5-11, 1913." The paper is published in the *Journal of the Society for 1913*.

*Proposition 1.—The Conditions necessary to maintain a Steady Atmospheric Current.*

The conditions which must be complied with if a steady current is to be persistently maintained must satisfy the first law, the law of relation of motion to pressure.

The law prescribes that the velocity  $V$  is related to the pressure gradient  $\gamma$ , density  $\rho$ , latitude  $\lambda$ , and the angular velocity of the earth's rotation  $\omega$ , by the equation

$$V = \gamma / (2\omega\rho \sin \lambda).$$

Provided that the latitude  $\lambda$  remains constant during the persistence of the current, this condition presents no difficulty; the flow will always be determined by the distance apart of the isobars, but the auxiliary condition that the current shall not change its latitude implies that the isobars are parallel to the circles of latitude. Hence we may infer that, neglecting a very small correction for curvature, a circulation round the pole along isobars parallel to the circles of latitude is a "steady" circulation which will be persistently maintained. The only forces which will interfere with it are frictional forces due to the relative motion of adjacent layers of air, and, except in the immediate neighbourhood of the ground where friction is aided by turbulent motion, these are extremely small. Hence a west-to-east circulation or an east-to-west circulation in the upper air, once steady will remain so, unless it is disturbed by changes of pressure-distribution.

But, on the contrary, when the air movement is from south to north or from north to south, or has any component which gives a motion across the circles of latitude, a change in  $\sin \lambda$  has to be dealt with.

*Motion from South to North.*

We propose to deal first with a current moving from south to north. We shall suppose the current to be uniform over the section *from the one-kilometre level upwards*. We leave out the lowest kilometre because we know that it is disturbed by quasi-frictional forces at the surface.

In this case the value of  $\sin \lambda$  is increasing, and therefore greater pressure-difference is required to get the same quantity of air through the same section. But the pressure-difference is limited by the isobars, which are by hypothesis supposed steady. Any convergence of the isobars themselves provides its own remedy, because the gradient velocity is inversely proportional to the distance. We have, therefore, only to deal with the change in  $\sin \lambda$  in the formula

$$V = \gamma / (2\omega\rho \sin \lambda).$$

Let  $L$  be the width of the current, and  $H$  its depth; then the flow over the whole section  $L \times H$  is  $LHV$ ; and by the equation of continuity this must be constant as the stream flows northward.

Now

$$LHV = \frac{HL\gamma}{2\omega\rho \sin \lambda},$$

and  $L\gamma$  is the pressure-difference,  $\Delta p$ , between the two sides of the current.  $LHV$  is constant; hence, differentiating, we get

$$0 = \frac{\partial H}{H} - \frac{\partial \rho}{\rho} - \frac{\partial \sin \lambda}{\sin \lambda}$$

or

$$\frac{\partial H}{H} = \frac{\partial \rho}{\rho} + \cot \lambda \partial \lambda.$$

Now  $\rho$  can only alter by variation of pressure, temperature, or composition; change of pressure is ruled out because the motion is along isobars; change of temperature will be very slight because there is no change of pressure, and there are no other causes of any appreciable change of temperature; and change of composition can only occur in consequence of condensation. By Law 5, in the absence of change of temperature no change of composition will occur. Hence

$$\partial \rho / \rho = 0,$$

and

$$\frac{\partial H}{H} = \cot \lambda \partial \lambda.$$

*In other words, the thickness of the moving layer must increase fractionally by the amount  $\cot \lambda \partial \lambda$  for the change of latitude  $\partial \lambda$ . If latitude is expressed in degrees and not in circular measure as differentiation supposes, we must introduce the factor  $\frac{\pi}{180}$ , and thus the formula becomes*

$$\frac{1}{H} \frac{dH}{d\lambda} = .0175 \cot \lambda.$$

Hence, in order that a current may persist over any stretch from south to north, *it is necessary that the thickness of the moving layer should increase fractionally to the extent of .0175 cot λ for every degree of latitude which it crosses.*

We have assumed the layer to be unlimited above, and limited below by the one-kilometre level. To provide for the additional air by increasing the height above the selected base-level would result in altering the pressure: that mode of operation is therefore excluded by the condition of maintenance of the current as steady. Consequently we must suppose the additional thickness to be provided by encroachment upon the lowest

kilometre : that region is already supposed to be occupied by an extension of the current which is disturbed by surface friction ; hence, unless there is a continual flow-off of air from below the one-kilometre level, the steady state cannot be maintained.

The south-to-north current implies a high pressure on the eastern side and a low pressure on the western side, and near the surface there is a component of flow from high to low across the isobars. Hence we may suppose a case in which the northward-flowing current is maintained steady by the flow-off from east to west in the surface layer. We proceed to calculate the amount of this east-to-west current which will suffice to draw off the increase of the current above 1 kilometre.

We suppose, for the purpose of calculation, that the east-to-west component is uniform over the lowest half kilometre of the western section. The fractional increase of thickness in the upper layer has been shown to be  $0.0175 \cot \lambda$  for each degree of advance northward. The increase of the thickness is the same over each elementary layer of height into which the whole thickness can be divided ; consequently the air to be removed is the fraction  $0.0175 \cot \lambda$  of the transverse vertical section at every level. If the removal is confined to the lowest half kilometre, which contains a fraction of the atmosphere approximately one-twentieth of the whole, it follows that a fraction  $20 \times 0.0175 \cot \lambda$  of the lowest half-kilometre layer has to be removed for each degree of advance northward.

For each *metre* of advance northward, therefore, a fraction  $\frac{20 \times 0.0175 \cot \lambda}{111.1 \times 10^3}$  of the lowest half-kilometre layer has to be removed ; and, similarly, for each *metre per second* of the wind velocity from south to north a fraction  $\frac{20 \times 0.0175 \cot \lambda}{111.1 \times 10^3}$  must be removed *every second*.

Suppose that the breadth of the advancing current which is supposed to be maintained steady is  $L$  kilometres, the westerly flow at the western end of the lowest half kilometre must carry away air at the rate of  $\frac{20 \times 0.0175 \cot \lambda}{111.1 \times 10^3} \times L$  kilometres per second, or there must be a cross component of wind there amounting to  $\frac{20 \times 0.0175 \cot \lambda}{111.1} \times L$  metres per second.

If the cross wind be referred to the width of a current expressed in degrees of longitude at the latitude  $\lambda$ , and if  $l$  be the width of the current in degrees, we get

$$L = 111.1 \cos \lambda l.$$

Whence it follows that in order to maintain a south-to-north current of  $V$

metres per second there must be a cross wind leaving the lowest half kilometre of  $\cdot35 \frac{\cos^2\lambda}{\sin \lambda} lV$  metres per second.

We have supposed the drainage to take place entirely in the lowest half kilometre, which represents one-twentieth of the atmosphere. The same result might be produced by a distributed cross-flow throughout the western vertical section of the moving air of  $\cdot0175 \frac{\cos^2\lambda}{\sin \lambda} lV$  metres per second.

We may therefore sum up the conclusion as follows:—

*In order that a current across circles of latitude from south to north with a breadth of  $l$  degrees of longitude may persist unaltered at any level, it is necessary that air should be drawn away from the moving air at that level to the extent of  $\cdot0175 \frac{\cos^2\lambda}{\sin \lambda} lV$  metres per second.*

The use of the surface layer, to draw off the excess of air which would otherwise prevent the persistence of a current across circles of latitude, is quite appropriate in the case of currents with a south-to-north component. According to the rider to Law 1, such a current certainly exists, and it only requires its magnitude to be adjusted in order that persistence may be secured. For a current extending over  $10^\circ$  of longitude in lat.  $45^\circ$  the cross component

CROSS SECTION OF 9 KILOMETRES [K TO 10K] OF  
A S. TO N. CURRENT, 5° WIDE MAINTAINED IN LAT. 45°

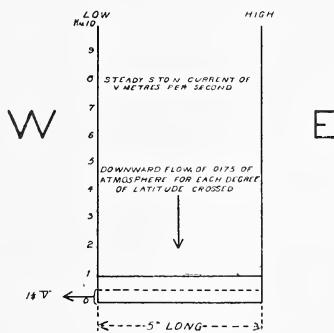


FIG. 1.

at the extreme west of the lowest half kilometre would have to be two and a half times the steady south wind above, and that hardly occurs in practice; but there are a variety of ways of accounting for any discrepancy between the calculated and observed cross-wind in case the south-to-north current is actually maintained. Hence the diagram, fig. 1, representing the conditions for maintenance of a south wind across a section of  $5^\circ$  of longitude is not unreasonable.

The representation is, moreover, borne out by the facts which are known as to the distribution of temperature in the atmosphere. For the seven kilometres between the 1-k. level and the 8-k. level the temperature on the "high" side is "too warm," and therefore represents the effect of a downward flow while the pressure is maintained.\* Hence it seems possible for the conditions for the maintenance of a south-to-north current to be realised in practice, though the adjustment would be delicate and might certainly be transient.

*Motion from North to South.*

Persistence in the reverse of the case just described, that is to say, in the case of a current flowing from north to south, is in one respect more difficult and in another more easy.

What we have to provide for here is not the thickening but the shrinkage of the current in consequence of the decrease of  $\sin \lambda$  as successive circles are crossed. The numerical result applies equally, but in the opposite sense. Thus a current of velocity  $V$  flowing from north to south requires that air should be *fed* with an inflow which, if distributed over the whole side, would be  $0.175 \frac{\cos^2 \lambda}{\sin \lambda} lV$  at any level at which the wind velocity is

$V$ , in order to avoid fractional shrinkage of  $0.175 \cot \lambda$  per degree of advance. It is more difficult to see how the air could be supplied; but the shrinkage of the current while the distribution of pressure which controls it is maintained presents little difficulty if the current in question may be supposed to remain an upper air-current and therefore subject only to the pressure-distribution appropriate to the current. To explain the persistence of a current in the lower layers would make greater demands upon one's ingenuity, because the introduction of the necessary air would, as a rule, alter the distribution of pressure below, and limitations to prevent that alteration would have to be invented. Hence the maintenance of a current from north to south at all levels requires some artifice for the continuous production of the necessary pressure-distribution. The difficulty is further aggravated by the fact that, just as in the case of the south-to-north current, there is a flow-off from "high" to "low" in the surface layers; but unfortunately it flows *away from where it is required* to make up the loss due to change of latitude, and consequently that loss as well as the loss by shrinkage has to be made good if the northerly current is to be maintained.

Putting the two currents side by side as in fig. 2, we see that the supply

\* See the paper in the *Journal of the Scottish Meteorological Society* already referred to.

for the north-to-south current may possibly come from the surplus of the south-to-north current, but it cannot be along the surface. It must be remembered that, so far as our information goes, we have no reason from observations for supposing that the relation between pressure and temperature in a northerly current is different from that in a southerly current, though the evidence is not quite conclusive, because the former has been less frequently the subject of investigation. The air supply ought, therefore, to be carried out in a similar manner in both cases. Persistence in this case, therefore, requires the surplus of the adjacent southerly current and *the outflow from the northerly itself* both to be delivered to the northerly current in the upper layers in order that the proper temperature distribution may be obtained.

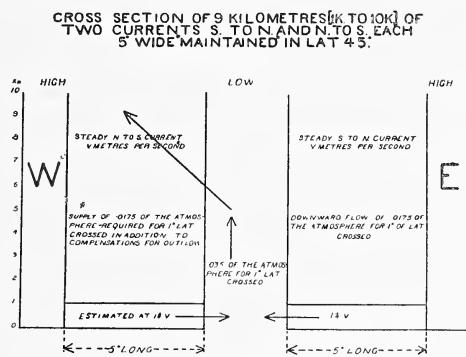


FIG. 2.

Such a combination of circumstances may fairly be regarded as exceptional, and therefore the maintenance of a northerly current must be regarded as exceptional.

#### *Changes from the Steady State.*

To complete the process of maintenance of the steady current from the north we should have to imagine the whole of the outflow in fig. 2 towards the "low" from both sides conveyed to the upper part of the northerly current, and thus transferred from low pressure to high pressure as well as from low level to high level. It is possible to make out a process with the aid of the law of convection if the two currents are at different temperatures. In such a case the surfaces of equal pressure may be so sloped as to produce an apparent flow across isobars from low to high; but we have no such obvious and automatic explanation to give in the case of the northerly current as in the case of the outflow of the southerly current. And, indeed, it was not intended to adduce the conditions for persistent maintenance with the object of claiming that they are generally satisfied in practice. On the contrary, the adjustment of the outflow in the southerly

current to the conditions of persistence must be fortuitous and unlikely to be maintained for long; the adjustment of conditions for the maintenance of a northerly current is even more fortuitous. The reason for setting out the conditions of maintenance is rather to show that natural conditions of atmospheric currents are not, as a rule, those of persistence but of change. If the conditions of persistence which have been set out are not realised, the currents will change, and by Law 1 changes in currents imply changes in the distribution of pressure. Consequently, an atmospheric system which includes northerly or southerly currents has within itself elements and causes of change in the distribution of pressure. It is therefore unnecessary to attribute all changes to outside causes. It is preferable to consider the causes of the changes which are inherent in cases in which we cannot suppose the conditions of maintenance satisfied, and to regard external causes of change which are known to exist as supplementary.

It follows that we have not to regard a quiescent atmosphere all over the globe as the starting-point of our explanation of the present condition, but we have rather to regard the circumstances of transition from one set of conditions to another.

We may add some notes upon practical cases.

#### *Persistent Southerly Current.*

The maintenance of a southerly current has been shown to be a question of adjustment of velocities, and a southerly current lends itself comparatively easily to persistence. Examples of a persistent southerly current across the parallels of Northern Europe furnish a well-recognised type of weather that seems to resist the incursions of cyclones from the west. A southerly current often extends throughout the vertical section of the atmosphere, as might be expected from the automatic thickening described above.

#### *Persistent Northerly Current.*

On the other hand, a northerly current requires constant reinforcement, and yet a northerly current, persistent for days over the North-Eastern Atlantic, is by no means unknown. It is possible that the necessary air in this case may be supplied by the gravitational flow of cold air off Greenland or Northern Siberia, which must contribute a large amount of air to the surface layers above the North-Eastern Atlantic.

#### *Replacement of a North-Easterly Current by a South-Westerly Current.*

An example of the disturbance of persistence frequently occurs in the case of a north-easterly current with a south-westerly current above it, a

case which is referred to in Mr Cave's book on the *Structure of the Atmosphere in Clear Weather* as a frequent precursor of weather of the thunder-storm type, accompanied by the setting in of the south-westerly wind. The distribution of temperature is such as to change the direction of the pressure-gradient near the surface. Consequently the outflow from high to low goes from under the upper "low" to under the upper "high." The necessity for the thickening of the southerly current is therefore not relieved by the outflow, but accentuated thereby. At the same time the north-easterly current has to get thinner, so it is gradually replaced by the south-westerly current settling down to the surface. The appropriate redistribution of pressure at the surface accompanies the redistribution of air-currents in the vertical section.

These examples are adduced because it seems not improbable that they give us the opportunity of watching the operation of the causes of change which are inherent in any actual state of atmospheric motion.

Let me summarise the attitude which seems to me to be appropriate for the meteorologist to take up in face of the complexities of the atmospheric circulation, by again referring to the position of the astronomer before the final enunciation of the laws of motion. Imagine the perplexity of the astronomer who, finding the heavenly bodies moving in all sorts of directions with all sorts of velocities, set himself to explain the motion which each possessed. To him the laws of motion bring the assurance that it is not necessary for him to explain why a body moves; it is the changes of motion which should occupy his attention. So the meteorologist, looking at the circulation of the atmosphere in obedience to the distribution of pressure, has not to ask himself why the pressure is high here or low there, but rather, "Is the distribution persistent, and if not, are the causes of change inherent in the existing circulation sufficient to account for the changes?" If it be said that, after all, the problem remains the same and the point of view is immaterial, it is right to remember that in astronomy the change in the point of view has simply reduced chaos to law.

From what has been already said, it appears that a steady state of persistent motion of the earth's atmosphere is in the highest degree improbable, because it can only occur in a combination of circumstances which are independently fortuitous; but it is desirable to call attention to a possible case of motion which is quasi-persistent in consequence of two concurrent and persistent infractions of the conditions of steadiness.

If we suppose the south-to-north and north-to-south currents of fig. 2 placed back to back so as to form an anticyclonic section instead of the cyclonic section represented in fig. 2, we find in juxtaposition a south-

to-north current which must get rid of air, and a north-to-south current which must have air in order to maintain itself, and all that is required in order to maintain both currents is a transverse flow of  $\cdot0175 \frac{\cos^2\lambda}{\sin\lambda} lV$  at any level where the current velocity is  $V$  from the south-to-north current to the north-to-south current. We cannot accept this transverse motion as a part of steady motion, because the motion would not be strictly speaking along the isobars as prescribed by Law 1. But if we could persistently take the momentum necessary for the perturbation of the steady motion in compliance with Law 1 out of the general west-to-east circulation, we

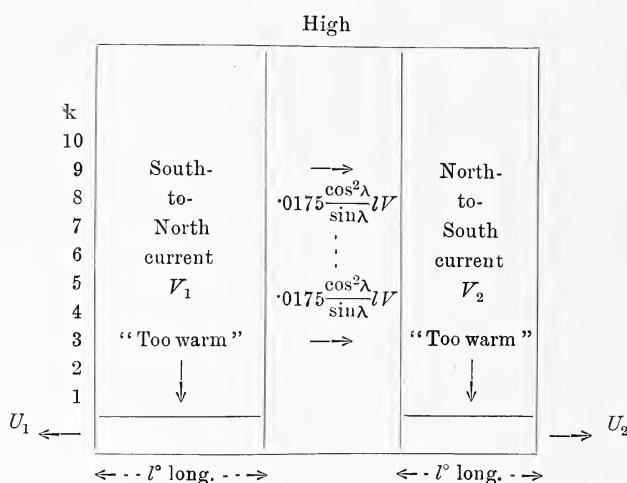


FIG. 3.—South-to-North current  $V_1$  supplying its own bottom outflow  $U_1$  and maintaining a parallel North-to-South current  $V_2$  and its bottom outflow  $U_2$  by transference of air across the "high" ridge.

could have both the southerly and northerly currents maintained. It is not unreasonable to suppose that, as a westerly circulation has to be diverted northward to produce the northward circulation, the westerly momentum at the various levels may produce the effect described. In this case we should have the permanence of the anticyclonic distribution maintained by the persistent infraction of the law of relation of pressure to wind. At the same time a flow-off at the bottom outwards in both cases has to be supplied, and in consequence there is a downward flow under permanent conditions of pressure over both sides of the ridge of "high" which would give the necessary warming of the air of a high-pressure region. Hence the case represented in fig. 3 seems to furnish a possible example of a high-pressure region maintained in a quasi-steady condition by a transfer of air across the isobars in consequence of the

uncompensated momentum; the flow-off on either side at the bottom from "high" to "low" denoted by  $U_1$  and  $U_2$  being provided by the adjustment of the currents  $V_1$  and  $V_2$ .

Whether or not this be a true explanation, it certainly agrees with common experience in regarding a high-pressure area as more easily maintained persistently than a "low."

*Propositions 2, 3, and 4.*

These propositions, which deal with the application of the formula for change of pressure-difference with height (the unit of height being the metre), viz.

$$\frac{d\Delta p}{dh} = 0.342 \frac{p}{\theta} \left( \frac{\Delta\theta}{\theta} - \frac{\Delta p}{p} \right),$$

to explain the dominance of the stratosphere and the lack of importance of the troposphere in the distribution of pressure at the surface, to compute the wind-velocity from the pressure-difference at any height and to explain the observed falling off of wind-velocity with height in the stratosphere, have been dealt with in the paper communicated to the Scottish Meteorological Society, and the work need not be repeated here, especially as Proposition 5 makes use of the same equations.

*Proposition 5.—The Calculation of the Distribution of Pressure and Temperature in the Upper Air from the Observations of Structure represented by Soundings with a Pilot Balloon.*

A pilot balloon gives primarily the horizontal direction and velocity of the wind at successive heights, so that we may suppose that we have the horizontal direction and velocity of the wind at each kilometre as the data for the calculation.

The first step is to resolve the wind-velocity into two components, west to east and south to north.

By the application of Law 1 we can then compute the pressure-difference for 100 kilometres in the south-to-north direction and the west-to-east direction.

Thus, if  $\Delta p$  is the pressure-difference for a distance  $L$  taken along the direction of the wind velocity  $V$ , if  $\theta$ , in absolute degrees, and  $p$ , in millibars, are the temperature and pressure,  $\lambda$  the latitude,  $\omega$  the angular velocity of the earth's rotation, and  $R$  the constant of the characteristic equation for air, we have

$$V = \frac{R}{2\omega \sin \lambda} \frac{\theta}{p} \frac{\Delta p}{L} = K \frac{\theta}{p} \frac{\Delta p}{L}.$$

And since both velocity and pressure-difference, or gradient, are vector  
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quantities, we get for the northward and westward components of the pressure-gradient per hundred kilometres

$$\Delta_s p = \frac{1}{K} \frac{p}{\theta} (\text{W to E})$$

and

$$\Delta_w p = \frac{1}{K} \frac{p}{\theta} (\text{S to N}),$$

where (W to E) and (S to N) indicate the components of the wind-velocity resolved in those two directions.

Now from a pilot balloon ascent we cannot get the value of  $p/\theta$  for the special occasion of the ascent, but there is really little variation from time to time of this ratio. For the greater part of the troposphere variations of pressure and temperature go together, and the whole range of variation of  $\theta$  for any particular time of year is less than 10 per cent., and the whole range of variation of  $p$  is of the same order. Consequently a mean value of  $p/\theta$  may be taken as a first approximation for the purposes of the calculation.

The following is a table of average values of  $p/\theta$ :

TABLE II.—TABLE FOR VALUES OF  $p/\theta$  AT DIFFERENT LEVELS—  
AVERAGE OF RESULTS IN “GEOPHYSICAL JOURNAL,” 1912.

Height, kilo- metres.	$p/\theta$ .						
20	.26	15	.53	10	1.18	5	2.11
19	.28	14	.64	9	1.35	4	2.35
18	.32	13	.75	8	1.52	3	2.61
17	.39	12	.87	7	1.70	2	2.91
16	.46	11	1.02	6	1.90	1	3.24
						Gd.	3.55

Having thus computed the pressure-difference for 100 kilometres, in two directions at right angles, for the level of each kilometre, we may next obtain by subtraction the change of pressure-difference for each kilometre. The use of the mean value for  $p/\theta$  will not altogether invalidate the process, because the variation from kilometre to kilometre depends generally on the ordinary diminution of pressure with height rather than on any extraordinary distribution of temperature.

Substituting the value of the rate of increase of pressure-difference per kilometre of height in the equation

$$\frac{d\Delta p}{dh} = 34.2 \frac{p}{\theta} \left( \frac{\Delta \theta}{\theta} - \frac{\Delta p}{p} \right)$$

and again assuming a value of  $\theta/p$ , we can compute  $\Delta\theta$  provided we have a value of  $\theta$  which can properly be substituted in the equation.

Here again we must have recourse to the mean value, as we have no observation of actual temperature at the time; but, again, the error made is not fatal to the practical success of the calculation, because  $\theta$  comes in as a factor which affects the scale of the variation; it does not affect the sign. By taking the mean value for the month instead of the actual value the error is probably less than 10 per cent., and the whole error of employing mean values for actual values probably amounts to less than 20 per cent.; and in considering the distribution of pressure and temperature in the upper air we are not yet in a position to reject observations and information which may be in error by as much as a fifth.

Consequently we may properly use the calculation here indicated to give at least a rough but working idea of the distribution of pressure and temperature at successive levels in the atmosphere when we know the velocity and direction of the wind there.

The errors in  $p/\theta$  and  $\theta$  are less important in considering the nature of the distribution, because the same values, right or wrong, are used for both components at the same level.

The following table of monthly averages gives values which may be used in the absence of any special information for the particular occasion:—

TABLE III.—AVERAGE TEMPERATURE AT DIFFERENT LEVELS FOR MONTHS.  
1. FOR BRITISH ISLES. TAKEN FROM "GEOPHYSICAL MEMOIRS," NO. 2 (W. H. DINES).

Height, kilo- metres.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
14	216°	217°	219°	221°	222°	223°	222°	221°	219°	217°	216°	215°
13	216	217	219	221	222	223	223	221	219	218	217	216
12	217	218	219	220	221	222	222	221	221	219	218	217
11	217	217	217	219	220	221	222	222	221	220	219	218
10	220	220	220	222	224	225	226	226	226	224	223	221
9	224	223	224	226	229	231	234	233	233	231	228	225
8	230	229	230	232	236	238	241	241	241	238	235	232
7	237	236	237	239	242	245	247	248	247	245	241	238
6	243	243	244	246	249	252	255	255	254	251	249	245
5	250	249	250	252	256	259	261	262	261	258	255	252
4	257	256	257	259	262	265	267	268	267	264	261	258
3	263	262	263	265	268	271	273	274	273	270	267	264
2	267	266	267	270	273	276	278	279	278	275	272	269
1	271	271	273	276	279	282	283	283	281	279	275	272
Gd.	276	276	277	282	285	288	289	289	286	283	280	277

I give in Table IV. a specimen of the calculation as applied to the results of a sounding with a pilot balloon on April 29, 1908.

TABLE IV.—COMPUTATION OF PRESSURE DISTRIBUTION AND TEMPERATURE DISTRIBUTION FROM A PILOT BALLOON ASCENT OF APRIL 29, 1908.

Height in kilo-metres.	Velocity (metres per sec.).	Direction.	$\frac{W \text{ to } E}{K} \times \frac{p}{\theta}$ $K = 25.4$ .	$\frac{W \text{ to } E}{K} \times \frac{p}{\theta}$ $= \Delta_N p.$	Change per k. (increase).	Incr. per k. $\frac{34.2}{34.2} \times \theta.$	$\frac{\text{Incr. per k.}}{34.2} \times \theta.$	$\frac{\left( \text{Incr. per k.} \right)}{34.2} \times \theta + \Delta_N p.$	$\frac{1}{\Delta_N \theta} \times 100.$
k.	m/s.								
6	20.5	300°	+17.75	+.70	1.88	+.26	+.0073	.248	+1.66
5	15.0	300	+12.99	+.51	2.08	+.29	+.0085	.254	+1.55
4	8.5	280	+8.37	+.33	2.33	+.77	+.0035	.261	+.91
3	6.5	265	+6.48	+.25	2.58	+.65	-.0064	.267	-1.71
2	8.0	250	+7.52	+.30	2.90	+.87	+.0091	.272	+2.48
1	5.0	240	+4.33	+.17	3.18	+.54	+.0023	.278	+.64
0	5.0	220	+3.21	+.13	3.50	+.46			
Height in kilo-metres.	Velocity (metres per sec.).	Direction.	$\frac{S \text{ to } N}{K} \times \frac{p}{\theta}$ $K = 25.4.$	$\frac{S \text{ to } N}{K} \times \frac{p}{\theta}$ $= \Delta_W p.$	Change per k. (increase).	Incr. per k. $\frac{34.2}{34.2} \times \theta.$	$\frac{\text{Incr. per k.}}{34.2} \times \theta.$	$\frac{\left( \text{Incr. per k.} \right)}{34.2} \times \theta + \Delta_W p.$	$\frac{1}{\Delta_W \theta} \times 100.$
6	20.5	300	-10.25	-.41	1.88	-.77	-.0044	.248	-1.09
5	15.0	300	-7.50	-.30	2.08	-.62	-.0140	.254	-3.56
4	8.5	280	-1.47	-.06	2.33	-.14	-.0056	.261	-1.47
3	6.5	265	+.057	+.02	2.58	+.05	-.27	.267	-2.11
2	8.0	250	+.274	+.11	2.90	+.32	+.0	.272	0
1	5.0	240	+.250	+.10	3.18	+.32	-.21	.278	-1.70
0	5.0	220	+.384	+.15	3.50	+.53			

I have used this method for the calculation of the distribution of pressure and temperature in the cases represented by photographs of models in Mr C. J. P. Cave's book on the *Structure of the Atmosphere in Clear Weather*,\* which includes that given in detail on p. 100. Some of the results are given below—the problem being understood to be stated thus: *Given the wind-velocity at any point, to find co-ordinates for drawing the isobar for the next higher millibar and the isotherm for the next higher degree of temperature.* It will be remembered that the isobar over the point of observation itself is to be taken parallel to the wind direction in accordance with Law 1, and the direction of the isothermal lines will be taken parallel to the line joining the computed co-ordinates, so that the distribution of pressure and temperature is to be represented each by two parallel lines, the co-ordinates giving their direction and their distance apart.

#### 1. SOUNDING OF MAY 5, 1909, 6H. 43M. P.M.

*“Solid Current”:* Wind approximately uniform in direction and velocity from 2 kilometres to 10 kilometres.

TABLE V.

Height. k.	Distance of next higher isobar in kilometres.		Distance of next higher isotherm in kilometres.	
9-10	143 N	233 E	93 N	93 W
8-9	143 N	181 E	1000 N	1250 E
7-8	123 N	291 E	454 S	54 E
6-7	114 N	292 E	137 N	74 W
5-6	99 N	141 E	100 S	139 W
4-5	77 N	110 E	832 N	58 E
3-4	67 N	187 E	303 S	909 W
2-3	58 N	144 E	769 N	196 W
1-2	54 N	353 E	270 N	49 E
0-1	...	...	...	...

In this case it is interesting first to notice the gradual separation of the isobars with increasing height and consequently diminishing density. This is the ordinary condition for the velocity remaining invariable with height.

Secondly, it is noteworthy that the separation of the isotherms is generally large and also very irregular, showing approximate equality of temperature in any layer, but great want of conformity between one layer and another. Such variations in the distribution of temperature may easily be accounted for by local convection producing changes of temperature and possibly clouds, and it leads us to reflect that the convection

\* Cambridge University Press, 1912.

which produces local clouds will also produce local modifications of temperature and consequently local modifications of pressure and wind velocity. If we ask whether such local variations of temperature and wind are at all probable, we have only to refer to the records of the ascents of registering balloons and of anemometers, or of pilot balloon ascents, to give an affirmative answer.

Nothing is more noteworthy than the irregular variations in temperature-difference as given by a pair of soundings with registering balloons, and the curious local irregularities of wind disclosed by pilot balloon ascents. Hitherto it has been customary, on quite general grounds, to regard them both as possibly due to the uncertainties of observation. We now see that they may equally well be important evidence of complication in the structure of the atmosphere.

Those whose temperament inclines them that way have still the possibility of uncertainties in observation to fall back upon; but the better plan would seem to be to arrange for simultaneous ascents of registering balloons and pilot balloons, so that the actual and computed distribution of temperature may be compared. The interesting feature of the comparison would be that, if the method of computation here indicated (with its acknowledged uncertainties in taking mean values for  $p/\theta$  and  $\theta$  instead of actual values) should prove serviceable, then one pilot balloon ascent gives for practical purposes almost as much information as three registering balloons.

Apart from the uncertainties which have been mentioned, the conclusions as to the distribution of temperature and pressure are incontrovertible by those who accept Law 1, and *per contra* if the conclusions are sustained Law 1 receives its most complete vindication.

## 2. SOUNDING OF SEPTEMBER 1, 1907.

*Westerly Wind rapidly increasing aloft.*

TABLE VI.

Height. k.	Distance of next higher isobar in kilometres.		Distance of next higher isotherm in kilometres.	
	k. 68 S	k. $\infty$ E or W	k. 86 S	k. 119 E
4	77 S	400 W	44 S	555 E
3	139 S	294 W	119 S	185 W
2	196 S	526 W	43 S	80 E
1				

The increase in the intensity of the pressure-distribution with height is clearly shown, and finds its explanation in a steep temperature gradient from south to north.

## 3. SOUNDING OF NOVEMBER 6, 1908, 10H. 55M. A.M.

*Reversal of Direction from E.S.E. in the lowest three kilometres  
to W.N.W. in the reach from 4 kilometres to 9 kilometres.*

TABLE VII.

Height. k.	Distance of next higher isobar in kilometres.		Distance of next higher isotherm in kilometres.	
8-9	185 S	356 W	96 S	312 W
7-8	204 S	356 W	416 S	770 W
6-7	200 S	416 W	294 S	189 W
5-6	233 S	435 W	139 S	625 E
4-5	344 S	665 W	101 S	109 W
3-4	5000 S	4000 E	119 S	270 W
2-3	588 N	416 E	286 S	108 W
1-2	100 N	142 E	24 S	65 W
0-1	77 N	172 E	34 N	40 E

The gradual diminution of velocity up to 4 kilometres, where the isobars become very wide apart, is well marked in the second and third columns; and it is seen that the reversal is to be accounted for by a rapid rise of temperature to the south-west in the second and third kilometres, with a similar distribution of temperature of less marked character in the higher layers.

It will be noticed that in the second and third kilometres, where the reversal is determined, the slope of temperature is opposite to the slope of pressure, a condition which we have already noticed as being characteristic of large change of pressure-difference with height. In the sixth kilometre the next higher isotherm is found a long way off on the east instead of on the west, as in the layers above and below. The change is not really very large, as the temperature conditions are nearly uniform in that region as regards the west-to-east direction, but it furnishes a reminder of the close association which we must expect to find between slight changes in temperature distribution and in the direction and force of the wind.

## 4. SOUNDING OF APRIL 29, 1908.

*North-Westerly Current in the Upper Layers crossing a Lower Current from the South-West.*

This is the example of which the details of the working are shown in the table on p. 100, and it is one of great interest, because it is characteristic

of the advance of a well-developed cyclonic depression from the westward. It has long been recognised, by seamen and other observers of weather, in observations of upper clouds which are seen to be moving from the north-west while the surface winds are coming from the south-west. It is one of the surest signs of the rainfall which occurs in the front of a cyclonic depression. The table already given shows the values of  $\Delta_N p$  and  $\Delta_W p$  for each kilometre level, and the values of  $\Delta_N \theta$  and  $\Delta_W \theta$  computed from the changes in the pressure-differences for successive kilometre steps. We may note here an ambiguity of notation, which we ought to find some means to remove, and which ought at least to be made clear. In the table  $\Delta p$  and  $\Delta \theta$  are used to indicate the *slope* of pressure and of temperature in the two directions N. and W. Thus in the table, when  $\Delta p$  or  $\Delta \theta$  is positive for a given direction, it is to be understood that it represents the *fall* of pressure in that direction. But the usual convention of the differential calculus is that an *increase* in the quantity represented is indicated by a positive value of the difference. The ambiguity arises from the use of the convenient symbol  $\Delta$  to denote the difference, while the meteorological practice is to think of gradient as represented by downward slope. I have not found any convenient new symbol to use instead of  $\Delta$  to indicate a negative difference, so the ambiguity remains for the present, though I feel that an apology is due for it.

In order to present in a table the corresponding values of  $\Delta p$  and  $\Delta \theta$  for the same level, I have taken the means of the two values of  $\Delta p$  for the top and bottom of the kilometre to which  $\Delta \theta$  refers. This practice is, perhaps, rather doubtful, but except in Table VI. it has been followed in the tables already given, so I adhere to it in this one.

Converting by simple inversion the figures for  $\Delta p$  and  $\Delta \theta$  per 100 kilometres into distances along the axis of the intercepts of the next higher isobar and isotherm respectively, we obtain the following:—

TABLE VIII.

Height. k.	Distance of next higher isobar in kilometres.			Distance of next higher isotherm in kilometres.	
	k. 84 S	k. 143 W		k. 60 S	k. 102 W
5-6	109 S	263 W		64 S	50 W
4-5	141 S	2000 W		135 S	132 W
3-4	131 S	526 E		244 N	125 W
2-3	141 S	312 E		93 S	909 E
1-2	200 S	232 E		270 S	222 W
0-1					

In this table the gradual conversion of a southerly component into a northerly component associated with higher temperature to the westward is very noticeable.

It will be seen that the isobars above 4 kilometres are, roughly speaking, at right angles to those in the lowest kilometre, which is, of course, in accordance with the wind observations; but that the isotherms, with some fluctuations, particularly in the second kilometre, are similarly arranged at the top and at the bottom. That is to say, the upper winds are flowing from the north-west with the higher temperature on the south-west side, while the lower winds are moving transversely from the south-west with a distribution of temperature parallel to that of the upper air, but in this case the isotherms are across the wind.

These results are represented in fig. 4, which was originally drawn to the same horizontal scale as the larger chart of the Daily Weather Report, and it is clear that in the lowest stage the columns of warmer air brought in by the south-westerly current are being carried underneath the parallel columns of the upper current. Up to 4 k., where the wind has become westerly, we have a distribution which produces the same effect. The wind is always carrying warmer air under colder air, and as, by Proposition 1, a southerly current tends to thicken and a northerly current to give way, the pushing under of the warmer air becomes more effective, until instability occurs and rainfall sets in. The irregularities which are shown in the distribution of temperature are probably due to previous convection.

We have here, therefore, the assurance of rainfall conditions as the south-westerly wind pursues its course under the north-westerly in front of the approaching depression. The rainy condition of that part of a depression is thus directly accounted for.

The characteristic rainfall of a cyclonic depression is generally associated with a general convergence of the surface isobars, but this hypothesis is difficult to follow into details, because the convergence is general over the area, while the rainfall is local. The analysis of the conditions of the upper air here set out shows that there is good reason for rainfall in the upper layers, to which the doctrine of general convergence cannot safely be held to apply.

To the examples which are taken from Mr Cave's work, I may add one for October 16, 1913, which was reported to me by Mr J. S. Dines in connection with his work for the branch Meteorological Office at South Farnborough.

On that day, at Pyrton Hill, where the sounding was made, there was

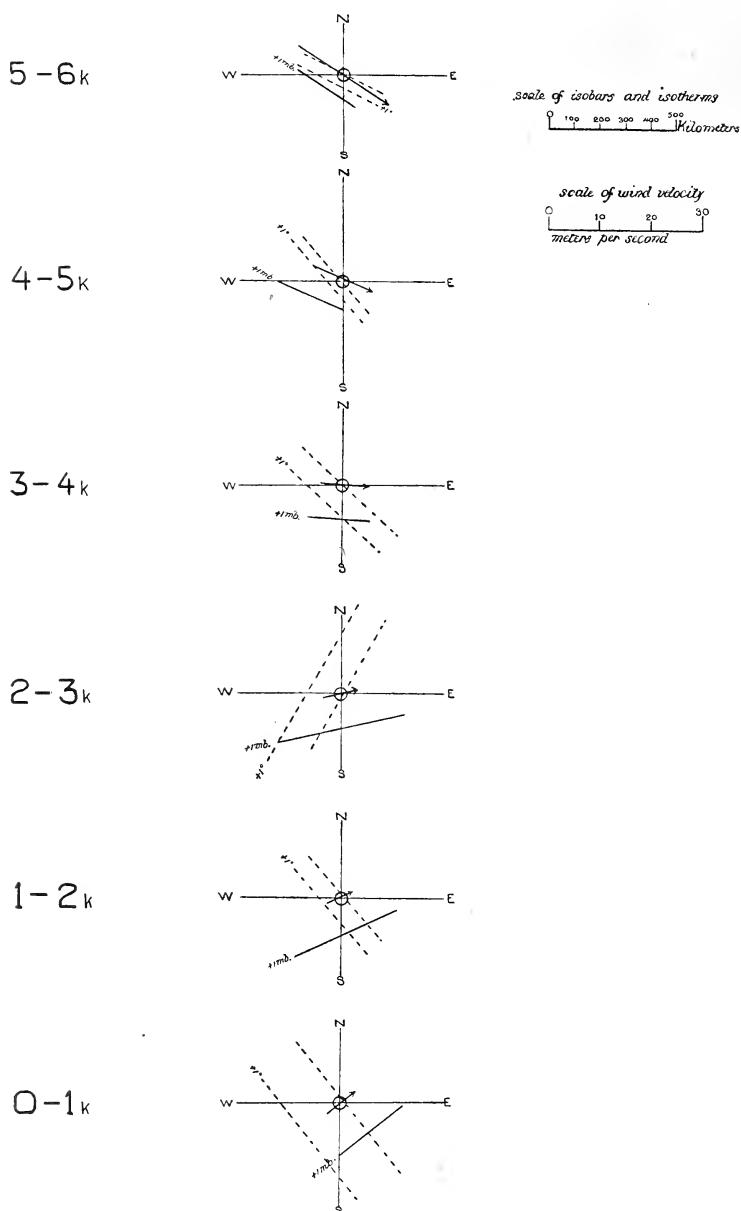


FIG. 4.—Pilot balloon sounding, April 29, 1908. North-west wind over south-west : characteristic of an advancing depression.

The arrow shows the direction and velocity of the wind ; the full line, the position of isobar *next above* that which passes through the station. The dotted line through the O shows the isotherm passing through the station ; the parallel dotted line, the isotherm for *one degree higher* than that of the station.

a sudden change of wind between 1100 and 1500 metres height from a reasonably steady wind from nearly due south into one almost as steady from due north, the change being accomplished within half a kilometre. The analysis in this case shows for the layer between 500 and 1100 metres a temperature distribution in isotherms nearly north and south with the warmer air on the east, and above 1500 metres an entirely different distribution with isotherms nearly east and west, and cold to the northward. The intermediate layer, 400 kilometres thick, showed a very rapid increase of temperature to the west—as much as 7° C. per hundred kilometres.

The complete arrest of the northerly current and production of a calm by the annihilation of the gradient between 1100 and 1500 metres is very remarkable, but nevertheless a real fact. The accompanying temperature difference is probably due to a strong temperature “inversion” at a height of about 1500 metres at the place of observation and of 1100 metres at a place 100 kilometres distant to the west. On that occasion it lasted for some time, as it was found an hour afterwards by a second balloon; but it must be remembered that it was a region of no velocity, and therefore the relatively warm and cold airs were not moving. In order to get them away, either convection must take place or a gradient must be created.

*Proposition 6.—The General Circulation of the Atmosphere in the Northern Hemisphere.*

The reasoning in this proposition is more general in form than that of the foregoing propositions. The extension of our knowledge tends more and more to strengthen the conclusion that the proximate cause of the variations of pressure in the region of the British Isles must be looked for in the layer at a height of about 7 to 9 kilometres; it is the layer of maximum wind-velocity just under the stratosphere, and it is also the layer within which must be located a rapid transition of slope of temperature. Below it, as set out in Lemma I., the slope of temperature follows the slope of pressure; above it, the slope is in the opposite sense. The mechanism by which the changes of pressure are produced is unknown; but this much is apparently true, that within the layer referred to, the relation between the pressure and temperature of the air at two places on the same level is that of adiabatic expansion. Above the critical layer where this relation holds, the air in the high-pressure area is “too cold,” and below it, for 5 or 6 kilometres at least, it is “too warm.”\* We may suppose that air becomes “too warm” by the dynamical warming of downward convection, and, perhaps, also

\* See *Journal Scottish Met. Soc.*, 1913, *loc. cit.*

that it becomes "too cold" by piling up under the stratosphere and readjustment of the several layers within the stratosphere, so that pressure on the sample which causes the bulging is reduced, while that over the surrounding regions is increased.\* Radiation is left out of account—whether rightly or wrongly, it is not possible at this stage to say.

The motion of the critical layer is on the average from west to east, but not invariably so, and apparently the temperature-relations which have been described are not dependent upon wind direction. Other phenomena, so far as they have been observed, seem to indicate a similar symmetry, but there is no sufficient evidence for supposing that the phenomena are necessarily centred locally. In fact, according to the distribution of isobars at 4 kilometres computed by Teisserenc de Bort (Lemma II.), the average motion does not differ much from a circulation round the pole which, once set up, might be persistent with little change if it was everywhere adjusted to the barometric gradient. The actual motion, however, certainly does change, and is, in fact, constantly changing.

Let us consider the conditions of Teisserenc de Bort's average isobars and the forces which are available to produce the perturbations of a supposed original circumpolar circulation indicated thereby. I have already remarked that, for such a circulation as that represented by Teisserenc de Bort, the isobars for 4 kilometres may fairly be accepted as applicable at 7 kilometres also, because the changes of pressure-difference between 4 kilometres and 7 kilometres are in ordinary circumstances very slight.

Taking the average map for January, it will be noticed that the isobars at 4 kilometres are clearly not circles round the pole. If they were so, a steady circulation would be a natural conclusion. It has been already postulated in Lemma II. that they are in reality indented ovals or approximate figures-of-eight with the lobes over the Asiatic and American continents and the inward bends over the two oceans. I purpose considering first the effect of convection as a possible cause of the deviation from the circular shape. The shape which we have to explain is exactly opposite of that which is often shown on synchronous charts of the distribution of pressure at the *surface* of the Northern hemisphere in winter, and which has "highs" over the continents and "lows" over the oceans. I remark in the first place that, to derive the figure-of-eight shape from the circular shape, one cannot rely simply upon the nutation of a west-to-east circulation round the pole; one must superpose either a pair of

\* See a note on the Perturbations of the Stratosphere in Publication 202 of the Meteorological Office.

anticyclonic systems, elongated north or south, over the oceans, or a pair of cyclonic systems over the continents, of which we can at present only determine the southern portions; or we might arrive at the actual shapes by adjustments of both kinds. If we assumed positions for the original circular isobars, it would be a simple matter to give numerical values of the superposed anticyclones or cyclones. But the circumpolar circular isobars are hypothetical, and, at the present stage, the numerical work indicated would be unremunerative. Let us assume, however, such an initial circumpolar system, and consider the physical forces which would disturb its motion.

The only force immediately at hand is that of gravity, due indirectly to the cooling of the surface air on the land and frozen sea in the arctic night operating in accordance with Law 3, the law of convection. This may produce a real effect of some magnitude on land-slopes. It is not, I think, necessarily effective over level surfaces, because there is no slope down which the cooled air can flow.

I have always hesitated about the common explanation of the trade-winds and other well-known phenomena based upon the reverse process of surface-heating. Surface-heating and surface-cooling necessarily produce a certain amount of expansion and contraction, but not necessarily any continuous convection current. Convection requires the juxtaposition of warm air and cold air, and, if the region is big enough, the result of surface-heating may easily give rise to a heated volume of air surrounded by isobars and air-currents that prevent any continuous process of general convection. Local convection there would be, but that need only extend high enough up to take up the day's heat. All the main air-currents of the globe have pressure-distributions to guide them. They cannot usefully be called convection currents.

So, if we had, say, a million square miles of level ice round the pole, I cannot see that the cooling of that area need produce any considerable effect upon the distribution of pressure; but if the cooling takes place on slopes, we at once get the force of gravity to help, and one can no more suppose the downward flow of the air to be stopped than the flow of a river to be permanently arrested. Hence there must be in winter a continual flow of air off the great land-areas of the Northern hemisphere if they have any slope. The air-fall off Greenland, for example, must be enormous. Every description by explorers in the Antarctic seems to support the suggestion of a great cold-air cascade from the Antarctic continent. How much flows, and where it flows to, I cannot say; ultimately it must find its way to warmer latitudes by some route or other;

but these air-flows must be a real cause of alteration in the distribution of pressure, and it is to the land-slopes which are losing heat that we may trace an indubitable influence, and therefore a disturbance of the uniformity of circulation. Apart from compensation, a flow-off of 1 metre thickness of air would mean a reduction of pressure by 0·1 millibar.\*

Similar phenomena must of course happen locally, and they are well known in mountainous regions, though we can hardly expect the smaller local examples to show much effect in the distribution of pressure over the globe.

But we may assume that cold land-slopes in winter are the cause of a constant abstraction of air from the lowest layers of the atmosphere in those regions. The cold air flows away by gravity, and since the *surface pressure* is apparently still maintained, the efforts to redress the loss of air have to be carried out in the upper atmosphere and in accordance with its laws; consequently we should expect to find a cyclonic circulation in the level in which the replacement is taking place. The cyclonic circulation may operate to prevent the pressure being made up overhead, but it cannot prevent the cold air from flowing downhill unless the reduction of pressure is enough to reduce the density by as much as the low temperature increases it, and this is a difficult task, for near sea-level it takes more than 3 millibars loss of pressure to make up for a single degree loss of temperature.

Hence we may suppose that the constant drainage of the land-areas would result in the superposition of a cyclonic distribution at high level over them, and the continental lobes of Teisserenc de Bort's isobars for the upper air may well be due to this cause.

But the cause is obviously a very variable one, depending upon the distribution of cloud and other circumstances. Statistically, its effect upon the circulation of the upper air is to exaggerate the pressure gradient for westerly winds over the temperate zones of the continents, and to diminish the gradient northward. Thereby we introduce into the circulation local accentuation of current, which must be disposed of by some dynamical process.

\* The facts which are here represented are sometimes taken as indicating the formation of anticyclones over the Arctic and Antarctic land-areas. When those areas are represented by plateaus 10,000 or 15,000 feet in height, the surface anticyclone may become merely a hypothetical construction supposed to occupy the space which is really occupied by land and not by air at all. To a considerable extent the great Asiatic and American anticyclones depend upon the reduction of observations to sea-level under conditions which can have no real existence. The mountain slope might possibly operate, in the maintenance of a cyclonic circulation in the upper air, much like the hole in the bottom of a basin, and the actual land-surface at the high level might therefore be a region of cyclonic circulation.

The next step in the consideration rests upon the fact that by superposing a cyclonic depression upon the circumpolar circulation we displace a part of that circulation to the southward and reduce the northern part. Taking the case of Teisserenc de Bort's map for January, the westerly run of isobars over America and Asia is about  $10^{\circ}$  to  $20^{\circ}$  of latitude lower than over the oceans, and these two positions of westerly circulation have to be connected by isobars which cross the parallels of latitude, and therefore have a south-to-north and a north-to-south component respectively. Therefore, they can only be maintained persistently under the conditions set out in Proposition 1. Now, it has been shown in the discussion of Proposition 1 that permanence of a quasi-steady character might be realised in the case of an anticyclonic ridge having a south-to-north current on its western side, and *vice versa*, provided that momentum was being taken out of the westerly circulation in order to provide a slight eastward deviation from the isobars setting to the north. Such a case would be fairly represented by the deviation from circular isobars shown over the oceans on Teisserenc de Bort's map for January, and hence the form of those isobars may be arrived at by the influence of a steady flow-off of air down the land-slope of the Arctic regions and the steady deviation of the wind from the direction of the south-west to north-west isobars on the western sides of the oceans in consequence of the momentum of the westerly circulation.

Meanwhile, what happens to the cold air which has run off the land-areas? That has to be steered about by the distribution of pressure in the upper air as modified by any special peculiarities of temperature in the lower regions, and all sorts of complications may arise from this cause. So far as it goes, its density tends to set up high pressure over the regions which it covers, and so to make a slope of pressure southward and cause easterly winds on its southern side. Whenever in a mass of air temperature-fall is in the opposite direction to pressure-fall, great change in the horizontal distribution of pressure underneath is the result, and many of our local variations of pressure may fairly be attributed to the reactions which these cold masses of air offer to the attempt (in the end futile) on the part of the upper air to steer them round the pole from west to east. By their eastward motion these masses of cold air are always reminding us that if left to themselves, without the overpowering guidance of the pressure-distribution of the upper air, they would form a circulation round the pole in opposition to the circulation of the upper air, with which they are in perpetual conflict.

## TURBULENT MOTION.

In the study which has been the subject of the foregoing pages we have always considered the motion of the air to be regulated by a distribution of pressure balanced by the rotation of the earth, except in regard to the surface layer and one other suggested exception when the momentum of the general westerly circulation was invoked. It should here be noted that by this limitation to what may perhaps be called "great circle motion," we are considering almost exclusively the circulation above that half of the earth's surface which is north of the northern tropic and south of the southern one. There is another section of meteorology which has to deal particularly with the region between the tropics where the beginnings of tropical revolving storms are to be found. These storms, which have a diameter of some hundred miles or more, as well as the tornadoes which have a diameter of perhaps a quarter of a mile, belong to the subject of turbulent motion, with which the eddies and whirls that are produced by obstacles on the surface of the ground are also associated. All these phenomena of turbulent motion, important as they sometimes are in real life and death, must be treated in a manner different from that of the present communication.

(Issued separately March 23, 1914.)

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6. COMMUNICATIONS NOT SUBMITTED FOR PUBLICATION, such as Demonstrations of Experiments, Statement of Scientific Problems, etc., may be received by the Council, and may also be selected for Special Discussion. The Council does not undertake to publish any notice of such communications in the Proceedings or Transactions of the Society.

X.—Enzymatic Peptolysis in Germinating Seeds. By Dorothy Court, B.Sc., Carnegie Research Fellow. *Communicated by Professor E. WESTERGAARD.*

(Read December 15, 1913. Revised MS. received February 10, 1914.)

IN a previous paper (*Proc. Roy. Soc. Edin.*, vol. xxxi. p. 342) a method was described for measuring small degrees of enzymatic peptolysis, and in a subsequent paper (*Proc. Roy. Soc. Edin.*, vol. xxxii. p. 251) the conditions were dealt with under which such experiments could be carried out with the greatest possible guarantee of sterility combined with the least interference with the reaction.

The intention was to employ these methods for the purpose of pursuing the main object of research—the activation of zymogens in germinating seeds—and it was accordingly decided to carry through a series of experiments on germinating barley. This material was selected on account of the readiness with which it may be obtained.

The presence of proteolytic and peptolytic enzymes in germinating barley has been previously described by Weis (*C. R. Carlsberg Lab.*, vol. v. p. 127), Vines (*Ann. Bot.*, xvi. 1), and Abderhalden and Dammhahn (*Zeitschrift für physiol. Chemie*, lvii.).

Weis, working with a watery extract of crushed germinated barley, found evidence of proteolytic as well as peptolytic activity, and it was therefore decided to use a similar extract in some preliminary experiments. For this purpose 900 grms. of material were crushed in a mincing machine and extracted with 700 c.c. chloroform water for twenty hours. The liquid was expressed in a hand-press, filtered, neutralised with sodium bicarbonate, and divided into three portions. One of these was made slightly acid (=·2 per cent. lactic acid), one was made alkaline (=·1 per cent.  $\text{NaHCO}_3$ ), and one remained neutral. One gramme of Pepton Roche was dissolved in 10 c.c. of each of these preparations, ·1 c.c. chloroform added, and the mixture incubated at  $37^\circ$ . It was somewhat surprising to find that though the digestion was carried on for several days no deposit of tyrosin was obtained.

At the same time another experiment was carried out with the same material for the purpose of determining the relative activities of the embryo and endosperm of the seed. The embryos were carefully dissected



out, ground with sand and a 1 per cent. solution of Pepton Roche, filtered, and the filtrates digested at 37° C., 1 per cent. chloroform being added. This procedure was also followed out with the residues and with a sample of the whole seed. In each of these cases, as before, a number of the digestions were allowed to remain neutral, while others were acidified and made alkaline respectively. An entirely negative result was obtained from these experiments also.

The experiments described above were repeated several times with different samples of material, the digestions were carried out within a wide temperature range (15°, 25°, 37°, 50°), and the period of incubation was extended to as much as three weeks. It thus became obvious that an invariable negative result could not be due to any accident, but to the absence of a peptase capable of splitting off tyrosin from Pepton Roche. It was therefore decided to carry through a final experiment in order to investigate the matter fully.

For this purpose a sample of germinating barley was ground up in a mincer, extracted for twenty-four hours with chloroform water, and pressed in a hand-press. The liquid was freed from suspended particles by means of a centrifuge. This extract will, in the following pages, be referred to as Extract A.

The residue from the press was then ground with sand and kieselguhr, with the addition of a little water, in the Buchner mortar, and then subjected to a pressure of 300 kg. per sq. cm. in the Buchner hydraulic press. The liquid obtained in this way, and freed from solid matter as before, will be referred to as Extract B.

Twelve flasks were made up, each containing 5 c.c. 20 per cent. Pepton Roche solution and 5 c.c. Extract A, while another twelve were similarly prepared with Extract B, 1 c.c. chloroform being added to each as anti-septic. Three of each series were digested at each of the following temperatures—15°, 25°, 37°, 50°. At the same time a similar number of flasks containing 5 c.c. 10 per cent. Pepton Witte solution instead of the Pepton Roche were placed at the same temperatures, a series of controls being prepared for the latter experiment by precipitating the material at once with excess of tannic acid. The digestions were examined from day to day, and whenever a deposit was found it was filtered off and the identity of the tyrosin established by means of Mörner's reaction, the corresponding Pepton Witte digestions being precipitated with tannic acid at the same time.

The first deposits of tyrosin were produced within six days in the digestions containing Extract B, at 25° and 37° respectively; the next ones

being formed after a period of fourteen days in the Extract B digestions at 15°. The remainder of the digestions gave negative results after three weeks' incubation, when the experiment was discontinued and the Pepton Witte digestions precipitated. The filtrates from these were used in the manner described by Weis (*l.c.*) for determining the degree of peptolysis which had taken place during incubation, expressed in terms of cubic centimetres of N/10 alkali, this latter figure representing the ammonia formed during the determination of the nitrogen contained in 5 c.c. of the filtrate, by Kjeldahl's method.

The results of the experiment may be seen in the following tables:—

EXTRACT A, WITH					
1. <i>Pepton Witte.</i>					
Temperature of Digestion.	Titrations.	Average.	Control.	Difference (indicating enzyme action).	
15°	1. 8·9, 2. 9·1, 3. 9·5	9·16	14·9	5·74	
25°	6·0, 5·5, 5·6	5·7	15·3	9·6	
37°	6·1, 6·1, —	6·1	15·3	9·2	
50°	8·3, 8·4, —	8·35	15·5	7·15	

2. *Pepton Roche.*

The whole of this series of digestions gave negative results.

EXTRACT B, WITH					
1. <i>Pepton Witte.</i>					
Temperature of Digestion.	Titrations.	Average.	Control.	Difference (indicating enzyme action).	
15°	1. 13·25, 2. 12·55, 3. 12·55	12·78	15·8	3·0	
25°	7·7, 7·4, 7·1	7·4	16·8	9·4	
37°	7·2, 8·2, 6·65	7·35	16·85	9·5	
50°	10·15, 9·0, —	9·55	16·0	6·45	

2. *Pepton Roche.*

15°. Positive result observed after fourteen days.  
 25°. Strongly positive result within six days.  
 37°. Positive result also within six days.  
 50°. Negative result.

The total result of the experiment may, for the sake of comparison, be expressed as follows:—

RESULTS OF PEPTOLYSIS.

Temperature.	Pepton Witte.		Pepton Roche.	
	A.	B.	A.	B.
15°	5·74	3·0	Negative.	14 days.
25°	9·6	9·4	"	6 "
37°	9·2	9·5	"	6 "
50°	7·15	6·45	"	Negative.

These results seem to indicate the presence in germinating barley of two different peptolytic enzymes, one of which can be readily extracted with water, while the other is apparently of the nature of an endo-enzyme and can only be obtained by destroying the cells of the seed tissues. The existence of these two enzymes is further indicated by the fact that their temperature curves differ materially. The optimum temperature for both seems to be between 25° and 37°. At 50°, however, while the hydrolysis of Pepton Witte proceeds vigorously, being considerably more marked than at 15°, the action on Pepton Roche seems to be inhibited, since no separation of tyrosin has ever been observed at this temperature. On the other hand, a slow but quite distinct action takes place at 15°.

The inhibition of the Pepton Roche digestion at 50° was further accidentally demonstrated in this way. A number of digestions which had been incubated at 15° and 50° for six days, with a negative result, were put aside and overlooked for a couple of weeks. It was then found that those which had been at 15° for the whole period gave a distinct deposit of tyrosin, while those which had previously been exposed to a temperature of 50° showed no such deposit. Apparently the Pepton Roche digestion is not only prevented at 50° but the activity completely destroyed, while, as has been previously demonstrated, the hydrolysis of Pepton Witte proceeds vigorously at this temperature.

The digestions were all examined for the presence of moulds or bacteria, partly by microscopic examination and partly by adding a drop of the material to sterile meat-extract gelatine and incubating at 20°. This examination invariably showed the absence of any development of bacteria or fungi. In a few cases only, an isolated *Penicillium* spore seemed to have survived.

For the sake of certainty on this point another experiment was devised. The barley was crushed, mixed with sand and kieselguhr, and ground in

the Buchner mortar. A suitable quantity of Pepton Roche solution was then added and the resulting mass subjected to a pressure of 300 kg. per sq. cm. in the Buchner press. The expressed liquid was freed from suspended particles by means of the centrifuge, and then passed through a Chamberland filter into sterilised Pasteur flasks, which were afterwards placed at the same temperatures as were employed before.

The results obtained from this experiment were similar to those obtained before with regard to the separation of tyrosin—a strongly marked reaction at 25° and 37°, a less marked but distinct reaction at 15°, and no reaction at 50°.

Part of the contents of the flask which had been incubated at 25°, and which had given the heaviest deposit of tyrosin, were transferred to a Pasteur flask containing sterilised glucose-Pepton Witte solution and further incubated at 25°. The contents of the flask were found to remain sterile throughout the whole period of incubation, which extended over several weeks, proving conclusively that the peptolysis was not due to any development of micro-organisms.

The presence in germinating barley of two distinctly different peptases having been thus established, the next step in the main research became that of ascertaining at what period the activation of the above-mentioned peptases takes place, in order that the conditions influencing the activation might be finally studied in detail.

For the purpose of elucidating this point, the peptolytic activity was determined from time to time in a sample of barley during germination and the results confirmed, firstly, with another quantity of the same sample, and secondly, by repeating and extending the experiment with a different sample.

The examination was in each case commenced with the ungerminated barley, and was continued in the first two instances for seventeen days, in the last instance for twenty-nine days. The germination took place under the conditions usually observed in the preparation of malt, and the samples were examined at intervals of from two to four days.

The water content was determined in every sample withdrawn for examination, by placing 5 grms. of the ground material in a weighing bottle, covering it with absolute alcohol, and drying it in a hot-water oven for twenty-four hours.

500 grms. of barley were disintegrated in a mincing machine and ground in a Buchner mortar with 500 grms. of sand and a suitable quantity of water to make a firm paste. This was placed in a Buchner press and subjected to a pressure of 350 kg. per sq. cm. for about one hour, when no

more liquid could be expressed, and the extract was finally made up with distilled water to 350 c.c.

The liquid was freed from suspended particles by means of a centrifuge, and was thereafter divided into two portions, one of which was mixed with a solution of Pepton Witte in such quantities as to give a concentration of 2 per cent. peptone, whilst 10 per cent. of Pepton Roche was dissolved in the other.

The liquids were then placed, in quantities of 10 c.c., in a number of small bottles, each of which received in addition 1 c.c. of chloroform. Excess of tannic acid and a trace of sodium acetate were added to half of those bottles containing Pepton Witte, while the remainder of these and all those containing Pepton Roche were placed in an incubator at 35° C.

The Pepton Witte digestions were withdrawn after forty-eight hours' incubation, precipitated with tannic acid and sodium acetate, and, along with the controls, which were precipitated before digestion, filtered and used for determining the amount of nitrogen contained in 5 c.c. of the filtrate, Kjeldahl's method being employed.

The increase in nitrogen, expressed in cubic centimetres of N/10 alkali, was, as before, taken as an indication of the amount of peptolytic activity during digestion.

This difference was, in all cases where ungerminated barley was employed, within the limits of experimental error, and the resting seed of barley may therefore be regarded as containing only in extremely small quantities, if at all, a peptase capable of catalysing the hydrolysis of the polypeptides contained in Pepton Witte.

A peptase of this nature was, however, found to be rapidly produced during germination, as will be seen below.

In the following tables, the first column shows the number of days from the time when the barley was steeped in water to the time when the sample was withdrawn for examination, while the second column gives the degree of peptolytic activity expressed in cubic centimetres of N/10 acid neutralised by the ammonia formed during the Kjeldahl process, and in each case corrected for the amount of moisture contained in the sample.

The Pepton Roche digestions were examined from day to day, and a note was made of the minimum number of days within which a deposit of tyrosin was formed.

Although there is no experimental evidence to show that under the conditions of the experiment, and especially considering its duration, the time required to produce a precipitate is inversely proportional to the degree of activity, it is nevertheless obvious that, the greater the peptolytic

activity, the more rapidly will the tyrosin be precipitated, and *vice versa*. The figures obtained in this way are therefore sufficient indication of the degree of activity, provided that they are expressed in a manner capable of direct comparison, and they are therefore, in the following tables, expressed in terms of a unit which, under the conditions of the experiment, would produce the first indication of tyrosin in 100 days. The amount of moisture contained in the samples has here, as in the case of the Pepton Witte digestions, been taken into consideration, the figures shown being calculated for dry material. These results are given in the third column.

TABLE I.

Number of days.	Pepton Witte.	Pepton Roche.
0 } in water	0·00	2·80
4 }	4·13	8·56
6	7·59	8·25
8	11·60	20·45
10	11·60	37·40
13	11·20	45·30
15	11·0	58·70
17	11·32	34·60

TABLE II.

0 } in water	0·00	2·45
4 }	6·24	7·88
6	9·16	12·40
10	11·50	24·90
12	12·05	24·40
13	14·60	43·35
15	13·27	34·60
17	13·48	24·06

TABLE III.

0 } in water	0·00	2·20
4 }	4·49	6·30
6	7·83	7·40
8	9·77	16·76
10	9·83	24·30
12	9·81	35·00
15	14·10	42·50
17	12·75	75·40
19	12·15	41·20
20	10·55	39·40
22	11·22	38·95
24	11·12	Lost.
26	13·72	31·57
29	12·84	25·50

It is obvious from the whole nature of the experiment, and from the manner in which the different samples were obtained, that a certain amount of irregularity must be expected in the results. Such fluctuations, however, have not been found to be nearly so serious as was anticipated. The fact that the maximum degree of activity is reached somewhat later in the third than in the first and second experiments is easily explained by the fact that an entirely different variety of barley was used, while the other irregularities are so small that they cannot obscure the evidence of the experiments, to the effect that, of the two forms of activity, the one rises sharply from nothing in the ungerminated seed till it reaches its maximum, after which it remains fairly constant during the remainder of the experiment. On the other hand, the activity in the second case rises comparatively slowly from slightly above zero in the ungerminated seed till it reaches its maximum a few days later than in the former case, after which it rapidly falls again.

The presence of both of these forms of activity in germinating barley having been thus demonstrated, it seemed desirable to investigate the existence of similar enzymes in material of widely different origin. For this purpose the strongly proteolytic and peptolytic enzyme, Bromelin, contained in the juice of the fruit of *Ananassa sativa*, was selected. In order to make the experiments more complete, it was decided to carry out parallel digestions, using as substrate in one case the alcohol-soluble protein of wheat, and in the other a solution of Pepton Witte.

The digestions were carried out partly in presence of the natural acidity of the juice, partly with a juice that had been neutralised, and partly with a portion made slightly alkaline with sodium bicarbonate. In each case 5 c.c. of the juice was employed, and three digestions were carried out with each substrate and each reaction.

The following series of digestions were accordingly prepared:—

- (a) 2 grms. protein + 5 c.c. water + 5 c.c. juice.
- (b) 5 c.c. 4 per cent. Pepton Witte solution + 5 c.c. juice.
- (c) 5 c.c. 10 per cent. Pepton Roche solution + 5 c.c. juice.

As usual, half of the digestions (a) and (b) were precipitated before digestion with tannic acid; the others, along with (c), being digested at 37°. After twenty-four hours (a) and (b) were withdrawn and precipitated, the nitrogen contents of the filtrates estimated, and the amount of peptolysis expressed as before. The results are given in the following table, and show that a very strong proteolysis and peptolysis had taken place. The corresponding Pepton Roche digestions, however, showed no deposit of tyrosin, even after three weeks' incubation, and it is therefore safe to conclude that bromelin does not decompose this polypeptid. Whether an enzyme capable of doing

so is present in the cells of the fruit, and might be extracted by the Buchner method, was not determined at the time.

## JUICE OF ANANASSA SATIVA ON

## 1. Protein.

Reaction of Medium.	Titrations. [c.c. N/10 NaOH]	Average.	Control.	Difference (indicating enzyme action).
Acid . . . . .	2·5, 2·55, —	2·52	8·5	6·0
Neutral . . . . .	6·6, 7·0, 5·7	6·46	8·1	1·64
Alkaline . . . . .	6·8, 6·6, 5·7	6·4	8·6	2·2

## 2. Pepton Witte.

Acid . . . . .	2·4, 2·4, —	2·4	5·6	3·2
Neutral . . . . .	4·4, 4·1, —	4·25	5·9	1·65
Alkaline . . . . .	5·2, 5·5, 6·0	5·56	6·0	·44

## 3. Pepton Roche.

The whole of this series gave negative results.

About the same time a number of fungi were gathered, ground to pulp with sand in a mortar, and the juice pressed out in the hand-press. The preparations thus obtained were used in a similar series of experiments, the only difference being that the digestions were in this case confined to the natural reaction of the extracts, in all cases slightly acid.

The results were as follows:—

Fungus.	Protein Digestions.				Pepton Witte Digestions.			
	Titrations.	Aver-	Con-	Diff.	Titrations.	Aver-	Con-	Diff.
Lycoperdon gemmatum	9·2, 9·35, 9·15	9·23	9·45	·22	5·6, 5·3, 5·2	5·36	8·4	3·0
Hyphaloma capnoides	8·75, 8·95, 8·85	8·85	9·5	·65	5·8, 5·9, 5·7	5·8	8·5	2·7
Hyphaloma trichaloma	9·15, 9·0, 9·2	9·12	9·4	·28	6·1, 6·1, 6·1	6·1	8·4	2·3
Russula emetica	9·1, 9·2, 9·1	9·13	9·3	·17	6·8, 6·7 —	6·75	8·6	1·85
Boletus badens	8·9, 8·7, 8·9	8·83	9·3	·5	5·3, 5·5, 5·4	5·4	8·3	2·9
Laccaria laccata	10·0, 9·85, 9·6	9·8	9·85	·05	6·9, 6·7, 7·0	6·86	8·8	1·94
Hydnnum repandum	9·3, 9·1, 9·2	9·2	9·3	·1	7·0, 6·6, 6·4	6·6	8·7	2·1
Amanita rubescens	7·0, 7·6, 7·5	7·36	8·9	1·54	3·9, 4·0, 4·15	4·0	7·7	3·7
Amentopsis strangulata	7·7, 7·65, 8·0	7·78	8·9	1·12	4·7, 4·7, 4·5	4·63	7·8	3·17

## Pepton Roche.

The whole of these gave negative results.

From this it will be seen that a distinct peptolytic action is found in all cases on Pepton Witte, and a slight action in some cases on the protein. The result of the Pepton Roche experiment remained negative after several weeks. Neither was it determined in this case, however, if the cell contents obtainable by the Buchner process would be capable of hydrolysing Pepton Roche, as the small quantities of the material available did not allow of the use of this method. It would, however, seem probable that a considerable proportion of the cell contents must have been liberated during the grinding, since sand was employed, and since the tissue of these fungi is by no means difficult to disintegrate. It would therefore seem almost safe to assume the entire absence of this enzyme in all the cases in question.

For further information, a series of experiments was carried out with the ordinary cultivated mushroom, which can be bought in quantities. The preparations were made as in the case of barley, Extract A being obtained by grinding in an ordinary mortar and expressing in a hand-press, while Extract B was obtained from the residue by the Buchner method. The following results were obtained :—

AGARICUS CAMPESTRIS.—EXTRACT A.					
Substrate.	Titration.	Average.	Control.	Difference.	
Protein . . . . .	5·3, 5·4, 5·3	5·36	6·8	1·44	
Pepton Witte . . .	1·7, 1·9, 1·5	1·7	6·0	4·8	
Pepton Roche . . .		Positive result obtained within 24 hours.			

EXTRACT B.					
Substrate.	Titration.	Average.	Control.	Difference.	
Protein . . . . .	8·8, 9·2, 9·15	9·05	9·2	.15	
Pepton Witte . . .	6·8, 6·4, —	6·6	7·3	.7	
Pepton Roche . . .		Positive result obtained only after 7 days' digestion.			

These results show at once the presence of a tyrosin-separating enzyme, and also that this, as well as the other forms present in this material, had been extracted in the first pressing, the amount remaining in the residue being doubtless removable by washing. To obtain confirmation of the presence of this enzyme in *Agaricus*, another quantity was ground with sand and kieselguhr and expressed in the Buchner press. Digestions similar to those just described were carried out with the following results :—

AGARICUS CAMPESTRIS—EXTRACT ON					
	Titrations.	Average.	Control.	Difference.	
Protein . . . .	5·9, 6·0, 6·1	6·0	7·3	1·3	
Pepton Witte . . .	2·8, 2·3, 2·35	2·5	6·3	3·8	
Pepton Roche . . .		Positive result—marked.			

A number of experiments similar to those just described have also been carried out, using as material *Saccharomyces cerevisiae*, *Penicillium glaucum*, *Aspergillus niger*.

In the case of *Saccharomyces cerevisiae* three experiments were carried out:—

I. Washed and pressed yeast was extracted for twenty-four hours with chloroform water, and the filtered liquid allowed to act on the protein, Pepton Witte, and Pepton Roche as before, at 37° C., using the same concentrations as in the previous experiments. Three digestions were carried out with each substrate, and in the case of the protein and Pepton Witte two controls were precipitated before digestion.

The Pepton Roche digestions did not give any precipitate of tyrosin within three weeks, thus indicating the absence of this form of activity in the extract. The other digestions were all precipitated after twenty-four hours' incubation and used for nitrogen determination in the usual manner, with the following results:—

	Titrations.	Average.	Control.	Difference.
Protein . . . .	9·3, 9·1, 9·15	9·18	9·10	.08
Pepton Witte . . .	7·9, 7·6, 7·8	7·77	8·5	.27

II. Washed and pressed yeast was ground with sand and kieselguhr in the Buchner mortar, the liquid expressed in the usual manner and freed from suspended particles by means of a centrifuge. Digestions similar to those described above were carried out, using 2 c.c. of the liquid in each case and adding distilled water to obtain the same concentrations of the substrate.

All the Pepton Roche digestions gave a strong deposit of tyrosin within twenty-four hours.

The other digestions were all precipitated after twenty-four hours' incuba-

tion, and the nitrogen content determined in the usual way. The results were as follows :—

	Titrations.	Average.	Control.	Difference.
Protein . . . .	8·7, 8·6, 8·5	8·6	9·3, 9·1	0·6
Pepton Witte . . . .	4·1, 4·15, 4·2	4·15	8·0, 8·2	3·95

III. A watery extract was first obtained, as in the first experiment, and the residue subjected to the Buchner method process in order to obtain the cell contents. The same conditions were observed as in the previous experiments, with the difference that the digestions were carried out with Pepton Witte and Pepton Roche only and that four temperatures were employed, viz. 15°, 25°, 37°, 50°.

The results will be seen from the following table :—

EXTRACT A, WITH PEPTON WITTE.				
Temperature.	Titrations.	Average.	Control.	Difference.
15°	7·9, 7·4, 8·2	7·83	7·9	·07
25°	8·3, 8·5, 7·9	8·23	8·5	·27
37°	8·0, 8·0, 8·0	8·00	8·0	·00
50°	7·9, 8·0, 8·0	7·97	8·1	·13

EXTRACT A, WITH PEPTON ROCHE.				
All results were negative.				

EXTRACT B, WITH PEPTON WITTE.				
Temperature.	Titrations.	Average.	Control.	Difference.
15°	6·0, 6·4, 5·9	6·10	7·60	1·50
25°	2·35, 2·5, 2·8	2·55	7·60	5·05
37°	2·4, 2·0, —	2·20	7·50	5·30
50°	3·6, 3·6, 3·3	3·76	7·40	3·94

EXTRACT B, WITH PEPTON ROCHE.				
15°. Slight positive result within twenty-four hours.				
25°. Strong " " "				
37°. " " "				
50°. Negative result at the end of three weeks.				

In the case of *Penicillium glaucum* and *Aspergillus niger*, pure cultures were developed in sterilised 10 per cent. malt extract in large flasks at room temperature. After several weeks a large growth had taken place,

and all the cultures were in a state of fructification. As much as possible of the liquid was then poured off and discarded, while the liquid remaining amongst the mycelium was expressed by means of the hand-press. This was retained for experiment as Extract A.

The mycelium was then ground in the usual way in the Buchner mortar and pressed in the Buchner press. In this way two liquids were obtained from each of the fungi—a medium, and a mycelium extract. Digestions were made as before with these preparations, using protein, Pepton Witte, and Pepton Roche as substrates. The results may be seen in the following tables:—

PENICILLIUM GLAUCUM—EXTRACT A (HAND-PRESS EXTRACT) ON					
	Titrations.	Average.	Control.	Difference.	
1. Protein . . .	8·3, 8·15, 8·15	8·2	9·45	1·25	
2. Pepton Witte . . .	8·4, 8·7, 8·2	8·46	8·9	·44	
3. Pepton Roche . . .		All negative.			

EXTRACT B (BUCHNER EXTRACT) ON					
	Titrations.	Average.	Control.	Difference.	
1. Protein . . .	6·4, 7·0, 6·7	6·7	8·8	2·1	
2. Pepton Witte . . .	4·8, 5·0, 5·2	5·0	8·6	3·6	
3. Pepton Roche . . .		All negative.			

ASPERGILLUS NIGER—EXTRACT A ON					
	Titrations.	Average.	Control.	Difference.	
1. Protein . . .	8·0, 7·0, 7·5	7·5	9·3	1·8	
2. Pepton Witte . . .	6·2, 6·1, 6·0	6·1	8·9	2·8	
3. Pepton Roche . . .		Negative result.			

EXTRACT B ON					
	Titrations.	Average.	Control.	Difference.	
1. Protein . . .	6·75, 6·9, 6·3	6·65	8·3	1·65	
2. Pepton Witte . . .	2·2, 2·5, —	2·35	6·8	4·45	
3. Pepton Roche . . .		Strongly marked positive result within 24 hours.			

In the apparently complete absence of the Pepton Roche hydrolysing activity in *Penicillium glaucum*, a striking contrast is shown with *Aspergillus*. In view of the close relationship existing between these fungi, it was thought desirable to ascertain if *Penicillium* could be brought to

produce this enzyme. A sterilised solution of sugar and Pepton Roche was made up and infected with a pure culture, and then incubated at 20° for several months. The development was, however, so small that it was quite impossible to carry out any examination of the mycelium, and no deposit of tyrosin appeared in the culture medium.

The results obtained with the watery extracts of *Saccharomyces* are so slight that they must be regarded as being within the limits of experimental error. It is, however, possible that more active preparations could have been obtained by prolonged extraction or by addition of sodium chloride as suggested by Vines (*Ann. Bot.*, vol. xviii. p. 289, 1904). The results are, however, in conformity with the statement made by that author, to the effect that a rapidly prepared watery extract of yeast has no proteolytic action, and with those of Geret and Hahn (Buchner, *Die Zymasegährung*, 1903, p. 287) that the Buchner extract possesses a much stronger proteolytic action than that exhibited by the living yeast towards its substrate, and that the proteolytic and peptolytic activity is due to a cell enzyme.

Whether this enzyme, as suggested by Geret and Hahn (*l.c.*), is of the nature of a tryptase has already been rendered doubtful by the works of Bokorny (Beihefte, *Bot. Centr.*, vol. xiii. p. 235, 1903), who suggested that an enzyme of the pepsin group is also present, and Vines (*l.c.*), who found evidence of the presence of an ereptase.

In this connection, the results obtained with the Buchner extract (Extract B), which are shown in the preceding tables, are of interest.

In the first place, it will be noticed that there is a strong peptolytic activity against Pepton Roche as well as against Pepton Witte. Secondly, the decomposition of Pepton Roche, as in the case of barley, is inhibited at 50°, whereas the hydrolysis of Pepton Witte proceeds vigorously at that temperature. Thirdly, assuming the activity on Pepton Roche to be inversely proportional to the time required for producing the first indication of tyrosin, this activity is, in the case of *Saccharomyces*, much more pronounced as compared with the action on Pepton Witte than in the case of barley, a fact which further supports the view of the non-identity of the two enzymes. Further, the proteolytic activity exhibited by the Buchner extract of *Saccharomyces* is very slight in comparison with the peptolytic activity, a fact which becomes even more striking when the results are compared with the corresponding figures obtained with *Penicillium* and *Aspergillus*. Finally, it would seem highly unlikely that the proteolysis and the peptolysis are catalysed by the same enzyme, as in that case the primary reaction would need to be accelerated in at least the same degree as the secondary one, which is obviously not the case.

Whether the comparatively slight proteolytic activity observed in the present investigation in the Buchner extract of yeast, and previously described by other observers (Geret and Hahn, Vines, etc.), is due to a trypsin or a pepsin is uncertain; but it is fairly evident that the peptolysis is almost entirely due to different agents, and it would further seem highly probable that these agents are similar to the two peptases found in germinating barley, if they are not identical with them.

*(Issued separately March 26, 1914.)*

XI.—A Study of the Curvatures of the Tasmanian Aboriginal Cranium. By L. W. G. Büchner, Victorian Government Research Scholar in the Anthropology Department of the University of Melbourne. *Communicated by Professor R. J. A. BERRY.* (With Three Folding Tables.)

(MS. received December 9, 1912. Read January 19, 1914.)

THE extinction of the Tasmanian aboriginal in 1876 closed, for all practical purposes, the further scientific study of this ancient and highly interesting race, and it appeared almost certain that our knowledge of this people would remain dependent on the earlier works of those who were fortunate enough to have studied them during life, and on the few remains housed in such fortunate centres as London, Paris, Edinburgh, Oxford, and Cambridge.

Fortunately, just at the moment when it seemed most improbable that any further specimens of Tasmanian crania would be discovered—the number known to be in existence up to 1909 having been given by Turner as seventy-nine,—Berry and Robertson published in the *Proceedings of the Royal Society of Victoria* (1) and the *Anatomischer Anzeiger* (2) an account of a further discovery of fifty-two. This discovery, important though it undoubtedly was, would not materially have greatly advanced Tasmanian craniology, had not the dioptrograph and diagraph just been invented. By the use of the former ingenious and accurate instrument, Berry and Robertson were enabled to record the whole of their fifty-two crania—forty of which were absolutely new to science—in such a way as to make any craniological investigations on these skulls available in any part of the world.

The great importance of this method was immediately realised, amongst others, by Professor Sergi of Rome, who hastened to avail himself of this unexpected increase in the wealth of Tasmanian material available, in order to study anew the form of the Tasmanian skull by means of his own highly original modes of investigation. The results have been made available to us in his recently published “*Tasmanier und Australier, Hesperanthropus tasmanianus, spec.*” (3).

The publication of Berry and Robertson's Atlas has also made it possible for any investigator to apply any of the recently introduced craniological and morphological methods of skull analysis to the Tasmanian cranium, quite apart from the possession of the skulls or otherwise, and

thus Tasmanian cranial work is no longer confined to those fortunate centres already mentioned, nor is it impossible now to apply modern methods to a race long extinct. It is therefore clear, in view of those enormous advantages, that Berry and Robertson are correct when they say "that all known existing Tasmanian crania, whether in Europe, America, or Australia, ought to be similarly recorded, and thus made available for study in all parts of the world, and for all time."

It will only be by the publication of similar works that any appreciable advance will be made in comparative craniological research. So many and varied methods of examination have been made on the Tasmanian and other crania, that it becomes imperative to secure some suitable method by which all the recorded observations may be referred to one common standard.

It is, therefore, most important that similar works on the European and other races should be published, so that a detailed system of comparative research may be instituted with the Australian and Tasmanian aboriginal crania.

The morphology and general characters of the Tasmanian crania have been the subject of research by such investigators as Barnard Davis (4), Topinard (5, 6), de Quatrefages and Hamy (7), Flower (8), Williamson (9), Wieger (10), Klaatsch (11), Garson (12), Harper and Clarke (13), Duckworth (14), and Turner (15). Still more recently, Berry, Robertson, and Cross (16, 17, 18) have made some important contributions to the subject, and have paid considerable attention to the biometric study of certain cranial observations based on Schwalbe's "form analysis." They selected this system of investigation in order "to determine with some degree of certainty the final position of the Tasmanian with reference to the anthropoids, *Pithecanthropus*, *Homo primigenius*, and *Homo sapiens*, both extinct and recent." They have succeeded, in some measure, in establishing the relative position of the Tasmanian aboriginal with the forms just quoted, by employing this investigational method. In view of these objects, it was absolutely necessary for Berry and Robertson to employ the glabella-inion plane as their working base-line, though they agree with Turner that this glabella-inion plane is not the best "from which to estimate the length of the cerebral part of the cranial cavity," for, in their opinion, the nasion-inion plane coincides more closely with the cerebral length than either the glabella-inion or Turner's nasio-tentorial plane.

As the nasion-inion is, therefore, important as a base-line, and as there is no reason why the present investigation should not employ it, I have directed some attention to it, as also to certain cranial proportions and

indices based on it, and referred to later. Two of these curvature indices, calculated in accordance with the procedure laid down by Schwalbe (19), have already been published by Berry and Robertson. They were estimated by taking the proportion which the length of the chord bears to the length of the arc, the latter being taken as 100. Klaatsch (20), on the other hand, in his work on the Australian and other skulls, estimates these indices of curvature quite differently, and says: "To properly appreciate a sloping forehead, the only part of practical importance is that between the glabella and bregma. The simplest way of determining it, though not employed so far as I am aware, is to measure the greatest distance of the curved surface of the frontal from the glabella-bregma line (*i.e.* the chord of frontal curvature), and to form an index comparing this greatest distance with the length of the glabella-bregma line."

By multiplying the length of the greatest distance of the chord from the arc by 100, and dividing by the length of the chord, he constructs his index of curvature for the ossa frontale, parietale, et occipitale. It will thus be seen that Klaatsch's index expresses the ratio of the maximum distance of arc from chord to the length of the chord, the latter being taken as 100.

I am not at all convinced that the above method will do all that Klaatsch endeavours to claim for it. It will, of course, be admitted that as a method of determining the amount of curvature it fulfils its purpose; but, in my opinion, it fails to express the degree of the recession of the forehead, for, as demonstrated by Schwalbe and others, the sloping forehead can only be estimated by angular measurements on a suitable base-line. It is, therefore, extremely difficult to see how Klaatsch's method of dealing with the chord of the os frontale and its distance from the arc without the use of any base-line whatsoever can express the recession or otherwise of the forehead.

This apart, it is an excellent method of determining the degree of curvature of the bone, and is probably preferable to Schwalbe's method, though, it may be noted, the degree of curvature of any cranial bone can now be estimated directly by means of Mollison's cyclometer.

Turner (21) and Cunningham (22) have also estimated the curvatures of various longitudinal osseous segments of the skull in a somewhat similar manner to Klaatsch, but do not construct an index of curvature. They simply record the greatest distance of the arc from its chord, and, in the case of the os frontale, prefer the nasion-bregma or total frontal arc and chord to the glabella-bregma measurements.



—THE INDIVIDUAL AND GENERALISED RESULTS OF THE EXAMINATION OF FIFTY-TWO TASMANIAN ABORIGINAL CRANIA

## RESULTS



Büchner →		Probable Sex →				
		Serial Number →		1	2	3
		Present Location of Specimen →		Tasmanian		
NATURE OF OBSERVATION.		Original Number of Specimen →		4288	4291	4300
1	Nasion-Inion Length.			177	182	178
2	Bregma Angle.			59	60	+ 64
3	Frontal Angle.			84·5	90	87
4	Lambda Angle.			— 78	80·5	82
5	Opisthion Angle.			35	— 31·5	...
6	Frontal Arc.			123	133	+ 143
7	Frontal Chord.			108	114	116·5
8	Greatest Distance of Arc from Chord.			25	27	29
9	Occipital Arc.			122	119	110
10	Occipital Angle.			113	— 112	...
11	Inion-Opisthion Chord.			56	55	...
12	Total Sagittal Curvature.			366	387	398
13	Total Longitudinal Circumference.			497	522	...
14	Vertical Transverse Arc.			280	297	309
15	Basal Transverse Diameter.			143	+ 145	137
16	Total Vertical Transverse Diameter.			423	442	446
17	Preauricular Curve.			226	239	260
18	Postauricular Curve.			286	+ 294	279
19	Total Horizontal Circumference.			512	533	539

The objects of the present research are:—

1. To record certain craniometrical curvilinear and angular measurements, the latter being based on the nasio-inion plane.
2. To estimate the degree of flattening, or otherwise, of the Tasmanian aboriginal crania.
3. To estimate the evolutionary position of the Tasmanian from a study of certain of his cranial curvatures.

It has already been pointed out that, as one of the main objects of the Tasmanian work of Berry and Robertson was a comparison of the Tasmanian evolutionary relationship with that of *Pithecanthropus erectus*, they were compelled to employ the glabella-inion plane as their base. As the present work is freed from this disability, one of its first objects is to restate certain already recorded Tasmanian measurements on the new base—the nasio-inion plane—a base which it has already been shown Berry and Robertson prefer, where possible. As it is not proposed to do more than record these figures, they are simply set forth in Table I, and will not herein be again referred to. Suffice it to state that there are now available on both base-lines a large number of Tasmanian measurements for future comparison of other races by subsequent observers, and that, in each instance, the number of Tasmanians so recorded is the largest on record.

The material on which the present work is based will be found in Berry and Robertson's "Dioptrographic Tracings in Four Normæ of Fifty-two Tasmanian Crania" (23). In Table I. the angular and certain curvilinear measurements are estimated from the median sagittal drawings, that is, a tracing in the norma lateralis. The remainder of the observations in Table I. were recorded by Professor Berry and Dr Robertson on the original crania, whilst they were engaged in their investigations in Tasmania in 1909, and they are now made available for scientific study for the first time. I have to express my thanks to these authors for permission to utilise their figures.

These observations, to the number of nineteen, are set forth in Table I., and are as follow:—

1. The nasion-inion length.
2. The bregma-nasion-inion angle. This angle corresponds with Schwalbe's bregma angle, which has already been recorded by Berry and Robertson on the Tasmanian crania which form the subject of the present research.
3. The frontal angle.
4. The nasion-inion-lambda angle. The lambda angle has already been recorded by Berry and Robertson, based on the glabella-inion plane.

5. The nasion-inion-opisthion angle. Also recorded by Berry and Robertson, as the opisthion angle, based on the glabella-inion plane.
6. The total frontal arc. Nasion to bregma.
7. The total frontal chord. Nasion to bregma.
8. The length of the greatest distance of the arc from the chord.
9. The length of the total occipital arc. Lambda to opisthion.
10. The occipital angle, enclosed by the lambda-inion and the inion-opisthion chords.
11. The length of the inion-opisthion chord.
12. The total sagittal curvature.
13. The total longitudinal circumference.
14. The length of the vertical transverse arc.
15. The length of the basal transverse diameter.
16. The length of the total vertical transverse diameter.
17. The length of the preauricular curve.
18. The length of the postauricular curve.
19. The length of the total horizontal circumference.

In Table I. the individual and generalised observations just referred to of fifty-two Tasmanian crania have been set forth. The probable sex, the serial number, the present location, and the original number of each skull are set forth in the four upper horizontal lines. In the two vertical columns on the left, the numbers and names of the observations recorded, and the nature of the observation, are set forth. In the vertical columns 1 to 52, inclusive, are set forth the individual measurements of each skull.

The male and female cranial observations have been recorded in separate columns. The four vertical columns immediately on the right of the male observations record the number of observations made, the average figures for each observation together with the minimum and maximum figures for that observation. The four vertical columns immediately on the right of the female observations record like results, and for both sexes combined the figures are set forth in the four vertical columns on the extreme right of the Table.

No. 48 has been shown to be a juvenile subject; all the observations recorded upon it have been uniformly omitted from the final results.

For the purposes of determining the range of variation of each observation, the minimum and maximum figures are denoted by means of a – or + sign.

Of the observations set forth in Table I., 4 and 5 and 8 to 12 are original; Nos. 13–19 are the original observations already referred to; Nos. 3, 6, and 7 have already been published by Berry and Robertson

(18), but they have been incorporated in the present work for necessary reasons. For further explanation of the observations of the median sagittal curvatures in Table I., the reader is referred to fig. 1, where the method of determining the various measurements is displayed.

As regards the degree of flattening or otherwise of the Tasmanian aboriginal crania, it is very important to notice that Duckworth (24), in his recently published (1912) *Prehistoric Man*, says, "The flatness of a cranial arc is but one of many characters awaiting research," and adds,



FIG. 1.—To illustrate the measurements employed.

"More research is needed." In the same work, he also states that "Dr Sera (25) has been led to pay particular attention to the remarkably flattened cranial vaulting" of certain crania previously mentioned in Duckworth's work. He also adds that, "as a rule, this flattening has been regarded as representative of a stage in the evolution of a highly developed type of human skull from a more lowly, in fact a Simian one. This conclusion is challenged by Dr Sera. The position adopted is that a flattened skull need not in every case owe its presence to such a condition as an early stage of evolution assigns to it. Environment, for which we may here read climatic conditions, is a possible and alternative influence. If sufficient evidence can be adduced to show that the flattened cranial

arc in the Neanderthal skull does actually owe its origin to physiological factors through which environment acts, the status of that type of skull in the evolutionary sequence will be materially affected. . . . The Gibraltar skull is flattened owing to its low place in evolution. But as regards the flatness of the brain case (called the platycephalic character) of the Neanderthal calvaria and its congeners (as contrasted with the Gibraltar specimen) Dr Sera suggests dependence upon the particular environment created by glacial conditions."

It is thus obvious that the degree of flattening or otherwise is, in view of modern opinion, an important present-day field of research, and its estimation for the Tasmanian is the chief object of the present work. The investigation of the problem is, however, very considerably handicapped by the fact that Sera's original paper is not available in Melbourne, in either its original form or in any adequate abstract. With this important reservation, I have estimated the degree of curvature, or flattening of the glabella-bregmatic arc of the frontal, total parietal arc, and superior occipital arc of the *os occipitale* by Klaatsch's "index of curvature," all the observations having been made upon the median sagittal plane of the Tasmanian life-size tracings already referred to. For a diagrammatic explanation of the observations thus recorded, the reader is referred to fig. 1.

The following twelve observations have thus been recorded:—

*Os Frontale.*

1. The length of the glabella-bregma arc.
2. The length of the glabella-bregma chord.
3. The length of the greatest distance of the arc from the chord.
4. The index of frontal curvature (Klaatsch).

*Os Parietale.*

5. The length of the bregma-lambda or parietal arc.
6. The length of the bregma-lambda or parietal chord.
7. The length of the greatest distance of the arc from the chord.
8. The index of parietal curvature (Klaatsch).

*Os Occipitale.*

9. The length of the lambda-inion or superior occipital arc.
10. The length of the lambda-inion or superior occipital chord.
11. The length of the greatest distance of the arc from the chord.
12. The index of occipital curvature (Klaatsch).

The individual measurements of each of the above, together with minimum, average, and maximum results for the whole series of fifty-two Tasmanian crania, are set forth in Table II., which is uniform throughout



TABLE III.—THE INDIVIDUAL AND GENERALISED RESULTS OF THE EXAMINATION OF FIFTY-ONE AUSTRALIAN ABORIGINAL CRANIA.

NATURE OF OBSERVATION.	Serial Number	Buchner.																																					Klaatsch.																		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	Min.	Ave.	Max.		
1 Glabella-Bregma Arc.		107	128	107	113	115	108	115	104	110	109	105	110	110	111	110	116	124	116	119	111	105	108	118	107	112	124	120	112	111	125	122	106	115	119	124	125	119	115	99	124	...	...	...	...	...	...	...	...	...	...	...	40	99	110	2	128
2 Glabella-Bregma Chord.		101	121	116	106	118	113	114	100	108	102	29	105	102	105	103	109	110	109	103	104	100	103	110	99	110	117	111	105	114	115	116	100	106	110	114	114	110	110	95	115	104	109	106	51	95	108	121									
3 Greatest Distance of Arc from Chord.		21	21	17	26	5	22	17	5	26	17	17	21	18	18	21	22	19	20	20	20	17	16	20	21	15	21	21	17	20	21	17	16	21	22	20	22	25	24	17	15	24	13	51	13	19	6	28									
4 Index of Frontal Curvature (Klaatsch).		207	173	147	192	186	154	243	17	157	205	181	176	205	209	203	165	189	174	191	192	17	165	181	212	136	179	189	161	175	208	146	16	198	20	175	214	172	136	136	186	125	2217	198	193	213	226	222	154	154	29	103	51	126	181	247	
5 Bregma-Lambda Arc		118	120	139	134	125	120	120	110	130	122	136	113	26	127	116	140	137	126	130	108	105	120	120	117	120	131	148	135	121	129	129	118	120	126	137	120	128	123	119	131	...	...	...	...	...	...	...	...	...	...	...	40	105	121	2	118
6 Bregma-Lambda Chord.		109	112	122	121	111	110	104	107	121	112	123	108	115	115	110	129	124	117	125	102	100	114	110	107	114	123	137	121	112	117	118	707	111	119	121	110	118	116	111	121	105	109	116	110	111	116	118	128	111	113	108	51	100	114	7	137
7 Greatest Distance of Arc from Chord.		22	21	25	27	26	24	20	18	5	23	22	28	20	25	25	21	26	26	23	29	18	17	21	23	22	24	27	29	23	23	24	24	23	22	305	25	24	20	21	24	21	24	18	17	24	25	24	28	23	22	20	51	17	23	2	30
8 Index of Parietal Curvature (Klaatsch).		201	186	204	223	252	217	192	172	194	196	226	186	216	217	19	201	208	196	232	176	17	184	209	211	192	197	239	205	196	203	224	207	184	252	227	203	172	189	198	26	22	155	153	216	203	218	207	194	188	61	153	202	252			
9 Lambda-Union Arc.		51	55	58	54	50	55	52	57	67	46	60	51	49	69	63	435	61	52	59	56	46	58	54	60	56	50	60	58	51	61	50	45	54	49	59	51	57	68	67	60	...	...	...	...	...	...	...	...	...	...	...	40	335	552	72	
10 Lambda-Union Chord.		50	54	54	52	56	69	55	45	59	51	49	55	69	42	58	51	46	56	51	56	62	45	58	57	48	51	50	60	55	45	52	49	55	51	56	66	65	58	69	62	67	71	57	66	65	52	56	61	51	12	55	71				
11 Greatest Distance of Arc from Chord.		5	55	8	65	3	95	7	5	5	4	6	8	9	4	10	6	45	7	3	45	5	6	9	6	8	4	7	6	4	7	75	35	7	3	7	4	6	8	85	65	6	7	11	10	8	6	5	8	51	9	61	11				
12 Index of Occipital Curvature (Klaatsch).		70	101	111	125	53	137	127	111	84	78	102	145	152	95	172	98	78	129	65	8	98	107	173	125	137	7	145	117	8	116	130	77	134	61	127	78	107	121	13	112	87	112	104	14	14	107	7	9	76	86	131	61	53	109	173	

L. W. G. BUNNKE.



		Unsexed—			
NATURE OF OBSERVATION.		Serial Number—			
		23	24	25	26
1	Glabella-Bregma Arc.	118	107	112	124
2	Glabella-Bregma Chord.	110	99	110	117
3	Greatest Distance of Arc from Chord.	20	21	15	21
4	Index of Frontal Curvature (Klaatsch).	18·1	21·2	18·6	17
5	Bregma-Lambda Arc.	120	117	120	131
6	Bregma-Lambda Chord.	110	107	114	125
7	Greatest Distance of Arc from Chord.	23	23	22	24
8	Index of Parietal Curvature (Klaatsch).	20·9	21·4	19·2	18
9	Lambda-Inion Arc.	56	50	60	58
10	Lambda-Inion Chord.	52	48	58	57
11	Greatest Distance of Arc from Chord.	9	6	8	4
12	Index of Occipital Curvature (Klaatsch).	+ 17·3	12·5	18·7	?

L. W. G. BÜCHNER.



TABLE II.—THE INDIVIDUAL AND GENERALISED RESULTS OF THE EXAMINATION OF FIFTY-TWO TASMANIAN ABORIGINAL CRANIA.

RESULTS.

Buchner—		No.	Sex.	Num.	Males.																								Females.																								J.		Both Sexes.										
Nature of Observation	Series				1	2	3	4	5	12	13	16	18	19	23	24	25	27	28	29	30	32	33	34	36	38	37	38	39	40	42	43	44	45	46	50	No.	Min.	Av.	Max.	6	7	8	9	10	13	14	15	17	20	21	22	26	31	41	47	19	61	62	48	No.	Min.	Av.	Max.	No.
1. The Glabella-Bregma Arc.		109	120	115	109	115	119	118	114	120	119	125	115	119	111	111	115	111	109	112	100	100	100	121	116	105	112	112	32	100	113·6	125	106	106	113	115	110	111	121	103	108	108	90	113	111	102	104	110	103	113	106	18	90	108·3	121	50	90	111·9	125						
2. The Glabella-Bregma Chord.		103	111	13	101	107	111	112	108	114	111	117	110	110	108	104	108	103	108	107	95	104	94	116	100	97	112	103	32	91	107·6	118	99	99	103	106	112	106	111	95	103	102	87	101	106	98	95	100	101	104	95	18	87	101·5	112	50	87	105·2	118						
3. The greatest Distance of Arc from Chord.		19	20	21	19	19·5	23	15	19	21	23	25	17	19	19	22	22	18	11	17	16	20	16	19	16	22	19	10	23	19	32	10	18·0	25	18	21	24	20	19	18	23	19	18	16	... 12·5	20·5	18	16	18	21	20	19	22	18	12·5	18·8	23	50	10	18·0	25				
4. The Index of Frontal Curvature (Klutschek).		18·4	18·1	17·8	18·8	18·2	29·7	15·1	17·5	18·1	20·7	21	15·4	17·2	17·5	20	18·2	17·4	15·4	15·7	18·2	18·2	16·7	16·5	10·3	18·6	15·7	18·2	17	18·0	17·4	10·3	20·6	18·4	32	10·3	17·2	21	+ 23·3	18·8	16·9	16·9	20·7	20	17·4	16·6	... 11·3	20·2	16·9	15·3	18·9	21	19·8	18·1	23·1	18	14·3	18·6	23·3	50	10·3	17·7	23·3		
5. The Bregma-Lambda Arc.		121	123	115	133	121	130	130	114	125	120	120	118	120	126	125	131	131	124	130	128	125	122	119	130	128	122	130	133	138	136	129	114	127·8	146	+ 132	120	122	119	121	120	127	112	124	120	126	129	122	127	124	128	118	120	109	111	19	112	124·6	132	47	112	145·8	115		
6. The Bregma-Lambda Chord.		111·5	121	117	118	109	117·5	115	103·3	117	118·5	...	99	110	114	111·5	111·5	111·5	110	114·5	116	116	113	114	109	119·6	107·5	126	108·5	116·5	120·5	123	123	30	99	115·3	127	118	108·5	111	107	116	99·5	111	118·5	106	111	111·5	115	11·3	107	110	100	09	18	90·6	109·8	118·6	48	90·5	113	127			
7. The greatest Distance of Arc from Chord.		23	26	27	25	25	25	20	20	25	23	...	22	22	24	25	26	24	21	24	22	25	26	24	21	23	27	21	26	26	26	26	23	30	20	24·0	28	20	25	23	26	23	22	24	20	...	23	24	23	23	23	19	21	18	19	22·3	28	18	19	23	23				
8. The Index of Parietal Curvature (Klutschek).		19·5	21·5	21	21·1	22·9	21·2	17·3	19·6	21·3	19·4	...	22·2	18·3	21	21·5	18·8	20·1	19·2	21·5	21·7	21·2	18·4	21·1	22·6	19·6	20	23	21·0	21·5	22·7	20·3	...	30	17·3	20·7	23	17·7	23	18·9	18·6	20·1	20	19·8	19·1	19·9	20·2	18·8	...	20·7	21·5	20	24·7	21·4	20·9	19	21·2	18	17·7	20·1	24·7	48	17·3	20·5	24·7
9. The Lambda-Iunior Arc.		51	65	77	57	60	73	65	70	67	50	...	60	68	56	61	+ 74	50	50	58	55	55	49	59	56	54	71	55	62	...	30	12	59·7	74	60	53	55	49	55	...	57	...	55	59	65	+ 67	50	62	48	18	47	54	39	16	47	55·2	67	16	47	57·6	74				
10. The Lambda-Iunior Chord.		62	62	56	55	58	69	63	67	63	49	...	19	65	54	58	+ 70	48	57	52	50	47	48	53	48	56	53	50	65	54	58	30	17	58·7	70	58	52	54	48	61	...	55	...	54	56	60	+ 63	58	47	56	59	36	16	40	52·5	60	46	46	56·1	70					
11. The greatest Distance of Arc from Chord.		8	7	6	6	5	5	5	5	9	9	4	4	4	4	9	7	8	+ 10	4	6	4	5	7	1	4	3	6	5	6	0	5	8	30	3	6·0	10	9	5	4	6	7	...	8	...	6	6	+ 12	7	8	3	5	2	16	2	6·5	12	46	2	6·1	12				
12. The Index of Occipital Curvature (Klutschek).		12·9	11·1	3	10·0	8·6	7·2	9·5	13·1	14·2	8·1	...	8·1	13·8	12·0	13·7	14·2	8·3	12·7	7	10·6	11	14·8	8·0	7·5	6·2	10·7	9·4	12	13·8	...	30	6·3	10·7	14·8	+ 15·6	9·6	7·4	10·4	12·9	...	14·5	...	11·1	10·7	20	19	15	13·7	6·3	10·8	4·3	9·4	6·2	16	4·9	11·9	19	46	4·3	11·1	20			



Büchner————→					
Nature of Observation.		Serial Number————→			No.
1	2	3			
1	The Glabella-Bregma Arc.	109	121	<sup>+</sup> 125	32
2	The Glabella-Bregma Chord.	103	111	<sup>+</sup> 118	32
3	The greatest Distance of Arc from Chord.	19	20	21	32
4	The Index of Frontal Curvature (Klaatsch).	18·4	18·1	17	32
5	The Bregma-Lambda Arc.	121	135	<sup>+</sup> 145	29
6	The Bregma-Lambda Chord.	111·5	121	<sup>+</sup> 127	30
7	The greatest Distance of Arc from Chord.	23	26	<sup>+</sup> 28	30
8	The Index of Parietal Curvature (Klaatsch).	19·9	21·5	22	30
9	The Lambda-Inion Arc.	64	63	57	30
10	The Lambda-Inion Chord.	62	62	56	30
11	The greatest Distance of Arc from Chord.	8	7	3	30
12	The Index of Occipital Curvature (Klaatsch).	12·9	11·1	5	30

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with Table I. As in Table I., No. 48 of Berry and Robertson's Atlas is omitted from the final results, as it is known to be a juvenile.

For comparative purposes I have recorded the same twelve observations on forty Australian aboriginal crania. The resulting figures are set forth in Table III., in precisely the same manner as those for the Tasmanian in Table II.

In the Australian table I have also included eleven crania from Klaatsch's work, and the entire table thus deals with fifty-one Australian crania, *i.e.* with precisely the same number as there are Tasmanians.

The source of my own forty original Australian crania is Berry and Robertson's "Dioptrographic Tracings in three Normæ of Ninety Australian Crania" (26), now in the press. As only forty of the original drawings have as yet been returned from the printer, it was not possible for me to utilise the whole ninety. For permission to avail myself of this work I have to thank Professor Berry and Dr Robertson. My Australian material will eventually be found, therefore, to include plates numbered 1 to 40, norma A, of the atlas of tracings just referred to. The whole of this series of Australian crania is quite new, and has not previously been recorded scientifically.

The indices of curvature of the several segments of the median sagittal curve of the fifty-one Tasmanian and fifty-one Australian crania may be summarised and compared with certain selected objects recorded by Klaatsch, as follow:—

Curvature index of glabella-bregmatic curve of the os frontale:—

1 <i>Pithecanthropus erectus</i> (Klaatsch)	. . . .	7·53
3 Spy-Neanderthal (Klaatsch)	. . . .	13·3
51 Australians (Klaatsch and Büchner)	. . . .	18·1
51 Tasmanians (Büchner)	. . . .	18·7

As Klaatsch's index of curvature expresses the ratio which the length of the greatest distance of the arc from the chord bears to the length of the chord, the latter being taken as 100, it follows from the above that the Tasmanian possesses the most highly curved glabella-bregmatic arc of any of the objects compared.

The curvature index of the os parietale, as worked out by Klaatsch's method, and compared with the same objects as before, is as follows:—

1 <i>Pithecanthropus erectus</i> (Klaatsch)	. . . .	9·68
3 Spy-Neanderthal (Klaatsch)	. . . .	17·04
51 Australians (Klaatsch and Büchner)	. . . .	20·2
51 Tasmanians (Büchner)	. . . .	20·5

Here again the Tasmanian possesses the most highly curved parietal arc, whilst the Australian again occupies third place, a little inferior to the Tasmanian. If these results be read in association with the known physiological functions of those portions of the brain which lie subjacent to the parietal arc, they become of real and striking significance.

Dealing in the same way with the superior occipital index of curvature, we achieve the following results:—

1 <i>Pithecanthropus erectus</i> (Klaatsch)	. . . .	4·76
51 Australians (Klaatsch and Büchner)	. . . .	10·9
51 Tasmanians (Büchner)	. . . .	11·1
3 Spy-Neanderthal (Klaatsch)	. . . .	14·17

Here the several objects have changed places—the Spy-Neanderthal group having the most highly curved superior occipital arcs, whilst the Tasmanian still retains his more advanced position over the Australian. It is difficult to account for the increased degree of superior occipital curvature in the Spy-Neanderthal group, but as regards the Australians and Tasmanians it is interesting to observe that, in all its segments, the median sagittal curvature of the Australian calvaria is less pronounced than in the Tasmanian, that is, the Australian has a flatter skull, as regards curvature, than has the Tasmanian.

In a previous publication (27) I recorded the range of variation on fifty-two Tasmanian crania for twenty-seven observations based on Klaatsch's cranio-trigonometrical methods. The figures expressive of this range of variation were so small as to warrant the conclusion which I then drew, that the Tasmanian is a pure race. By totally different methods Berry and Robertson (28), in a memoir as yet unpublished, but now ready for the press, have arrived at identical conclusions.

I have, therefore, again recorded the range of variation for the twelve observations set forth in Table II. of the present work, in order to ascertain if my former conclusions would be sustained.

The results attained from the present and previous works just referred to are, for the Tasmanian, as follows:—

Present Work.		Previous Work.	
Degree of Sagittal Curvature.		Twenty-seven Cranio-trigonometrical Observations.	
Males . . . .	8·9	Males . . . .	7·9
Females . . . .	7·8	Females . . . .	7·5
Both sexes . . . .	10·3	Both sexes . . . .	9·9

The range of variation of the combined observations, from these two works, is therefore, for the Tasmanian, as follows:—

Males . . . . .	8·4
Females . . . . .	7·6
Both sexes . . . . .	10·1

The manner of estimating the range of variation by a single figure was dealt with in my previous work. The result is so surprisingly low as to justify the statement already made, that the Tasmanian is a homogeneous race.

Passing now to the third and last purpose of the present work, namely, an estimate of the evolutionary position of the Tasmanian, as deduced from a study of the relative degree of curvature of the various segments of the calvaria as herein described, I propose to deal with the subject on somewhat similar lines to those adopted by Berry and Robertson.

It will be remembered that these authors, in conjunction with Dr Cross, introduced some strikingly original methods, in their attempt to place the Tasmanian in his correct evolutionary position as compared with certain supposed lower morphological forms. Their work was based solely on twenty-seven of the "form analysis" measurements of Schwalbe, and it seems to me desirable to ascertain if their final conclusions will be sustained by like methods based on completely different observations.

With this object in view, I shall, therefore, deal with the degree of flattening of the skull as studied in this work, and I shall employ as objects of comparison the crania of the chimpanzee, *Pithecanthropus erectus*, Gibraltar, Spy-Neanderthal, Brüx, Galley Hill, Brünn, Cro-Magnon, Australian, Tasmanian, and European.

The sources from which I have obtained the necessary data are as follow:—

For the anthropoid I have utilised certain observations already published by Berry and Robertson (18).

For *Pithecanthropus* I have utilised the necessary observations already recorded by Klaatsch (20) in his memoir on the Australian skull.

For the Gibraltar skull the observations have been calculated from the diagrams furnished by Sollas (29).

For the Spy-Neanderthal group the observations have been calculated from median sagittal diagrams in Schwalbe's monograph on *Pithecanthropus erectus* (19).

For the Brüx, Galley Hill, Brünn, and Cro-Magnon crania the observations have been calculated by me from median sagittal diagrams furnished by Schwalbe (30) and Klaatsch (31).

The Australian and Tasmanian measurements are the original contributions to the subject of the present work.

For the European the observations have already been recorded by Klaatsch.

In grouping these several objects of comparison together for purposes of calculation, I have regarded the Neanderthal and Spy crania as a homogeneous group, and have dealt with the Galley Hill, Brüx, and Brünn crania in a like way, and for like reasons. To the former procedure there

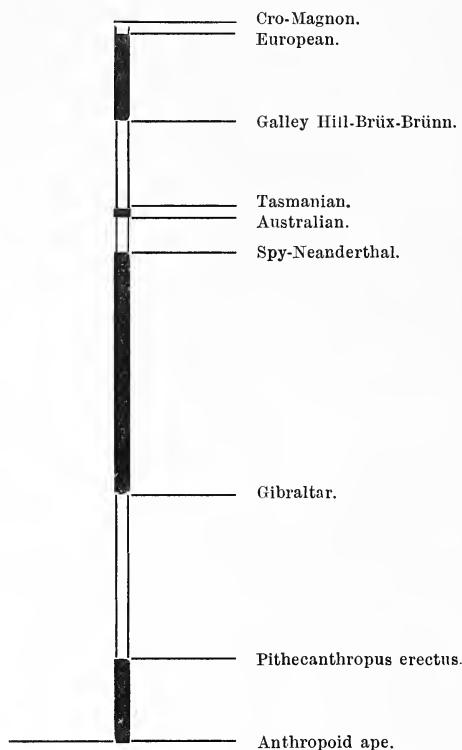


FIG. 2.

can be no objection, and for the inclusion of the Galley Hill skull with those of Brüx and Brünn I have been largely influenced by the recently expressed opinions of Duckworth (24).

For the mathematical estimation of the relative evolutionary positions of the Tasmanian and the other objects of comparison, I have adopted the ingenious methods introduced by Cross. I have not, however, deemed it necessary to prolong the calculations beyond what Cross terms the "composite order." The working of the method is illustrated in Tables IV., V., and VI.

The final result is displayed graphically in fig. 2, and numerically in Table VI.

TABLE IV.

Nature of Observation.			Anthropoid.	Pithecanthropus.	Spy-Neanderthal.	Gibraltar.	Galley Hill-Briix-Briinn.	Australian.	Tasmanian.	Cro-Magnon.	European.
1	Glabella-Bregma Arc .	92	110	124	...	135	110·2	119·9	138	...	
2	Glabella-Bregma Chord .	87	93	108	...	119	108	105·2	123	110	
3	Greatest Distance of Arc from Chord .	...	7	14·3	...	20·7	19·6	18·8	24	24	
4	Index of Frontal Curvature .	...	7·53	13·13	...	17·5	18·1	18·7	19·5	21·8	
5	Bregma-Lambda Arc .	62	103	119·6	...	126·6	121·2	125·8	135	133	
6	Bregma-Lambda Chord .	...	93	112	...	116·3	114·7	113	123	112	
7	Greatest Distance of Arc from Chord .	...	9	18·3	...	21	23·2	23·3	26·5	24	
8	Index of Parietal Curvature .	...	9·68	17·04	...	17·3	20·2	20·5	21·6	21·4	
9	Lambda-Inion Arc .	...	30	57·3	58·5	60	55·7	58·5	63	...	
10	Lambda-Inion Chord .	...	42	54	38	60·6	55·2	55·5	60·75	63	
11	Greatest Distance of Arc from Chord .	...	2	7·6	5	8·2	6·1	6·1	8·2	11	
12	Index of Occipital Curvature .	...	4·76	14·17	13·1	13·3	10·9	11·1	13·4	17·46	

TABLE V.

Nature of Observation.			Maximum.	Anthropoid.	Pithecanthropus.	Spy-Neanderthal.	Gibraltar.	Galley Hill-Briix-Briinn.	Australian.	Tasmanian.	Cro-Magnon.	European.
1	Glabella-Bregma Arc .	46	0	18	32	...	43	18·2	19·9	46	...	
2	Glabella-Bregma Chord .	36	0	6	21	...	32	21	18·2	36	23	
3	Greatest Distance of Arc from Chord .	...	17	...	0	7·3	...	13·7	12·6	11·8	17	17
4	Index of Frontal Curvature .	14·3	...	0	5·6	...	10	10·6	11·2	12	14·3	
5	Bregma-Lambda Arc .	73	0	41	57·6	...	64·6	59·2	63·8	73	71	
6	Bregma-Lambda Chord .	30	...	0	19	...	23·3	21·7	20	30	19	
7	Greatest Distance of Arc from Chord .	...	17·5	...	0	9·3	...	12	14·2	14·3	17·5	15
8	Index of Parietal Curvature .	11·92	...	0	7·36	...	7·62	10·52	10·82	11·92	11·72	
9	Lambda-Inion Arc .	33	...	0	27·3	28·5	30	25·7	28·5	33	...	
10	Lambda-Inion Chord .	25	...	4	16	0	22·6	17·2	17·5	22·8	25	
11	Greatest Distance of Arc from Chord .	9	...	0	5·6	3	6·2	41	4·1	6·2	9	
12	Index of Occipital Curvature .	12·7	...	0	9·41	8·34	8·54	6·14	6·34	8·54	12·7	

Concerning the Tasmanian and Australian, it will be seen that these results confirm absolutely the conclusions previously drawn by Berry, Robertson, and Cross; and it will also be subsequently found that, as regards the placing of the Australian on the minus side of the Tasmanian, these results confirm those about to be published by Berry and Robertson (28).

The Gibraltar skull appears herein between *Pithecanthropus erectus* and the three Spy-Neanderthal crania. This lowly position may, however,

TABLE VI.

Nature of Observation.			Number.	Anthropoid.	Pithecanthropus.	Spy-Neanderthal.	Gibraltar.	Galley Hill-Brüx-Brunn.	Australian.	Tasmanian.	Cro-Magnon.	European.
1 Glabella-Bregma Arc . . .	6	0	2·35	4·17	...	5·61	2·37	2·60	6·00	...		
2 Glabella-Bregma Chord . . .	7	0	1·17	4·08	...	6·22	4·08	3·54	7·00	4·47		
3 Greatest Distance of Arc from Chord . . .	6	...	0	2·58	...	4·84	4·45	4·16	6·00	6·00		
4 Index of Frontal Curvature . . .	6	...	0	2·35	...	4·20	4·45	4·70	5·04	6·00		
5 Bregma-Lambda Arc . . .	7	0	3·93	5·52	...	6·19	5·68	6·12	7·00	6·81		
6 Bregma-Lambda Chord . . .	6	...	0	3·80	...	4·66	4·34	4·00	6·00	3·81		
7 Greatest Distance of Arc from Chord . . .	6	...	0	3·19	...	4·11	4·87	4·90	6·00	5·14		
8 Index of Parietal Curvature . . .	6	...	0	3·70	...	3·84	5·30	5·45	6·00	5·90		
9 Lambda-Inion Arc . . .	6	...	0	4·96	5·18	5·45	4·67	5·18	6·00	...		
10 Lambda-Inion Chord . . .	7	...	1·12	4·48	0	6·33	4·82	4·90	63·8	7·00		
11 Greatest Distance of Arc from Chord . . .	7	...	0	4·35	2·33	4·82	3·19	3·19	4·82	7·00		
12 Index of Occipital Curvature . . .	7	...	0	5·19	4·60	4·71	3·38	3·49	4·71	7·00		
Total . . . .	...	0	8·57	48·38	12·11	60·98	51·60	52·23	70·95	59·12		
Possible Maximum . . .	...	20	77	77	27	77	77	77	77	65		
Relative Position . . .	...	0	·111	·628	·315	·792	·670	·678	·921	·912		

be due to the fact that I have only dealt with four measurements, inasmuch as it is well known that the Gibraltar calvaria is imperfect; or, on the other hand, it may be really due to the lowly position claimed for this skull by Keith and others.

That the Galley Hill-Brüx-Brunn group appear on the plus side of the Tasmanian-Australian series need cause no surprise, because they are herein dealt with as a group, and not singly as in Cross's work. Even the latter observer placed one of them on the plus side and the other two on the immediate minus side of the Tasmanian.

Working, then, on totally different craniological lines, it is sufficiently obvious that, as regards the Tasmanian-Australian type—the “*Hesperanthropus tasmanianus*, spec.” of Sergi,—the present work sustains the thesis of Berry and Robertson that the “lately extinct Tasmanians recall the mental level of eolithic man in Britain we can quite believe; but that either the Australian or the Tasmanian carries us back nearly to the Neanderthal physical type, we must, as the result of the present investiga-

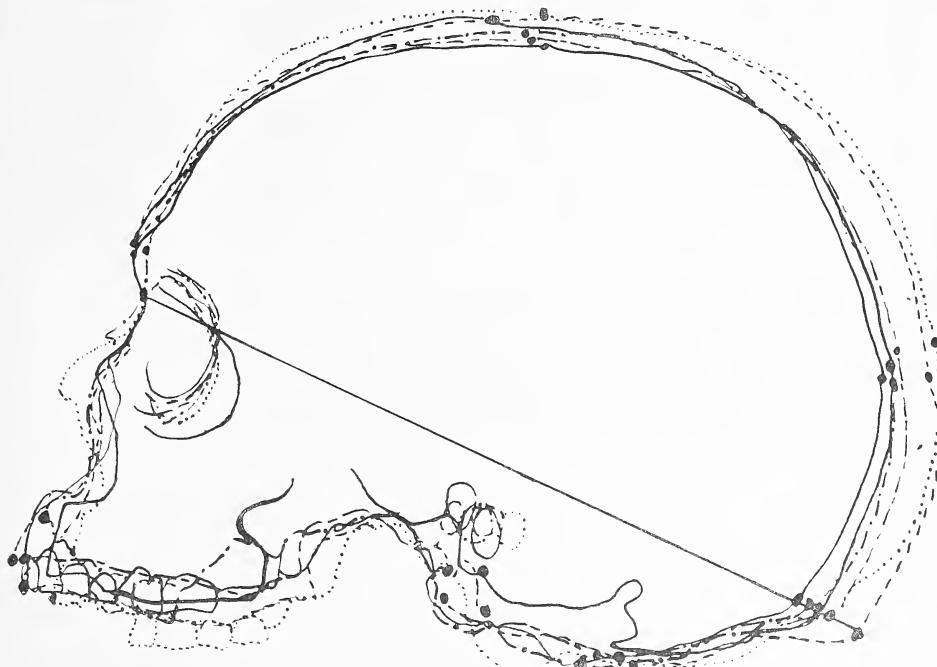


FIG. 3.—Six Tasmanian Skulls superposed on the Nasion-Inion plane.

tion, deny, because the physical construction of the Tasmanian is herein certainly shown to go back only so far as the Galley Hill type at furthest, and more than this cannot be maintained with any degree of scientific certainty.”

#### REFERENCES.

- (1) BERRY, R. J. A., and A. W. D. ROBERTSON, “Preliminary Communication on Fifty-three Tasmanian Crania, forty-two of which are now recorded for the first time,” *Proc. Roy. Soc. of Vict.*, vol. xxii., N.S., pt. i., 1909, pp. 47-58.
- (2) BERRY, R. J. A., and A. W. D. ROBERTSON, “Preliminary Account of the Discovery of Forty-two hitherto unrecorded Tasmanian Crania,” *Anat. Anz.*, Bd. xxxv. pp. 11-17, 1909.

(3) SERGI, G., "Tasmanier und Australier, *Hesperanthropus tasmanianus*, spec.," *Archiv für Anthropol.*, Neue Folge, Bd. xiii., Heft 3, 1912.

(4) BARNARD DAVIS, "Osteology and Peculiarities of the Tasmanians," *Natuur. Kund. Verhandl. der Hollandsche Maatschap. der Wetenschap*, 3 verz., Deel ii., No. 4, Haarlem, 1874.

(5) TOPINARD, P., "Etude sur les Tasmaniens," *Mém. de la Soc. d'Anthropologie*, tome iii. p. 307, 1872.

(6) TOPINARD, P., "Examen des mesures craniométriques adoptées par le Thesaurus Craniorum de Barnard Davis et en particulier de celles de la série des Tasmaniens," *Revue d'Anthropologie*, tome ii. p. 99, 1873.

(7) QUATREFAGES and HAMY, *Crania Ethnica*, Texte et Atlas, Paris, 1882.

(8) FLOWER, W. H., *Osteological Catalogue, Museum Royal College of Surgeons of England*, pt. i., "Man," London, 2nd edition, 1907.

(9) WILLIAMSON, G., "Observations of the Human Crania in the Museum of the Army Medical Department, Fort Pitt, Chatham," *Dub. Journ. Med. Science*, vol. xxiii., vol. xxiv. p. 42, May and August 1857.

(10) WIEGER, G., *Katalog der anthropologischen Sammlung des Anatomischen Institute zu Breslau*, Festgabe, Braunschweig, 1884.

(11) KLAATSCH, H., "Bericht ueber einen anthrop. Streifzug nach London," *Zeit. f. Ethnologie*, Heft 6, p. 875, 1903.

(12) GARSON, J. G., Chapter on "Osteology" in Ling Roth's *The Aborigines of Tasmania*, 1899.

(13) HARPER, W. E., and A. H. CLARKE, "Notes on the Measurements of the Tasmanian Crania in the Tasmanian Museum, Hobart," *Papers and Proc. Roy. Soc. Tasmania*, 1897, pp. 97-110.

(14) DUCKWORTH, W. L. H., "Craniological Notes on the Aborigines of Tasmania," *Journ. Anthropol. Inst.*, vol. xxxii. p. 177, 1902; and *Studies from the Anthropological Laboratory*, Cambridge, 1904.

(15) TURNER, Sir W., "The Aborigines of Tasmania: pt. ii., The Skeleton," *Trans. Roy. Soc. Edin.*, vol. xlvi., pt. iii. (No. 16), pp. 411-454, 1910.

(16) BERRY, R. J. A., A. W. D. ROBERTSON, and K. S. CROSS, "A Biometrical Study of the Relative Degree of Purity of Race of the Tasmanian, Australian, and Papuan," *Proc. Roy. Soc. Edin.*, vol. xxxi., pt. i. (No. 2), 1910.

(17) CROSS, K. S., "On a Numerical Determination of the Relative Positions of certain Biological Types in the Evolutionary Scale, and of the Relative Values of various Cranial Measurements and Indices as Criteria," *Proc. Roy. Soc. Edin.*, vol. xxxi., pt. i. (No. 4), 1910.

(18) BERRY, R. J. A., and A. W. D. ROBERTSON, "The Place in Nature of the Tasmanian Aboriginal as deduced from a Study of his Calvaria: pt. i., His Relations to the Anthropoid Apes, *Pithecanthropus*, *Homo primigenius*, *Homo fossilis*, and *Homo sapiens*," *Proc. Roy. Soc. Edin.*, vol. xxxi., pt. i. (No. 3), 1910.

(19) SCHWALBE, G., "Studien über *Pithecanthropus erectus* Dubois," *Zeitschrift für Morphologie und Anthropologie*, Bd. i., Heft 1, p. 16 et seq., 1899.

(20) KLAATSCH, H., "The Skull of the Australian Aboriginal," *Reports from the Path. Lab. of the Lunacy Dept. N.S.W.*, vol. i. p. 45 et seq.

(21) TURNER, Sir W., "The Craniology, Racial Affinities, and Descent of the Aborigines of Tasmania," *Trans. Roy. Soc. Edin.*, vol. xlv., pt. ii. (No. 17), pp. 365-403, 1908.

(22) CUNNINGHAM, D. J., "The Australian Forehead," *Anthropological Essays presented to E. B. Tylor*, 1907.

(23) BERRY, R. J. A., and A. W. D. ROBERTSON, "Dioptrographic Tracings in Four Normæ of Fifty-two Tasmanian Crania," *Trans. Roy. Soc. Vict.*, vol. v., 1910.

(24) DUCKWORTH, W. L. H., *Prehistoric Man*, Cambridge, 1912.

(25) SERA, Dr, *Archivio per l'Antropologia e per la Etnologia*, xl., fasc. 3-4, quoted by Duckworth under (24).

(26) BERRY, R. J. A., and A. W. D. ROBERTSON, "Dioptrographic Tracings in Three Normæ of Ninety Australian Crania," now in the press, *Trans. Roy. Soc. Vict.*

(27) BÜCHNER, L. W. G., "An Investigation of Fifty-two Tasmanian Crania by Klaatsch's Cranio-trigonometrical Methods," *Proc. Roy. Soc. Vict.*, vol. xxv., new series, pt. i., pp. 122-135, 1912.

(28) BERRY, R. J. A., and A. W. D. ROBERTSON, "The Place in Nature of the Tasmanian Aboriginal, as deduced from a Study of his Calvaria: pt. ii., His Relation to the Australian Aboriginal," *Proc. Roy. Soc. Edin.*, vol. xxxiv., pt. ii., No. 12, 1914.

(29) SOLLAS, W. J., "On the Cranial and Facial Characters of the Neanderthal Race," *Phil. Trans.*, B, vol. excix., 1907.

(30) SCHWALBE, G., "Das Schadelfragment von Brüx und verwandte Schädel-formen," *Zeit. für Morph. und Anthropol.*, Sonderheft, Mai 1906.

(31) KLAATSCH, H., "Die Fortschritte der Lehre von den fossilen Knochen-resten des Menschen in den Jahren 1900-1903," *Merkel und Bonnet's Ergebnisse der Anatomie und Entwicklungsgeschichte*, vol. xii. pp. 545-551, 1903.

(Issued separately April 28, 1914.)

XII.—The Place in Nature of the Tasmanian Aboriginal as deduced from a Study of his Calvaria.—Part II. His Relation to the Australian Aboriginal. By Richard J. A. Berry, M.D. Edin., Professor of Anatomy in the University of Melbourne; and A. W. D. Robertson, M.D. Melb., Government Research Scholar in the Anatomy Department of the University of Melbourne. (With One Folding Table.)

(MS. received December 9, 1912. Read January 19, 1914.)

#### INTRODUCTION.

IN December 1910 we published, in conjunction with Dr K. Stuart Cross, in the *Proceedings of the Royal Society of Edinburgh* (1, 2, 3, 4) a series of four papers dealing with the relations of the Tasmanian aboriginal to *Pithecanthropus erectus* and to primitive man generally. In an earlier publication, published in the *Transactions of the Royal Society of Victoria* (5), we also made available the material upon which our Tasmanian work was based. In the present publication we propose to deal with the question of the relationship of the Tasmanian aboriginal to the Australian, with a view to deciding, if possible, the vexed questions as to whether the Tasmanian and the Australian are one and the same race, or, if not, if the Australian is a homogeneous or a heterogeneous race.

#### LITERATURE.

In one of our previous communications (2) we have dealt fairly exhaustively with the views of the two opposed schools into which the study of the Australian aboriginal has divided scientific ethnologists. On the one hand there are Keane, Flower and Lydekker, Topinard, Tylor, Curr, de Quatrefages, and Mathew, who hold the Australian to be an impure race—that is, to have resulted from a cross; on the other hand there are Klaatsch, Schoetensack, and other German savants, who hold that the Australian is a pure type and that the Tasmanian is but an insular variation of that type. This subject has also been still further dealt with by one of us in another publication (6), so that it is unnecessary here to pursue the question further. The more recent literature bearing on this question will be dealt with as occasion demands in the subsequent parts of this paper.

## SOURCES OF THE MATERIAL.

For the purely Australian part of the present investigation we have availed ourselves of 100 Australian aboriginal crania, none of which have ever previously been examined by any scientist. Of these, numbers 1 to 50, both inclusive, are from the Anatomy Museum of the University of Melbourne: the remaining 50 are from the National Museum, Melbourne; and for their use we have to thank the Director of the Museum, Professor Spencer, as also his assistants, Messrs Kershaw and Walcott.

Of these 100 crania it is most important to note that all with the exceptions of numbers 43 to 50, both inclusive, are Victorian crania; the eight exceptions are from Queensland. It follows therefore that 92 per cent. of our Australian crania are derived from sources in the vicinity of the Murray River, or roughly from a district south of the thirty-fifth parallel of latitude; the importance of this lies in the fact that there cannot be any question of racial impurity due to admixture with the Malay element, which is not infrequently the case with Australian crania derived from the Northern Territory or other portions of the Australian Continent in the vicinity of the Malay Peninsula.

For the purposes of comparison with the Tasmanian, our material is naturally that of our recent Tasmanian work, to which reference has already been made.

For other comparative purposes, to which reference will subsequently be made, we have availed ourselves of material derived from the Catalogue of the Royal College of Surgeons of London. The material so utilised comprised 19 Andamanese Islander crania, and 90 crania of modern Italians.

In addition to this we have also availed ourselves of certain data published by Schwalbe for the Spy-Neandertal group of crania.

## TECHNIQUE.

In the case of the 100 Australian aboriginal crania dioptrographic tracings in four normæ were recorded of all by means of Martin's dioptrograph, each skull being orientated in the Frankfort plane in the Kubuskraenophor. Selections from these tracings are now in the printer's hands, and will be published in due course.

## OBSERVATIONS RECORDED.

On the dioptrographic tracings there have been recorded the measurements of the 27 observational counts previously employed by us in the

Tasmanian work, and to which the reader is referred. These, it will be remembered by those who have seen that work, are the data employed by Schwalbe in his examination of the Pithecanthropus, Spy, Neandertal, and other calvaria. To these 27 observations there have been added, in the case of the Australian, 5 other observations employed by Klaatsch (7), as follows:—

- A. The nasio-inion length.
- B. The glabella-lambda length.
- C. The lambda-glabella-inion angle.
- D. The distance of the bregma foot-point from the glabella on the glabella-lambda line.
- E. The bregma foot-point-glabella-lambda index—that is, the proportion which the distance of the bregma foot-point from the glabella on the glabella-lambda line bears to the glabella-lambda length, the latter being taken as 100.

The complete series of measurements employed will be readily seen in fig. 1.

In addition to the foregoing 32 observational points of the form analysis of the Australian aboriginal skull, we have also recorded and employed for purposes of comparison a second series of ordinary craniological observations as follows:—

1. Maximum cranial length.
2. Maximum cranial breadth.
3. The cephalic index.
4. Cranial height.
5. The height index.
6. The basi-nasal length.
7. The basi-alveolar length.
8. The alveolar index.
9. The nasal height.
10. The nasal width.
11. The nasal index.
12. The orbital width.
13. The orbital height.
14. The orbital index.

The necessary figures for the above in the cases of the Australian and Tasmanian have been obtained from our own original material. In the cases of the Andamanese Islanders and the modern Italians they have been obtained from the Catalogue of the Royal College of Surgeons of London.

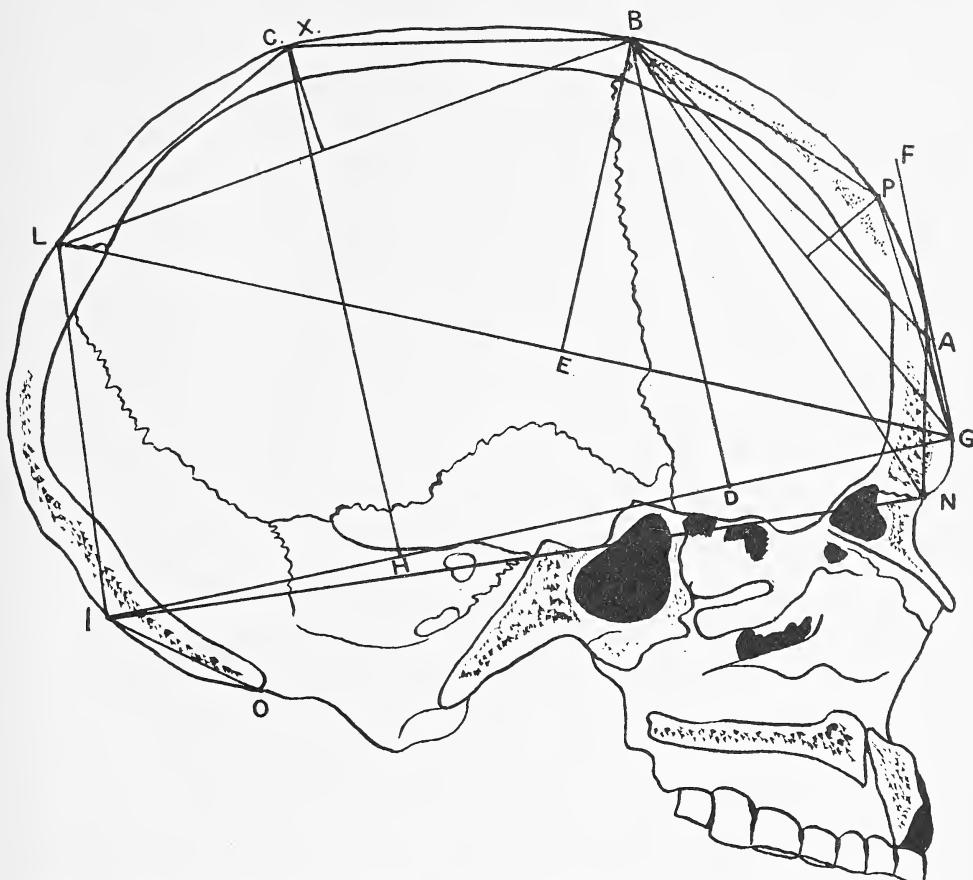


FIG. 1.—Median Sagittal Section through an Adult Male Australian Aboriginal Skull. (Victoria No. 18. From the Anatomical Museum of the University of Melbourne.) To illustrate Schwalbe's form analysis of the skull, as employed in the present investigation.

- N. The nasion.
- G. The glabellar point.
- A. The upper limit of the glabellar curve.
- P. The maximum point of the frontal curvature.
- B. The bregma.
- C. The maximum point of the calvarial height.
- X. The maximum point of the parietal curvature.
- L. The lambda.
- I. The inion.
- O. The opisthion.
- H. The calvarial height foot-point.
- G.I. The glabella-inion length.
- G.L. The glabella-lambda length.
- N.I. The nasio-inion length.
- C.H. The calvarial height.
- D. The bregma foot-point on the glabella-inion line.
- E. The bregma foot-point on the glabella-lambda line.
- G.H. The distance of the calvarial height foot-point from the glabella.
- G.D. The distance of the bregma foot-point from the glabella on glabella-inion line.
- G.E. The distance of the bregma foot-point from the glabella on glabella-lambda line.
- B.G.I. The frontal angle.
- F.G.I. The parietal angle.
- N.B. The frontal chord.
- N.A. The glabellar chord.
- A.B. The cerebral chord.
- B.L. The parietal chord.
- G.P.B. The angle of frontal curvature.
- B.X.L. The angle of parietal curvature.
- L.I.G. The lambda angle.
- L.G.I. The lambda-glabella-inion angle.
- O.I.G. The opisthionic angle.

## THE 32 FORM ANALYSIS MEASUREMENTS OF THE AUSTRALIAN SKULL.

For the display of the 32 observational counts made upon each one of the hundred Australian aboriginal crania with which this memoir deals, we propose, for purposes of comparison, to adopt the same procedure as employed by us in our former work upon the Tasmanian (3). The individual results of the entire series of 100 crania are, therefore, set forth in a table of measurements (Table XXVIII.). This table will form a valuable means of comparison and contrast with the similar table published by us for the Tasmanians (3), the more so as the two tables deal with what is probably the largest consecutive series of Tasmanian and Australian crania yet dealt with, namely, 52 Tasmanian and 100 Australian crania.

In the Tasmanian work just referred to, in addition to publishing a complete table of all measurements, we dealt with each observational count separately. This procedure was adopted in order to form a first estimation of the evolutionary position occupied by the Tasmanian under each observational count. We regard it as important to form a like opinion for the Australian, so that it is necessary, even at the risk of reiteration, to record the same tables here with the Australian included. We have, however, taken the opportunity whenever it was afforded, to increase the numbers of the objects of comparison. In the several tables now to follow the results are set forth, just as they were for the Tasmanian, in a progressive series from the lowest figure to the highest, or in the reverse way according to the scale of evolution, and each table also shows not only those with which the comparison is made, but also those which are excluded from the comparison.

TABLE I.—COMPARISON OF THE CALVARIAL HEIGHT (KALOTTENHÖHE).

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . . .	..	48·5	..
2. <i>Pithecanthropus erectus</i> . . . . .	..	62	..
3. Gibraltar . . . . .	..	85	..
4. Brüx . . . . .	..	85	..
5. Three Spy-Neandertal . . . . .	81	85·3	88
6. Four Kalmucks . . . . .	88	90·7	94
7. One hundred Australians, unsexed . . . . .	79·5	95	108
8. Galley Hill . . . . .	..	97	..
9. Forty-eight Tasmanians, unsexed . . . . .	87	97	108
10. Eight Veddahs . . . . .	94	99·2	107
11. Thirty-four Europeans, unsexed . . . . .	91	99·9	115
12. Twenty-three Dschagga negroes . . . . .	84	100	115·5
13. Egisheim . . . . .	..	100	..
14. Cro-Magnon . . . . .	..	101	..
15. Brünn . . . . .	..	103	..
16. Stängenäs . . . . .	..	106	..

TABLE II.—ANGLE OF FRONTAL CURVATURE MEASURED ON THE GLABELLA-BREGMA CHORD.

	Minimum.	Average.	Maximum.
1. An adult Gibbon . . . . .	..	160	..
2. Three Spy-Neandertal . . . . .	150	153·3	159
3. <i>Pithecanthropus erectus</i> . . . . .	148·5	153·2	158
4. One hundred Australians, unsexed . . . . .	123·5	139·6	153
5. Fifty Tasmanians, unsexed . . . . .	131·5	139·5	149
6. Seven Europeans . . . . .	127	135·4	148
7. Four Dschagga negroes . . . . .	122	131·5	136·5

Gibraltar, Brüx, Galley Hill, Brünn, Cro-Magnon, Veddahs, Kalmucks, Egisheim, and Stängenäs absent.

TABLE III.—COMPARISON OF THE CALVARIAL HEIGHT-BREADTH INDEX.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . . .	..	42·9	..
2. <i>Pithecanthropus erectus</i> . . . . .	..	46·6	..
3. Three Spy-Neandertal . . . . .	55·4	56·7	57·9
4. Gibraltar . . . . .	..	57·4	..
5. Four Kalmucks . . . . .	62·1	63·3	64·8
6. Brüx . . . . .	..	63·3	..
7. Cro-Magnon . . . . .	..	66·8	..
8. Forty-four Tasmanians, unsexed . . . . .	65·9	72·2	79·2
9. Four Europeans, unsexed . . . . .	69	72·4	76·2
10. One hundred Australians, unsexed . . . . .	60·2	72·7	85·4
11. Brünn . . . . .	..	74·1	..
12. Galley Hill . . . . .	..	74·6	..
13. Four Veddahs . . . . .	69·6	76·9	82·9

Dschagga negroes, Egisheim, and Stängenäs absent.

TABLE IV.—COMPARISON OF THE BREGMA ANGLE.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . . .	..	39·5	..
2. <i>Pithecanthropus erectus</i> . . . . .	34	37·5	41*
3. Three Spy-Neandertal . . . . .	45	47·5	50·5
4. Gibraltar . . . . .	50	50·5	51
5. Brüx . . . . .	..	51·1	..
6. Galley Hill . . . . .	..	52	..
7. Stängenäs . . . . .	..	52·5	..
8. Brünn . . . . .	..	54	..
9. Cro-Magnon . . . . .	..	54	..
10. One hundred Australians, unsexed . . . . .	49	54·7	60
11. Forty-five Tasmanians, unsexed . . . . .	51·5	56	64
12. Four Kalmucks . . . . .	55	56·5	57
13. Egisheim . . . . .	..	58	..
14. Twenty-four Dschagga negroes . . . . .	53	58·6	63·5
15. Forty Europeans . . . . .	54	59·9	68

Veddahs absent.

\* Employed for comparative purposes.

TABLE V.—COMPARISON OF THE CALVARIAL HEIGHT INDEX (KALOTTENHÖHEN-INDEX.)

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . .	..	34·2	..
2. An adult male chimpanzee . . . .	..	35·1	..
3. Three Spy-Neandertal . . . .	40·9	44·9	47
4. Gibraltar . . . .	..	45·4	..
5. Brünn . . . .	..	47·6	..
6. Galley Hill . . . .	..	48·2	..
7. Cro-Magnon . . . .	..	50	..
8. Brünn . . . .	..	51·2	..
9. One hundred Australians, unsexed . .	44·9	53	61·5
10. Four Kalmucks . . . .	52·8	54·5	84·9
11. Stängenäs . . . .	..	54·6	..
12. Egisheim . . . .	..	55·5	..
13. Forty-four Tasmanians, unsexed . .	48·3	56·1	62·7
14. Eight Veddahs . . . .	54·6	58·4	62·9
15. Twenty-three Dschagga negroes . .	52·1	59·8	67·1
16. Thirty-two Europeans . . . .	54·4	59·8	66·2

TABLE VI.—COMPARISON OF THE CALVARIAL HEIGHT HALF-SUM GLABELLA-INION LENGTH PLUS BREADTH INDEX.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . .	..	38·6	..
2. <i>Pithecanthropus erectus</i> . . . .	..	39·4	..
3. Three Spy-Neandertal . . . .	47	48·9	50
4. Gibraltar . . . .	..	50·7	..
5. Galley Hill . . . .	..	50·8	..
6. Brüx . . . .	..	54·8	..
7. Cro-Magnon . . . .	..	57·2	..
8. Four Kalmucks . . . .	57·1	58·7	60·2
9. Brünn . . . .	..	60·5	..
10. One hundred Australians, unsexed . .	52·3	61·3	69·9
11. Forty-four Tasmanians, unsexed . .	55·2	63	69·5
12. Five Europeans, unsexed . . . .	60·9	65·8	69·8
13. Four Veddahs . . . .	61·2	66·6	71·5

Egisheim, Stängenäs, and Dschagga negroes absent.

TABLE VII.—COMPARISON OF THE LENGTH OF THE PARIETAL ARC.

	Minimum.	Average.	Maximum.
1. An adult female chimpanzee . . . .	..	62	..
2. <i>Pithecanthropus erectus</i> . . . .	93	103	113
3. Brüx . . . .	..	108	..
4. Gibraltar . . . .	..	111	..
5. Seventeen Maories, unsexed . . . .	101	117	127
6. Three Spy-Neandertal . . . .	119	119·6	120
7. Forty-eight Tasmanians, unsexed . .	112	125·8	145
8. One hundred Australians, unsexed . .	109	125·9	147
9. Galley Hill . . . .	..	132	..
10. One European . . . .	..	133	..
11. Cro-Magnon . . . .	..	135	..
12. Brünn . . . .	..	139·5	..

Egisheim, Stängenäs, Dschagga negroes, Veddahs, and Kalmucks absent.

TABLE VIII.—COMPARISON OF THE FRONTAL ANGLE.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . . .	..	52.5	..
2. An adult male chimpanzee . . . . .	..	56	..
3. Three Spy-Neandertal . . . . .	57.5	64.8	70
4. Gibraltar . . . . .	..	73	..
5. Brüx . . . . .	..	74.7	..
6. Brünn . . . . .	..	75	..
7. Galley Hill . . . . .	..	82	..
8. Cro-Magnon . . . . .	..	83	..
9. Four Kalmucks . . . . .	80	85.2	91
10. One hundred Australians, unsexed . . . . .	71	85.2	100
11. Forty-four Tasmanians, unsexed . . . . .	72	86	96
12. Egisheim . . . . .	..	89	..
13. Stängenäs . . . . .	..	92.5	..
14. Forty Europeans, unsexed . . . . .	73	92.5	103
15. Twenty-four Dschagga negroes . . . . .	88	100.3	110

Veddahs absent.

TABLE IX.—COMPARISON OF THE BREGMA FOOT-POINT POSITIONAL INDEX.

	Minimum.	Average.	Maximum.
1. An adult male gibbon . . . . .	..	63.4	..
2. <i>Pithecanthropus erectus</i> . . . . .	39.7	44.1	50.2
3. Stängenäs . . . . .	..	38.9	..
4. Brüx . . . . .	..	37.3	..
5. Three Spy-Neandertal . . . . .	34.8	36.6	40.1
6. Gibraltar . . . . .	..	35.2	..
7. Galley Hill . . . . .	..	34.3	..
8. One hundred Australians, unsexed . . . . .	29.2	34.1	38.8
9. Brünn . . . . .	..	34	..
10. Forty-four Tasmanians, unsexed . . . . .	26	33.5	40.6
11. Egisheim . . . . .	..	33.3	..
12. Four Kalmucks . . . . .	30.1	32.8	37.4
13. Cro-Magnon . . . . .	..	32.6	..
14. Twenty-four Dschagga negroes . . . . .	26.6	32.1	37.2
15. Forty-five Europeans, unsexed . . . . .	22.2	30.4	35.7

Veddahs absent.

TABLE X.—COMPARISON OF THE LAMBDA ANGLE.

	Minimum.	Average.	Maximum.
1. Nearest anthropoids . . . .	43	55.5	68
2. <i>Pithecanthropus erectus</i> . . . .	..	66	..
3. Neandertal . . . .	..	66.5	67
Spy 1 . . . .	67	68	..
4. Gibraltar . . . .	..	69	..
5. Cro-Magnon . . . .	..	70	..
6. Galley Hill . . . .	..	74	..
7. Brünn . . . .	..	78	..
8. One hundred Australians, unsexed . .	70	79.5	90
9. Brüx . . . .	..	80	..
10. Forty-six Tasmanians, unsexed . .	74	80.5	88
11. Stängenäs . . . .	..	81.5	..
12. Modern Man . . . .	78	81.5	85

Egisheim, Dschagga negroes, Veddahs, and Kalmucks absent.

TABLE XI.—COMPARISON OF THE OPISTHIONIC ANGLE.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . .	..	64	..
2. Nearest anthropoids . . . .	50	59.5	69
3. Spy 1 . . . .	..	54	..
Neandertal . . . .	..	51.5	..
4. Galley Hill . . . .	..	42	..
5. Brünn . . . .	..	42	..
6. Thirty-eight Tasmanians, unsexed . .	34.5	40.6	47
7. One hundred Australians, unsexed . .	31	40	51.5
8. Gibraltar . . . .	..	36	..
9. Recent Man . . . .	31	35.5	40
10. Cro-Magnon . . . .	..	34	..

Egisheim, Stängenäs, Brüx, Dschagga negroes, Veddahs, and Kalmucks absent.

TABLE XII.—COMPARISON OF THE LENGTH OF THE FRONTAL ARC.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . .	..	92	..
2. <i>Pithecanthropus erectus</i> . . . .	100	110	120
3. Four Kalmucks . . . .	110	115.2	120
4. Three Spy-Neandertal . . . .	115	124	133
5. Seventeen Maories, unsexed . . . .	116	125	135
6. Five Europeans, unsexed . . . .	121	125.6	130
7. Forty-seven Tasmanians, unsexed . .	113	126	143
8. Gibraltar . . . .	..	126	..
9. One hundred Australians, unsexed . .	116	126.8	143
10. Brüx . . . .	..	135	..
11. Galley Hill . . . .	..	135	..
12. Brünn . . . .	..	135	..
13. Cro-Magnon . . . .	..	138	..

Egisheim, Stängenäs, Dschagga negroes, and Veddahs absent.

TABLE XIII.—COMPARISON OF THE LENGTH OF THE CHORD OF THE PARS CEREBRALIS OF THE OS FRONTALE.

	Minimum.	Average.	Maximum.
1. An adult female chimpanzee and a gibbon	..	55	..
2. Gibraltar	..	82	..
3. Spy-Neandertal	77	83·6	87
4. <i>Pithecanthropus erectus</i>	80	87·5	95
5. Eleven Europeans, unsexed	87	92·1	101
6. Fifty Tasmanians, unsexed	73	93·7	106·5
7. Galley Hill	..	95	..
8. Five Dschagga negroes	94	95·8	97
9. One hundred Australians, unsexed	85	95·9	112
10. Brünn	..	96	..
11. Cro-Magnon	..	97·5	..
12. Brüx	..	99	..

Egisheim, Stängenäs, Kalmucks, and Veddahs absent.

TABLE XIV.—COMPARISON OF THE LENGTH OF THE CHORD OF THE OS FRONTALE.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee	..	87	..
2. Four Kalmucks	98	103·8	107·5
3. <i>Pithecanthropus erectus</i>	96	104	112
4. Fifty Tasmanians, unsexed	97	109·5	120
5. Seventeen Maories, unsexed	103	110	119
6. One hundred Australians, unsexed	100	110·8	124
7. Gibraltar	..	111	..
8. Five Europeans, unsexed	109	112·5	118·5
9. Three Spy-Neandertal	108	114	119
10. Brüx	..	114	..
11. Galley Hill	..	120	..
12. Brünn	..	123	..
13. Cro-Magnon	..	123	..

Egisheim, Stängenäs, Dschagga negroes, and Veddahs absent.

TABLE XV.—COMPARISON OF THE PARIETAL FRONTAL ARC INDEX.

	Minimum.	Average.	Maximum.
1. Brüx	..	80	..
2. An adult female chimpanzee	..	82·6	..
3. <i>Pithecanthropus erectus</i>	71·1	85·8	102·7
4. Gibraltar	..	88	..
5. Seventeen Maories, unsexed	81	93·3	104
6. Three Spy-Neandertal	89·4	96·8	104·3
7. Galley Hill	..	97·7	..
8. Cro-Magnon	..	97·8	..
9. One hundred Australians, unsexed	87·7	99·3	113·9
10. Forty-five Tasmanians, unsexed	85·8	99·7	114·1
11. Brünn	..	103·3	..
12. One European	..	109·9	..

Egisheim, Stängenäs, Dschagga negroes, Veddahs, and Kalmucks absent.

TABLE XVI.—COMPARISON OF THE DISTANCE OF THE BREGMA FOOT-POINT FROM THE GLABELLA.

		Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i>	.	72	81.5	91
2. Brüx	.	..	75	..
3. Three Spy-Neandertal	.	67	72.3	81
4. An adult male chimpanzee	.	..	72	..
5. Galley Hill	.	..	69	..
6. Brünn	.	..	67.5	..
7. Gibraltar	.	..	66	..
8. Cro-Magnon	.	..	66	..
9. One hundred Australians, unsexed	.	51.5	61.2	74
10. Forty-four Tasmanians, unsexed	.	45	58.7	71.5
11. Four Kalmucks	.	51.5	54.6	61
12. Twenty-four Dschagga negroes	.	41	53.9	62.5
13. Thirty-five Europeans, unsexed	.	40	51.3	61

Egisheim, Stängenäs, and Veddahs absent.

TABLE XVII.—COMPARISON OF THE LENGTH OF THE PARIETAL CHORD.

		Minimum.	Average.	Maximum.
1. Brüx	.	..	100.5	..
2. <i>Pithecanthropus erectus</i>	.	..	104	..
3. Seventeen Maories, unsexed	.	92	104	110
4. Three Spy-Neandertal	.	104	107.7	113
5. Gibraltar	.	..	108	..
6. Forty-eight Tasmanians, unsexed	.	99.5	113	127
7. One hundred Australians, unsexed	.	98	114.6	137
8. Galley Hill	.	..	120	..
9. Cro-Magnon	.	..	123	..
10. Brünn	.	..	127.5	..

Anthropoids, Egisheim, Stängenäs, Dschagga negroes, Veddahs, Kalmucks, and Europeans absent.

TABLE XVIII.—COMPARISON OF THE CURVATURE INDEX OF THE OS FRONTALE.

		Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i>	.	93.3	94.6	96
2. An adult male chimpanzee	.	..	94.5	..
3. Three Spy-Neandertal	.	89.4	92.8	93.9
4. Brünn	.	..	91.1	..
5. Four Kalmucks	.	88.3	90.1	92.8
6. Five Europeans, unsexed	.	87.4	89.5	91.1
7. Cro-Magnon	.	..	89.1	..
8. Galley Hill	.	..	88.8	..
9. Gibraltar	.	..	88	..
10. One hundred Australians, unsexed	.	81.3	87.4	90.8
11. Forty-seven Tasmanians, unsexed	.	81.4	87.1	97.5
12. Stängenäs	.	..	85.2	..
13. Brüx	.	..	84.4	..

Egisheim, Dschagga negroes, and Veddahs absent.

TABLE XIX.—COMPARISON OF THE DISTANCE OF THE FOOT-POINT OF THE CALVARIAL HEIGHT FROM THE GLABELLA.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . . .	70	80·5	91
2. An adult male gorilla . . . . .	..	84	..
3. Four Kalmucks . . . . .	76	86·3	95
4. Forty-one Europeans, unsexed . . . . .	78	95·8	112·5
5. One hundred Australians, unsexed . . . . .	88	101·1	123
6. Forty-five Tasmanians, unsexed . . . . .	85	101·9	115·5
7. Gibraltar . . . . .	..	105·7	..
8. Brünn . . . . .	..	110	..
9. Three Spy-Neandertal . . . . .	103	111	123
10. Brüx . . . . .	..	111	..
11. Galley Hill . . . . .	..	111	..
12. Cro-Magnon . . . . .	..	121·5	..

Egisheim, Stängenäs, Dschagga negroes, and Veddahs absent.

TABLE XX.—COMPARISON OF THE GLABELLA-CEREBRAL CHORD INDEX.

	Minimum.	Average.	Maximum.
1. Gibraltar . . . . .	..	43	..
2. An adult female orang . . . . .	..	40	..
3. Three Spy-Neandertal . . . . .	34·4	39·6	43·1
4. Cro-Magnon . . . . .	..	32·3	..
5. Brünn . . . . .	..	31·2	..
6. <i>Pithecanthropus erectus</i> . . . . .	25·2	27·6	30
7. Egisheim . . . . .	..	27·5	..
8. Five Dschagga negroes . . . . .	23·3	27·4	30·3
9. Eleven Europeans, unsexed . . . . .	21·4	26·6	31·8
10. Fifty Tasmanians, unsexed . . . . .	17·6	25·5	35·6
11. Galley Hill . . . . .	..	25·2	..
12. Brüx . . . . .	..	24·2	..
13. One hundred Australians, unsexed . . . . .	15·6	23·6	33·5
14. Stängenäs . . . . .	..	18·6	..

Kalmucks and Veddahs absent.

TABLE XXI.—COMPARISON OF THE MAXIMUM BREADTH.

	Minimum.	Average.	Maximum.
1. An adult male chimpanzee . . . . .	..	113	..
2. Four Veddahs . . . . .	123	129.7	135
3. Brüx . . . . .	..	130	..
4. Galley Hill . . . . .	..	130	..
5. One hundred Australians, unsexed . . .	120	130.7	143
6. <i>Pithecanthropus erectus</i> . . . . .	..	133	..
7. Nineteen Andamanese Islanders, unsexed	128	133	141
8. Forty-eight Tasmanians, unsexed . . .	120	134.7	145
9. Fifteen Maories, unsexed . . . . .	128	136	141
10. Brünn . . . . .	..	139	..
11. Five Europeans (Germans), unsexed . . .	137	142.4	149
12. Ninety Europeans (Italians), unsexed . . .	124	142.5	155
13. One hundred and seventy-six Europeans (Scotch), unsexed . . . . .	128	143.6	159
14. Four Kalmucks . . . . .	140	146	148
15. Gibraltar . . . . .	..	148	..
16. Three Spy-Neandertal . . . . .	146	150.3	153
17. Cro-Magnon . . . . .	..	151	..

Egisheim, Stängenäs, and Dschagga negroes absent.

TABLE XXII.—COMPARISON OF THE CURVATURE INDEX OF THE OS PARIETALE.

	Minimum.	Average.	Maximum.
1. Gibraltar . . . . .	..	97.2	..
2. Three Spy-Neandertal . . . . .	90.4	93.7	96.3
3. Brüx . . . . .	..	93	..
4. <i>Pithecanthropus erectus</i> . . . . .	..	92?	..
5. Brünn . . . . .	..	91.3	..
6. Cro-Magnon . . . . .	..	91.1	..
7. One hundred Australians, unsexed . . .	81.6	91	103.6
8. Galley Hill . . . . .	..	90.9	..
9. Forty-seven Tasmanians, unsexed . . .	83.8	90	97.6

Anthropoids, Egisheim, Stängenäs, Dschagga negroes, Veddahs, Kalmucks, and Europeans absent.

TABLE XXIII.—COMPARISON OF HALF THE SUM OF THE GLABELLA-INION LENGTH PLUS THE BREADTH.

	Minimum.	Average.	Maximum.
1. An adult female gorilla . . . . .	..	129	..
2. Four Veddahs . . . . .	142.5	149.5	153.5
3. Forty-four Tasmanians, unsexed . . . . .	140.5	154	164.5
4. Four Kalmucks . . . . .	148.5	154.3	161
5. Brüx . . . . .		155	157.5
6. One hundred Australians, unsexed . . . . .	143	155.05	166
7. Five Europeans, unsexed . . . . .	153	156.8	159
8. <i>Pithecanthropus erectus</i> . . . . .	..	157	..
9. Galley Hill . . . . .	..	165.5	..
10. Gibraltar . . . . .	..	167.5	..
11. Brünn . . . . .	..	170	..
12. Three Spy-Neandertal . . . . .	172	174.3	177
13. Cro-Magnon . . . . .	..	176.5	..

Egisheim, Stängenäs, and Dschagga negroes absent.

TABLE XXIV.—COMPARISON OF THE CALVARIAL HEIGHT FOOT-POINT POSITIONAL INDEX.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . . .	38.6	44.4	50.2
2. Four Kalmucks . . . . .	43.6	52.6	56.8
3. Brünn . . . . .	..	54.7	..
4. Galley Hill . . . . .	..	55.2	..
5. Three Spy-Neandertal . . . . .	52	55.7	60.8
6. Forty-one Europeans, unsexed . . . . .	48.8	56.2	66.9
7. One hundred Australians, unsexed . . . . .	44.8	56.3	65.3
8. Gibraltar . . . . .	..	56.5	..
9. Forty-four Tasmanians, unsexed . . . . .	53.1	59	64.8
10. Brüx . . . . .	..	60 (61.6)	..
11. Cro-Magnon . . . . .	..	60.1	..
12. An adult female gorilla . . . . .	..	61.8	..

Egisheim, Stängenäs, Dschagga negroes, and Veddahs absent.

TABLE XXV.—COMPARISON OF THE GLABELLA-INION LENGTH.

	Minimum.	Average.	Maximum.
1. An adult male gorilla . . . . .	..	147	..
2. Four Kalmucks . . . . .	157	166·2	174
3. Twenty-three Dschagga negroes . . .	145	167·4	180
4. Thirty-one Europeans, unsexed . . .	155	168	184
4. One hundred and fifty-four Europeans, unsexed . . . . .	154	169·3	199
5. Eight Veddahs . . . . .	160	169·8	176
6. Forty-four Tasmanians, unsexed . . .	157	173·1	188
7. One hundred Australians, unsexed . . .	162	179·5	196
8. <i>Pithecanthropus erectus</i> . . . . .	..	181	..
9. Brüx . . . . .	..	185 (180)	..
10. Gibraltar . . . . .	..	187	..
11. Three Spy-Neandertal . . . . .	196	198·6	202
12. Galley Hill . . . . .	..	201	..
13. Brünn . . . . .	..	201	..
14. Cro-Magnon . . . . .	..	202	..

Egisheim and Stängenäs absent.

TABLE XXVI.—COMPARISON OF THE LENGTH OF THE CHORD OF THE PARS GLABELLA OF THE OS FRONTALE.

	Minimum.	Average.	Maximum.
1. An adult female orang . . . . .	..	20	..
2. One hundred Australians, unsexed . . .	15	22·3	29·5
3. Forty-nine Tasmanians, unsexed . . .	18	23·8	29
4. <i>Pithecanthropus erectus</i> . . . . .	..	24	..
5. Brüx . . . . .	..	24	..
6. Galley Hill . . . . .	..	24	..
7. Eleven Europeans, unsexed . . . . .	19·5	24·5	28
8. Five Dschagga negroes . . . . .	27	26·4	28·5
9. Brünn . . . . .	..	30	..
10. Cro-Magnon . . . . .	..	31·5	..
11. Three Spy-Neandertal . . . . .	30	33·1	37·5
12. Gibraltar . . . . .	..	36	..

Egisheim, Stängenäs, Kalmucks, and Veddahs absent.

TABLE XXVII.—COMPARISON OF THE ANGLE OF PARIETAL CURVATURE.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . . .	..	150	..
2. Chimpanzee . . . . .	..	149	..
3. Three Spy-Neandertal . . . . .	142·5	142·6	143
4. Galley Hill . . . . .	..	139	..
5. One hundred Australians, unsexed . . .	125	135·7	145
6. Brünn . . . . .	..	135	..
7. Forty-nine Tasmanians, unsexed . . .	125·5	134·3	141·5
8. Cro-Magnon old man . . . . .	..	134	..
9. One European (Schwalbe) . . . . .	..	129	..

Gibraltar, Brüx, Kalmucks, Veddahs, Dschagga negroes, Egisheim, and Stängenäs absent.

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TABLE XXXIX.

Stängenäs.		Brünn.		Cro-Magnon.		Egisheim.		Dschagga Negroes.	
No.	r.								
1	.9749	14.62	0	3.43	9.28	9.36	10.73	11.82	12.33
2	.9602	5.76	0	1.37	.88	4.63	6.85	4.12	4.15
3	.9512	11.41	0	1.25	4.87	7.83	5.76	10.94	9.90
4	.9212	12.90	1.15	0	7.49	6.15	11.15	10.33	10.65
5	.9379	14.07	0.50	0	4.83	6.46	4.11	8.02	9.06
6	.9314	11.17	0	0.32	4.86	5.80	5.45	6.82	7.56
7	.9179	9.18	0	0.86	5.80	5.99	3.32	8.83	8.83
8	.9216	12.90	0.95	0	5.53	6.97	10.17	9.42	11.04
9	.8508	11.91	0	6.97	9.11	9.64	9.64	10.58	10.50
10	.8781	9.66	0	3.90	5.01	4.08	4.08	6.89	6.88
11	.8711	7.84	1.18	0	7.32	6.21	6.21	5.75	6.12
12	.8382	9.22	0	3.61	6.81	8.62	6.41	6.45	6.94
13	.8234	9.06	0	6.69	5.56	9.06	5.89	8.42	8.23
14	.7845	8.63	0	4.07	5.75	6.47	4.02	5.70	7.91
15	.7473	7.47	0.65	1.45	2.00	4.20	2.56	4.83	4.42
16	.6999	8.40	2.64	0	4.31	1.81	2.56	7.48	5.65
17	.7045	5.64	..	0.73	1.56	0	1.50	3.73	2.94
18	.7044	8.45	0.08	0	5.47	8.45	1.49	5.97	4.80
19	.5974	6.57	0.36	0	4.04	4.89	0.93	3.30	4.89
20	.5526	7.18	0.88	4.54	0	5.54	1.00	5.71	5.24
21	.4585	5.50	0	2.89	5.07	2.46	5.40	4.78	2.56
22	.4465	3.57	..	2.58	0	2.09	1.74	3.08	3.13
23	.4613	5.54	0	3.26	4.49	3.03	2.58	2.95	3.04
24	.1589	1.75	1.75	0	1.22	1.57	1.13	0.82	1.19
27	.9495	7.60	0.36	0	1.15	2.68	2.68	5.18	3.98
B	.5539	4.99	..	0	1.15	2.69	2.74	1.46	4.41
Total	.	220.99	10.66	51.92	124.43	111.57	96.92	163.39	154.24
Pos. Max.	..	206.79	220.99	207.63	209.79	220.99	143.05	220.99	215.23
Rel. Pos.	..	-0.515	-2.349	-5.485	-5.931	-5.049	-6.775	-7.393	-7.166

TABLE A.—COMPARISON OF THE NASIO-INION LENGTH.

	Minimum.	Average.	Maximum.
1. Four Kalmucks . . . .	154	162·5	169
2. Twenty Europeans, unsexed . . . .	154	168	178
3. <i>Pithecanthropus erectus</i> . . . .	..	168	..
4. Forty-four Tasmanians, unsexed . . . .	144	169·7	183
5. One hundred Australians, unsexed . . . .	157	173·8	191
6. Gibraltar . . . .	..	182·1	..
7. Cro-Magnon old man . . . .	..	194·2	..
8. Three Spy-Neandertal . . . .	192	196·3	199

Anthropoids, Brüx, Galley Hill, Veddahs, Dschagga negroes, Egisheim, Brünn, and Stängenäs absent.

TABLE B.—COMPARISON OF THE GLABELLA-LAMBDA LENGTH.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> . . . .	..	171	..
2. Forty-eight Tasmanians, unsexed . . . .	162	173·2	189
3. Gibraltar . . . .	..	177	..
4. One hundred Australians, unsexed . . . .	161	178·6	194
5. Egisheim . . . .	..	185	..
6. Brüx (Schwalbe) . . . .	..	185	..
7. Three Spy-Neandertal (Klaatsch) . . . .	185	185·3	186
8. Cro-Magnon old man (Klaatsch) . . . .	..	193	..
9. Galley Hill . . . .	..	194	..
10. Brünn . . . .	..	197	..

Anthropoids, Gibraltar, Kalmucks, Veddahs, Europeans, and Dschagga negroes absent.

TABLE C.—COMPARISON OF THE LAMBDA-GLABELLA-INION ANGLE.

	Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> (Klaatsch) . . . .	..	10	..
2. Three Spy-Neandertal (Schwalbe) . . . .	15	15·8	16·5
3. Galley Hill (Klaatsch) . . . .	..	17	..
4. Brünn (Klaatsch) . . . .	..	17	..
5. One hundred and sixty-eight Australians (Berry, Robertson, and Klaatsch), unsexed . . . .	13·5	17·1	22·5
6. Cro-Magnon old man (Klaatsch) . . . .	..	17·5	..
7. Fifty-three Tasmanians (Berry, Robertson and Klaatsch), unsexed . . . .	15	18	23
8. Brüx (Schwalbe's estimation) . . . .	..	20	..
9. Forty-three ancient Egyptians (Schwalbe) . . . .	15	20·7	29
10. Twenty-five Dschagga negroes (Schwalbe) . . . .	17	21·5	28
11. Thirty-five Europeans (Schwalbe) . . . .	17·5	22	30
12. Gibraltar (Sollas) . . . .	..	24 ?	..

Anthropoids, Kalmucks, Veddahs, Egisheim, and Stängenäs absent.

TABLE D.—COMPARISON OF THE DISTANCE OF THE BREGMA FOOT-POINT FROM THE GLABELLA ON THE GLABELLA-LAMBDA LINE.

		Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i> (Klaatsch)	.	..	79·5	..
2. Gibraltar	.	..	82·2 ?	..
3. Forty-eight Tasmanians, unsexed	.	65	82·3	96
4. One hundred Australians, unsexed	.	73	85·3	101
5. Galley Hill (Klaatsch)	.	..	91·5	..
6. Cro-Magnon old man (Klaatsch)	.	..	91·5	..
7. Brünn (Klatsch)	.	..	92·7	..
8. Three Spy-Neandertal (Klaatsch)	.	..	98	..
9. Brüx (Schwalbe estimated)	.	..	100·5	..

Anthropoids, Kalmucks, Veddahs, Europeans, Dschagga negroes, Egisheim, and Stängenäs absent.

TABLE E.—COMPARISON OF THE BREGMA FOOT-POINT GLABELLA-LAMBDA INDEX.

		Minimum.	Average.	Maximum.
1. <i>Pithecanthropus erectus</i>	.	..	46·4	..
2. Gibraltar	.	..	46·4 ?	..
3. Forty-eight Tasmanians, unsexed	.	40·1	46·7	51·8
4. Brünn	.	..	47	..
5. Galley Hill	.	..	47·1	..
6. One hundred Australians, unsexed	.	40·9	47·2	53·1
7. Cro-Magnon old man	.	..	47·4	..
8. Three Spy-Neandertal	.	..	52·8	..
9. Brüx	.	..	54·3	..

Anthropoids, Kalmucks, Veddahs, Europeans, Dschagga negroes, Egisheim, and Stängenäs absent.

In the foregoing tables there are several points to which we should like to direct particular attention.

Firstly, as regards the tables, Nos. I. to XXVII. are those already employed in our previous Tasmanian work (3). Tables A to E inclusive comprise the additional observations which we then stated we intended to employ in our Australian researches, and which are, therefore, now included. It is a matter of congratulation that we are enabled to incorporate the five additional observations not previously made upon the Tasmanian. Mr L. W. G. Büchner, a recently appointed Victorian Government Research Scholar in the Physical Anthropological Laboratory of the University of Melbourne, has been engaged in an examination of the Tasmanian facial

skeleton, and has kindly furnished us with the necessary Tasmanian data under Tables A to E inclusive for comparison with the Australian.

Secondly, as regards the numerical order of the Tables. In our previous Tasmanian work, Tables I. to XXVII. were simply set forth in the accidental order in which the observations were recorded. Our fellow-worker, Dr K. Stuart Cross, subsequently showed (4) that these morphological observations were not all of equal morphological value. He set them forth in their correct order of value, and in this work we have arranged Tables I. to XXVII. in Dr Cross's order. Table II. of the Tasmanian work becomes therefore now Table I., Table XVII. similarly becomes Table II., and so on as determined by Dr Cross. Tables A to E still retain their accidental order of observation.

Thirdly, as already stated, additional objects of comparison have been incorporated in the present tables whenever it was possible to secure accurate data. It is, however, a matter for regret that so few figures dealing with the morphological form analysis of the human skull on the lines advocated by Schwalbe and Klaatsch are as yet available, and hence, although extended, the present tables are not even yet as complete as we should have desired. If our colleagues in Europe would undertake the necessary researches on British long and round barrow skulls, on modern Europeans, on pre-historic and modern Egyptians, and so forth, the problem of the true place in Nature of the Australian and Tasmanian aboriginal inhabitants would be rendered simpler. These difficulties notwithstanding, a comparison of our previously published Tasmanian tables with those now incorporated will show that the latter have been increased. Thus the Egisheim and Stängenäs primitive crania have been included (Frédéric, 8) whenever the necessary figures were available. The 17 Maories first mentioned in Table VII. are from the work of Mollison (9). The Andamanese Islanders and the 90 Italians incorporated in Table XXI. are from the Royal College of Surgeons Catalogue of Osteology, and have been worked out therefrom by ourselves. The 176 Europeans (Scotch) mentioned in the same table are from the work of Sir William Turner (10).

#### THE RELATIVE EVOLUTIONARY POSITIONS OF THE AUSTRALIAN AND THE TASMANIAN.

In our previous Tasmanian communication we stated that we thought the Australian would stand on the plus side of the Tasmanian—that is to say, we confidently expected to find that the Australian would be a more highly evolved morphological type than the Tasmanian. This expectation

has not, however, been realised. Mr W. M. Holmes, of the Natural Philosophy Department of this University, has been good enough to apply Cross's mathematical formula to the results of the observations recorded in the several tables. He finds the Australians are represented by the figure 0·739, and the Tasmanians by 0·779. The complete results are graphically represented in fig. 2, and numerically in Table XXIX. An examination of fig. 2 will demonstrate that the Australian, as regards his skull type, is less highly evolved, morphologically, than is the Tasmanian. How far this result agrees with one's preconceived conceptions, it is difficult to say; but probably an extract from *Nature* of the 14th July 1910, taken from a review of Professor Keith's Hunterian Lectures on the *Anatomy and Relationships of the Negro and Negroid Races*, best reconciles the position. It is there stated that "an analysis of the cranial features of the aborigines of Tasmania and of Australia shows that we have in these two races an early stage in the differentiation of the negro and negroid races of mankind. The Tasmanian is the most primitive type of *negro* yet discovered; the Australian, on the other hand, although deeply pigmented and less Simian in some features than the Palæolithic European, is the most primitive representative of the *negroid* race. Negroid as he is, the native Australian represents a stage in the evolution of the dominant non-negroids of the northern hemisphere. It is a remarkable fact that the negro and negroid races occur side by side, not only in Australasia, but in Asia proper and in Africa."

If this be the case, our results would appear to harmonise with the views expressed in the above quotation, for our work simply shows that, as regards his cranium at all events, the negroid Australian has not progressed quite so far in the evolutionary scale as has the Tasmanian negro.

#### ARE THE AUSTRALIANS AND THE TASMANIANS ONE AND THE SAME RACE?

If we have interpreted the above extract from *Nature* correctly, it would appear to be the opinion of the reviewer that the Australians and the Tasmanians are, in effect, different types, if not, indeed, different races. Sir William Turner, too, would appear to hold the same view, for, in his "*The Craniology, Racial Affinities, and Descent of the Aborigines of Tasmania*" (11), he states: "From the consideration of these characters the skulls support the opinion, based on the study by so many observers of the external features, that the existing aborigines of Australia are distinct from the Tasmanians, although the presence, in a proportion of the natives

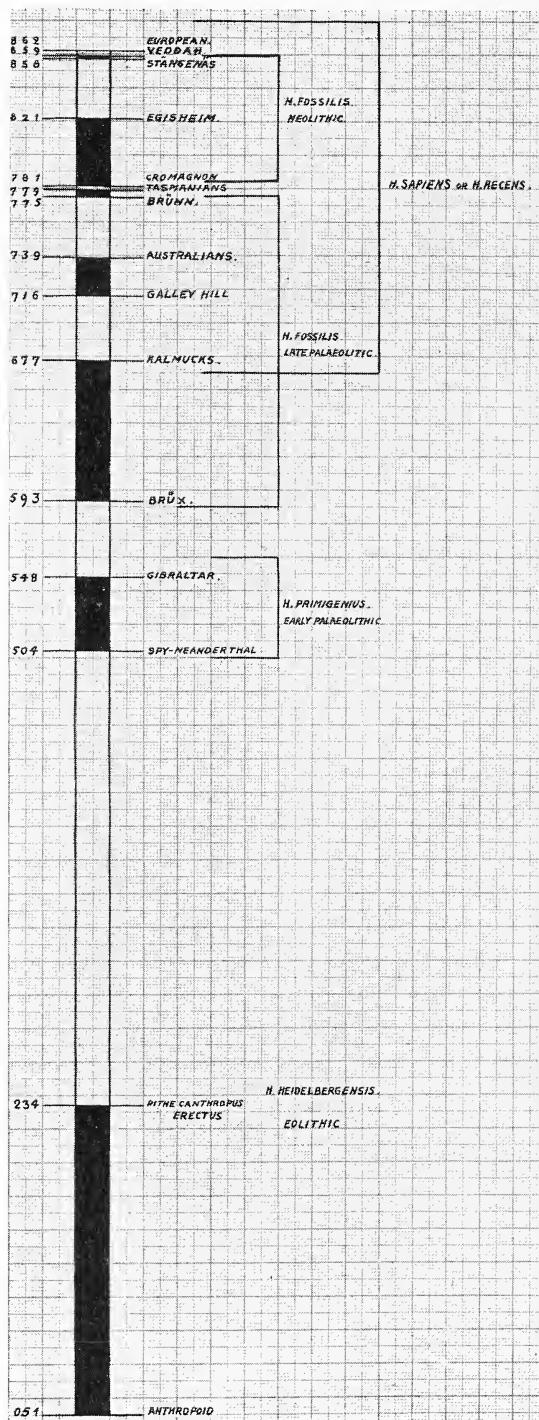


FIG. 2.—To illustrate the place in nature of the Australian and Tasmanian Aborigines, and the comparison of both with prehistoric and recent forms of man.  
On the extreme left are the figures indicative of the mathematical positions of the several objects. These have been worked out from the data of the paper by Mr W. M. Holmes, M.A., B.Sc., of the University of Melbourne, with Dr Gross's formula. The evolutionary intervals indicated by the figures are correctly spaced in the diagram. On the right is shown the classification of Prehistoric and Modern Man as given by Duekworth in his *Prehistoric Man*.

of South and West Australia, of skulls in which the height was less than the breadth, the not infrequent sunk sagittal suture, the more marked parietal eminences, and the antero-posterior parietal depressions, point to a possible amount of intermixture and racial affinity of these Australian tribes with the Tasmanians."

The Breslau school of anthropologists apparently hold the directly opposite view, for Basedow (12), a pupil of Professor Klaatsch, states that "the few superficial characteristics of the Tasmanian skull are not sufficient proof of his different origin from the Australian. It appears much more probable that in consequence of the comparatively recent separation of Tasmania from the mainland the Tasmanians have from that time first inherited their superficial differential characteristics." Basedow concludes by stating quite bluntly that the "Tasmanian was an insular form of the genuine Australian." The pupil's view is apparently held by the master, for we find Klaatsch stating that "the Tasmanians do not show any nearer relationships to other races than the Australians." He adds that the separation of the two races probably occurred a very long time ago. Alsberg (13) would appear to agree with Klaatsch.

Between the two extreme views above quoted there appears to be an intermediate opinion represented by Haddon, Keane, and many other anthropologists. For the illustration of this school of thought one characteristic quotation must suffice, though it would be easy to multiply examples. Haddon (14), in his *Races of Man and their Distribution*, published in 1911, says: "It is generally believed that Australia was originally inhabited, or at all events in parts, by Papuans or Negritoës, who wandered on foot to the extreme south of that continent. When Bass' Strait was formed, those who were cut off from the mainland formed the ancestors of the Tasmanians, who never advanced beyond an early stage of Stone-Age culture. Later, a pre-Dravidian race migrated into Australia, and overran the continent and absorbed the sparse aboriginal population. Since then they have practically remained isolated from the rest of the world. Their languages bear no relation to the Austronesian or Oceanic linguistic family." A somewhat similar view was advanced by one of us in 1909 (Berry, 6).

Whether the divergent views as to the commonality of race of Australians and Tasmanians be quoted *in extenso* or but briefly as above, it is clear that all are based on either pure theory or on certain slight superficial osteological resemblances or differences according to the opinions of the author.

We now propose to submit this question of the community of race or other-

wise of the Australian and Tasmanian to a somewhat severe proof. Whatever may be the result, it must be remembered that it is the first time that such an attempt has been made on the lines of strictly severe scientific analysis; and further, that in submitting the question to such proof, we are now enabled to deal with the largest numbers of Australian and Tasmanian crania which have ever yet been employed; and lastly, that the application of such an analysis to what previously has been mere theory is due to the introduction of some ingenious craniological methods by Dr Th. Mollison, formerly of Zurich, and now of Dresden.

In 1908 Mollison published in the *Zeitschrift für Morphologie und Anthropologie* his "Beitrag zur Kranieologie und Osteologie der Maori" (9). In this paper he introduced for the first time what he then termed the "Abweichungsindex." The object of what in English we should term the "variation index" is to discover if two skulls belong to one and the same race or not, and the procedure adopted is as follows:—

Multiply the distance of the individual from the average value of the standard type group by 100, and divide the product by the sum of the extremest distance of the group on the minimum or maximum side of the variation breadth. Thus a standard group of skulls (see fig. 3) has an average greatest breadth value of 135, and a cephalic index average value of 76. Another skull to be compared with this group has a greatest breadth of 126 and a cephalic index of 64. The question to be answered by the variation index is, Does this skull belong to the same race as the standard group or not? The distance of the doubtful skull from the average value of the standard group is for the greatest breadth 9, and for the cephalic index 12. Multiply these figures, both of which are on the minimum range of variation side of the standard group, by 100, and divide by the greatest range of variation of the standard group on the maximum or minimum side as the case may be: in this case the minimum side. The correct figures will therefore be found in the example quoted by subtracting for the greatest breadth 129 from 135, and for the cephalic index 70 from 76. In each instance the difference is 6. If this calculation be worked out in the manner indicated, it will be found that the variation index for the imaginary object to be compared with the standard group is, for the greatest breadth 150, and for the cephalic index 200.

To compare the variation indices of a large number of characteristics, draw a straight line which shall be supposed to pass through the average figures of each characteristic. Parallel to this draw two lines at arbitrary distances and supposed to represent the minimum and maximum range of variation of each observation from the average for the same; in each

instance the distance of the minimum or maximum parallel from the line of average measurement is supposed to represent 100 per cent.

The variation index for the object to be compared is now set at its correct distance from the average line on the plus or minus side. Connect these several points together, and a graph is at once constructed which, if

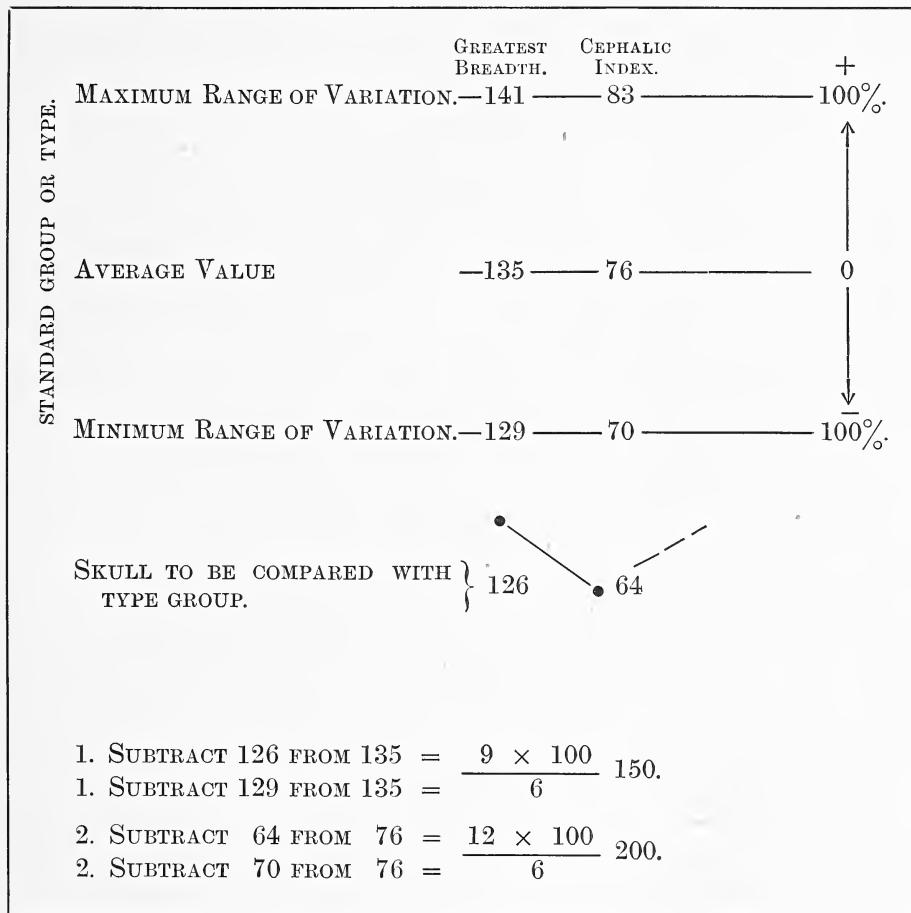


FIG. 3.—To illustrate Mollison's Variation Index.

it lies within the parallels, is proof that the compared stock is of the same race as the original group. If it be outside, then the compared object and the original group are not of the same species. A glance at the figure referred to (fig. 3) will illustrate the whole method of working Mollison's variation index, and will also demonstrate the fact that the supposititious skull to be compared with the crania of the standard group or type is of a different race. Mollison's original paper contains some highly instructive examples of the working of this index. Amongst other things he was

enabled to prove that a skull in the Zurich collection alleged to be that of a Maori was not a Maori skull, but that of an Australian aboriginal.

It must not be supposed that in adopting Mollison's method we are breaking new ground. His method has already been adopted by Czekanowski (15) for the solution of a problem almost precisely similar to that with which we are confronted—namely, the racial affinities of the Central African pygmies; by Oppenheim, in his "Zur Typologie des Primaten-craniums" (16); and by Radlauer (17); and we believe that we are correct in stating that all these authors have accepted the correctness of Mollison's methods and have abided by the conclusions to which that method led them.

In fig. 4 the Tasmanian is taken as the basis. The 32 morphological observations made by us upon the cranium are indicated by the numbers 1 to 27, and the letters A to E, inclusive. The centre line in that figure labelled "Tasmanian average" is supposed to represent the average values for these 32 observational counts on the Tasmanian cranium. At an arbitrary distance from this centre line are drawn two parallel lines which indicate the maximum and minimum ranges of variation of the Tasmanian from the average, and which are uniformly treated throughout as being equal to 100 per cent. The Australian "variation index" is plotted in for each of the 32 counts of the investigation, and is indicated in the figure by the dotted line. It will be clearly evident that the Australian "variation index" falls altogether within the Tasmanian maximum and minimum ranges of variation, thus proving conclusively, according to Mollison, that Australian and Tasmanian are one and the same race.

Mollison, in the paper already referred to, speaking of his variation index, says, "Schon für die Vergleichung in einem einzelnen Merkmal ist dieses Verfahren zweckmassig. Sien voller Wert zeigt sich aber erst dann, wenn wir eine grössere Reihe von Merkmalen vor uns haben, bezüglich deren wir die Stellung des Individiums zu der Grupp beurteilen sollen."

As Mollison is clearly of opinion that his method will always furnish more accurate results if the procedure be made to include as large a number of observations as possible, we have thought it desirable to supplement these 32 morphological observations of the form analysis of the skull by an additional series of craniological observations specially recorded by us on both Australian and Tasmanian for this special purpose and which have little or nothing to do with the form analysis of the skull.

These additional observations are 14 in number as given on page 146, and have been specially selected by us for two reasons: firstly, because they have nothing to do with the measurements and angles concerned in our investigation of the form analysis of the skull; and secondly, because

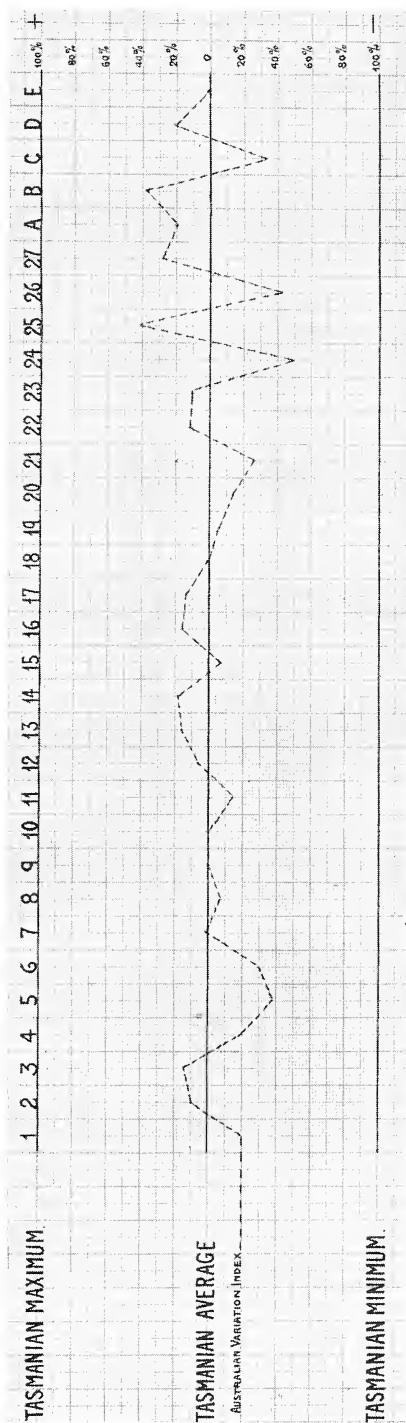


FIG. 4.—The Australian Variation Index plotted out upon the Tasmanian as the basis.

they are those uniformly recorded in the Osteological Catalogue of the Royal College of Surgeons of England, and have, therefore, enabled us to make use of any further comparative data which we desired—an opportunity of which, as will be seen later, we have availed ourselves.

It will be noticed that one measurement, and one only, is common to the 32 form analysis counts and the 14 general craniological observations, and that is the maximum breadth. We have, however, availed ourselves of an increased number of Tasmanian skulls under this count from one of our previous communications, and this fact explains the slight divergence in the results obtained.

The total number of observations now available—namely, 46, composed

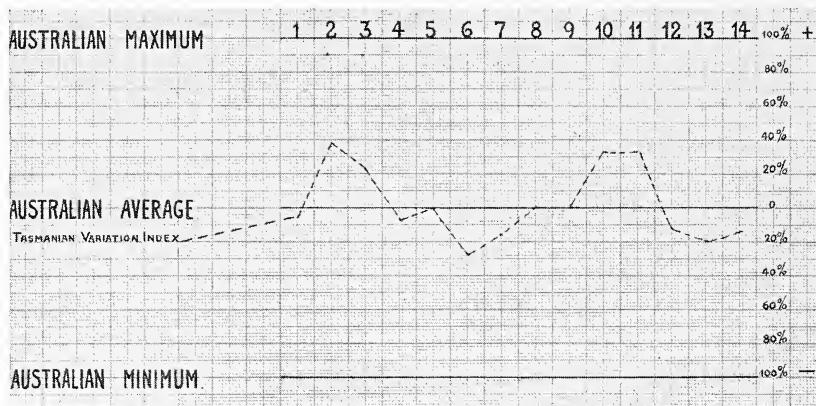


FIG. 5.—The Tasmanian Variation Index for 14 craniological observations plotted out upon the Australian as the basis.

of the 32 form analysis counts and the 14 craniological observations—is the largest number which has yet been employed for the working out of Mollison's variation index. Mollison himself only employed some 24 observations, Czakanowski about 28, Oppenheim 8, and Radlauer 20.

In fig. 5 these 14 general craniological observations are set out with the Australian as the base. The Tasmanian variation index is plotted out in a dotted line, and the results conform in every respect to those already obtained by the 32 form analysis figures. In all cases the variation index falls altogether within the maximum and minimum range of variation, thus again proving conclusively, according to Mollison's method, that Australian and Tasmanian are one and the same race.

**THE PHYSICAL RELATIONS OF THE AUSTRALIAN AND TASMANIAN TO  
THE SPY-NEANDERTAL RACE.**

It has repeatedly been stated that the Australian aboriginal is, in his physical construction, so closely related to the Spy-Neandertal men as to justify the assumption that they are of the same race, or at all events have been derived from the same stock. Thus Klaatsch (18) says, "The general formation of the skull capsule and the form of the facial skeleton have led him to the conclusion that the Australian and the Neandertal race have many features in common as heirlooms from a common primitive stock from which the Australian and the Neandertal race have developed in different directions."

von Luschan (19) probably shares this belief, for he says, "The primitive qualities or characters (*Eigenschaften*) of the Australian show a considerable amount of similarity with similar characteristics of the oldest European man, which possibly underlies a real relationship."

Stratz (20), in advancing his ideas for the correct racial divisions of mankind, says that amongst the "protomorphic" races he regards the modern Australian as amongst those which stand nearest to the oldest monogenetic common primitive stock.

Schoetensack (21), in advancing his theory that man originated in Australia, states that in Pliocene times the fauna of Australia contained not even one single dangerous rival for the evolution of man. Many human varieties could have developed themselves under these conditions, and the modern Australian is even to-day extraordinarily rich in varieties; the closely allied form relations of the oldest European human species (Spy-Neandertal) turns one's thoughts towards the emigration of such a variety.

Alsberg (13) agrees with Klaatsch as to the primitive nature of the Australian and points to the Neandertal species as a step in the ancestral series.

In view of the above opinions—and they could be multiplied—we thought that it would be of some interest to apply Mollison's variation index to the alleged relationship of Australian and Spy-Neandertal. For this purpose we have employed the 32 form analysis observations only, as it is obvious that the 14 general craniological characteristics, including as they do orbital and nasal measurements and indices, are not available for the Spy-Neandertal group.

In fig. 6 the variation index of the three Spy-Neandertal crania is plotted out upon our 100 Australians as a basis. A glance at the resulting

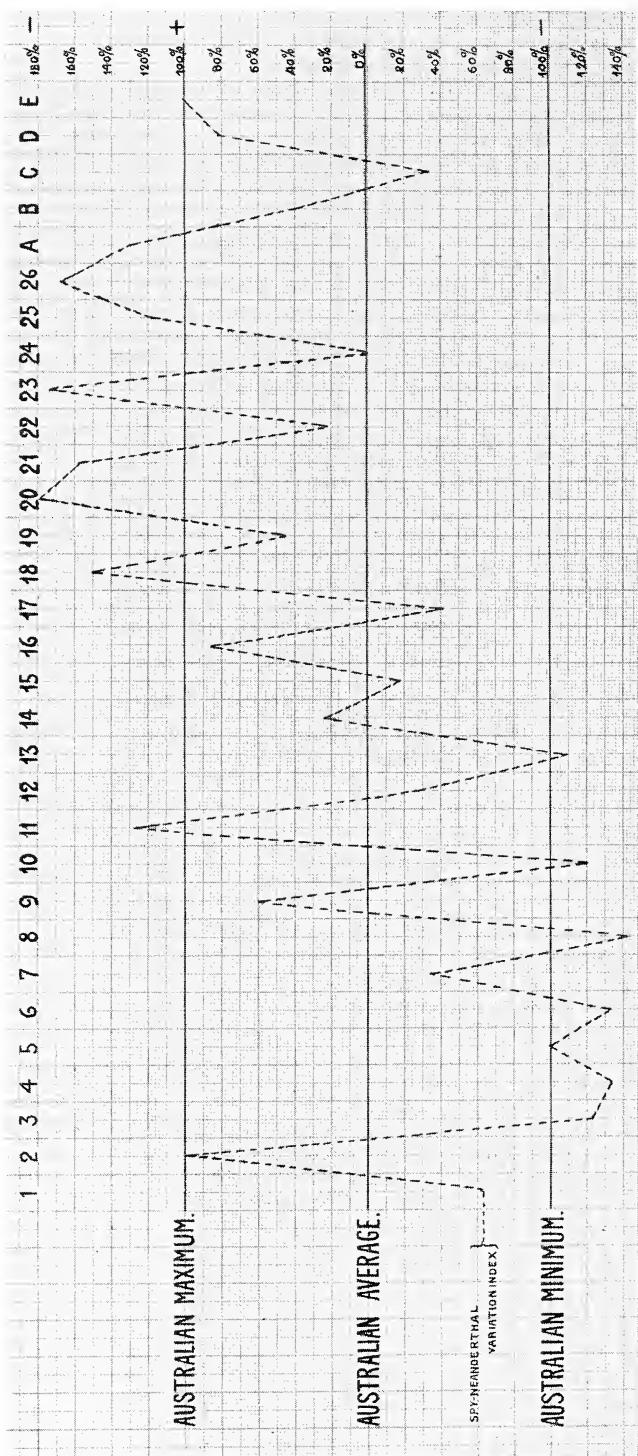


Fig. 6.—The Spy-Neandertal Variation Index plotted out upon the Australian as the basis.

graph will show that here a totally different result is produced than in the similar graphs for Tasmanians and Australians. If Mollison's index be a correct guide to this question of similarity of race, it is clear that here we are dealing with two entirely different races. The Spy-Neandertal variation index graph is seen to be highly irregular, and is by no means confined within the maximum and minimum range of Australian variation.

This fact is still more strikingly brought out in fig. 7, where the Tasmanian is taken as the basis, and on it are plotted out the variation indices for the Australian and the Spy-Neandertal group. The variation index for the Australian is seen to be very uniform and to fall altogether within the Tasmanian range of variation, whilst that for the Spy-Neandertal group is most irregular and is altogether outside the range of variation more often than within it. This figure conclusively demonstrates one or other of two things: either that Mollison's variation index is not a reliable guide to the differentiation of race, or that the views quoted as to the unity of type of Australian and Spy-Neandertal are erroneous. It must, we think, be admitted by any fair-minded critic that one or other of the two things must therefore, in future, be eliminated from scientific discussion.

A closer glance at the variation index for the Spy-Neandertal race on either the Australian or the Tasmanian (figs. 6 and 7) will show that, notwithstanding the marked irregularity of the graph, the index occasionally falls within the range of variation. In the case of both the Australian and the Tasmanian this happens 13 times out of 32. If Mollison's variation index be, indeed, any reliable guide to this question of differentiation of race, then the most we can admit for the alleged relationship of Australian to Spy-Neandertal is that they have something in common, but that that something is so little that the view of commonality of race between the two must be abandoned. We agree therefore with Schwalbe (22), who says, "The Australians are certainly a primitive race, but have nothing whatever to do with *Homo primigenius*."

#### THE RELATIONSHIP OF AUSTRALIAN AND TASMANIAN TO A SUPPOSED PURE RACE LIKE THE ANDAMANESE.

The next use which we propose to make of Mollison's variation index is to test the relationship, if any, of the Tasmanian and Australian with a supposed homogeneous race like the Andamanese Islanders. This will be a useful comparison, because it has been thought by some observers that the Australians are very closely akin to these Islanders. Quite apart

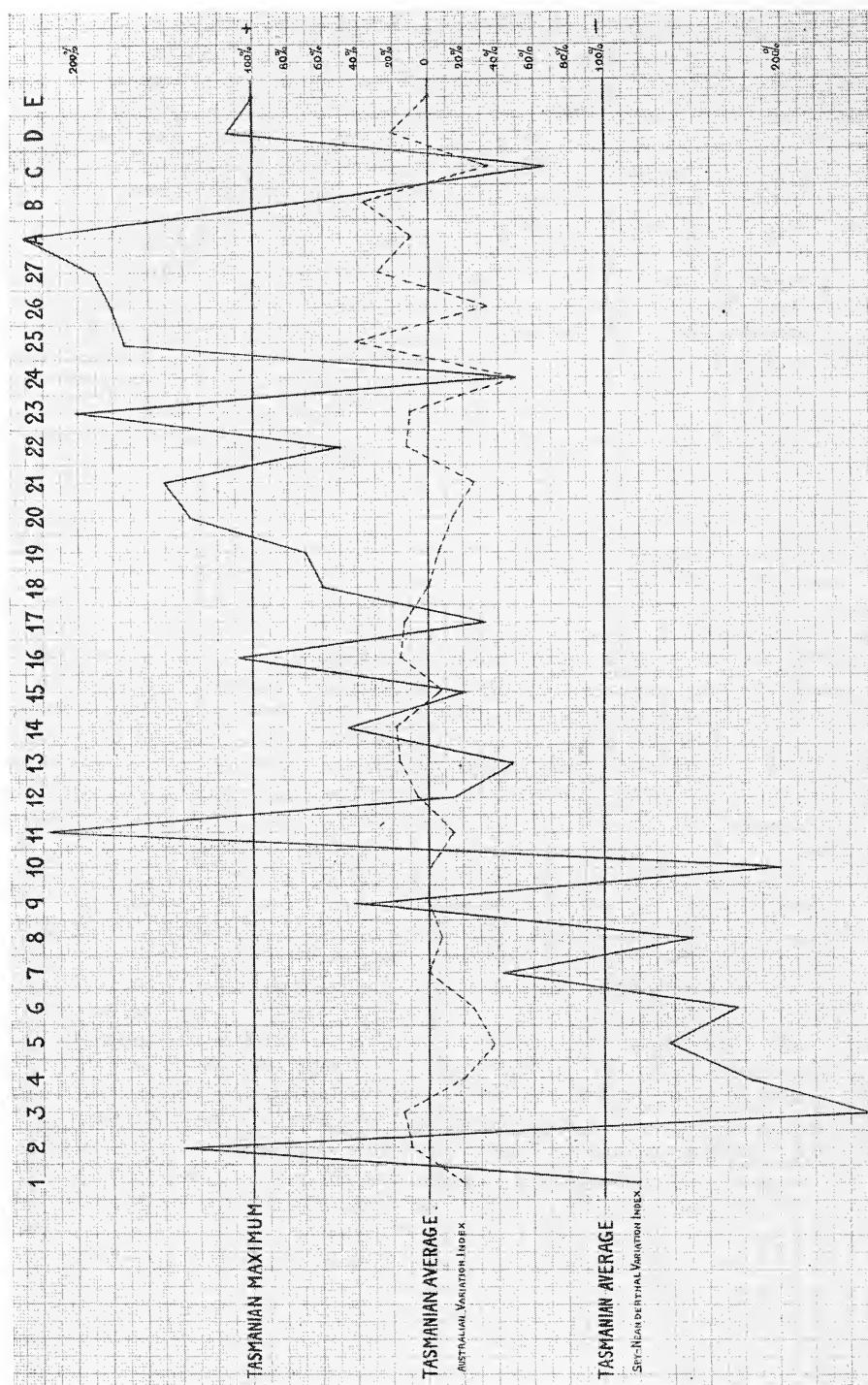


FIG. 7.—The Australian Variation Index and the Spy-Neandertal Variation Index plotted out upon the Tasmanian as the basis.

from this the comparison will be of interest, inasmuch as certainly both Tasmanians and Andamanese have long been isolated from contact with other races.

For the purposes of this comparison we have been compelled to make use of the 14 general craniological observations only, the reason being that we have had no means of access to the necessary figures for the form analysis of the Andaman Islanders skull. In fact we do not believe such figures exist. The data that we have been able to employ are derived from the Osteological Catalogue of the Royal College of Surgeons of England. They comprise 19 skulls of Andamanese upon which we have utilised the necessary figures for the 14 general craniological observations.

In fig. 8 the Andamanese Islander is utilised as the base. Upon this are plotted out the variation indices for both Australian and Tasmanian. The resulting graphs are again highly irregular and, according to Mollison, are sufficient proof that the Australian-Tasmanian race is something different from the Andamanese Islander. A closer inspection of the graphs will, however, show that the Australian-Tasmanian variation indices fall within the Andamanese range of variation 7 times out of 14—that is, in 50 per cent. If this index of Mollison tells us anything at all, it is that, notwithstanding the difference in race, the Australian-Tasmanian race is more nearly related to the Adamanese Islander than to the Spy-Neandertal group.

#### THE RELATIONSHIP OF AUSTRALIAN AND TASMANIAN TO A HETEROGENEOUS RACE LIKE THE MODERN ITALIAN.

The last use which we propose to make of Mollison's variation index is to test the racial relationships of the Australian and Tasmanian with an admittedly heterogeneous race like the modern Italian. We have selected the modern Italian for this comparison for three reasons: firstly, because the modern Italian is admittedly and undoubtedly a mixed or impure race; secondly, because modern Italians and the indigenous inhabitants of Australia and Tasmania cannot possibly have any racial relationships in common; and thirdly, because the Catalogue of the Royal College of Surgeons of England enabled us to command a sufficiently large number of Italian data, namely 90, to eliminate the possibility of error due to the use of insufficient numbers.

In this comparison we have again been restricted to the 14 general craniological observations, and for the same reasons as in the case of the Andamanese Islanders.

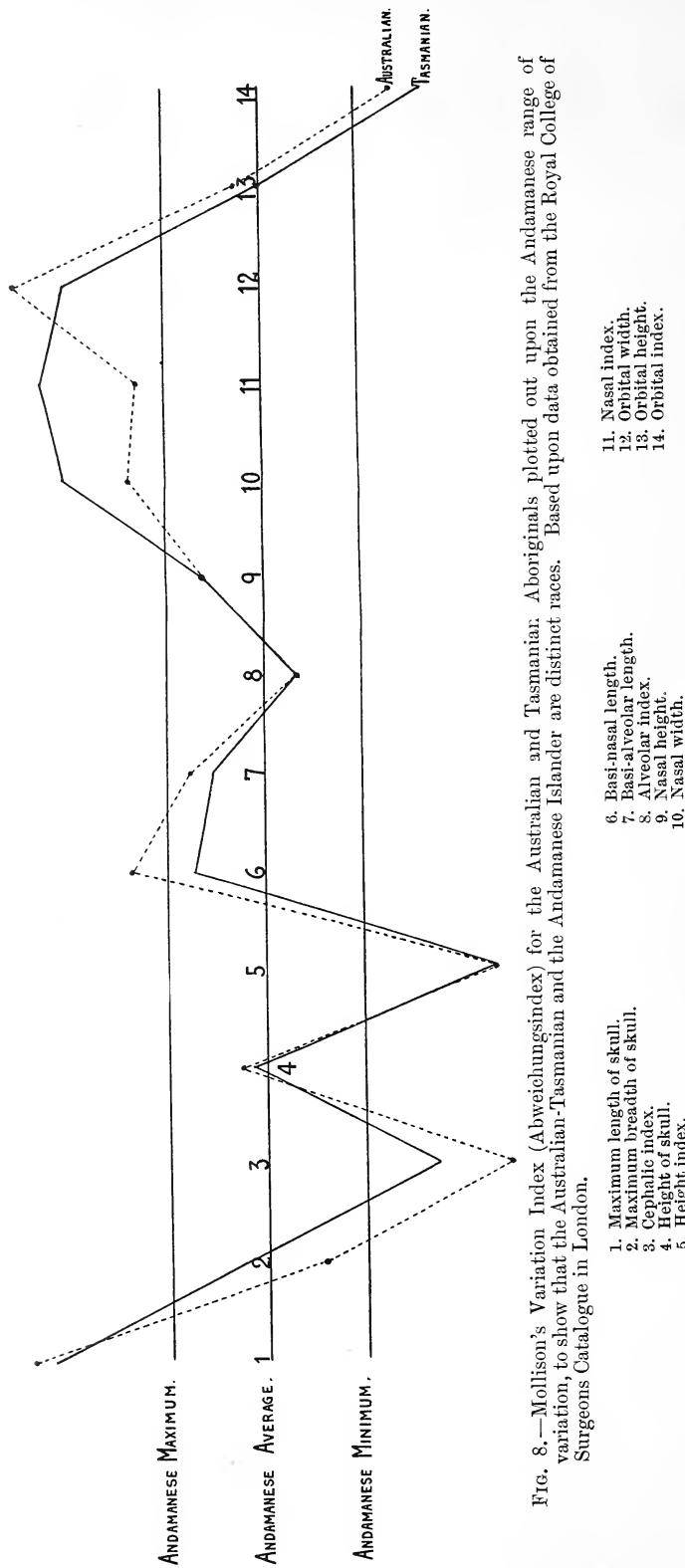


FIG. 8.—Mollison's Variation Index (Abweichungsindex) for the Australian and Tasmanian Aborigines plotted out upon the Andamanese range of variation, to show that the Australian-Tasmanian and the Andamanese Islander are distinct races. Based upon data obtained from the Royal College of Surgeons Catalogue in London.

In fig. 9 the modern Italian is utilised as the base, and upon it is plotted out the Australian variation index. As the latter falls within the extreme ranges of variation of the former, the graph would lead us to believe, if Mollison's index be indeed a reliable guide to racial affinities, that the Australian aboriginal and the modern Italian are one and the same race—a conclusion which, we take it, will not be credited by any anthropologist, certainly not by ourselves.

It may well be, that, in view of this extraordinary result, some anthropologists will be inclined to discredit Mollison's index altogether and refuse to accept it as a means of distinguishing diverse racial types. On the other hand, there are the undoubted facts that in the hands of Mollison

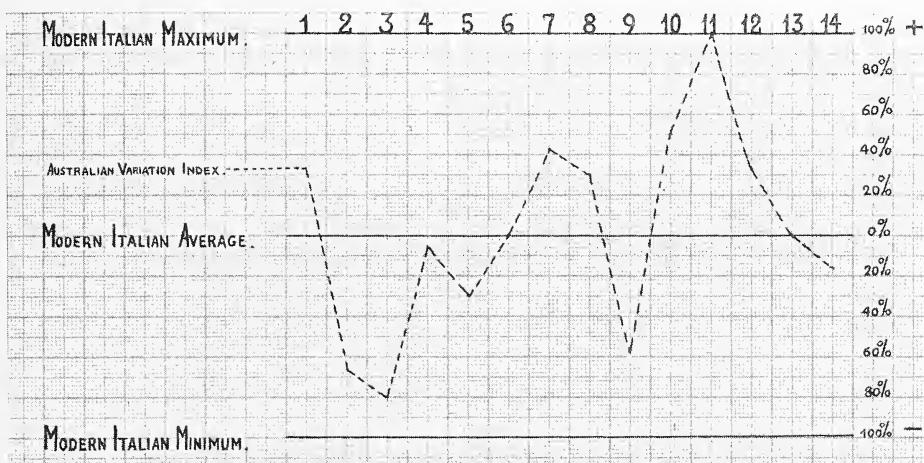


FIG. 9.—The Australian Variation Index for 14 craniological observations plotted out upon the Modern Italian as the basis.

himself, and in the work of Czekanowski, Oppenheim, Radlauer, and ourselves it has given results which confirm the conclusions attained from other sources and which would lead to the supposition that it is a fairly accurate method of eliminating racial types one from the other; but the Italian-Australian-Tasmanian comparison just instituted seems to prove conclusively that the index is not an infallible guide and that its findings must be regarded with a considerable amount of caution. For ourselves, for reasons to be presently adduced, we have been led to the conclusion that Mollison's variation index is only a reliable guide to racial difference provided the range of variation of one, at least, of the racial types compared is but small. It is perfectly obvious to anyone who has worked with the index that as the range of variation increases it gradually encroaches on the variation index and eventually necessitates the latter falling within

the former and thus gives rise to the *reductio ad absurdum* results of our modern Italians, Australians, and Tasmanians.

#### THE RANGE OF VARIATION.

We now propose to consider the very important question of the range of variation in the Australian, Tasmanian, and the other homogeneous and heterogeneous races selected by us for comparison, in order to see what light such a study throws on the vexed question of the purity of origin or otherwise of the Australian aboriginal.

Almost every author who has investigated Australian osteology has been impressed with the great range of variability displayed by his results. Thus Klaatsch (23) says that the study of the variability of the Australian is of the greatest significance. Wetzel (24), from his researches as to the amount of variation in the vertebral column of the Australian, came to the conclusion that the total amount of variation in the osseous vertebrate column of the Australian is considerably greater than in the European, but that, on the other hand, the variation in individual sections is less in the Australian than in the European, and that females are the least variable. Stratz (20), in advancing his theory of the racial division of mankind, states that the protomorphic races—especially the Australians—are characterised by great individual variability. Schoetensack (21) also states that the modern Australian is even to-day extraordinarily rich in varieties. It is unnecessary to multiply examples of the current belief that the Australian is extraordinarily rich in his range of variation, because, as will now be shown, our investigations confirm this view.

Of the Tasmanian there are necessarily fewer opinions, for the sufficient reason that he was extinct before it was realised how important this study of the range of variability is. In discussing, therefore, the range of variation exhibited by the Tasmanian we are practically breaking new ground, apart, of course, from the generally accepted belief that as a homogeneous race the Tasmanian would tend to display a less extended range of variation than in undoubtedly heterogeneous races.

#### COMPARISON OF THE RANGE OF VARIATION IN SUPPOSED PURE RACES LIKE THE ANDAMANESE AND TASMANIANS WITH AN ADMITTED HETEROGENEOUS RACE LIKE THE MODERN ITALIAN, WITH THE OBJECT OF ESTABLISHING THE PLACE OF THE DOUBTFUL AUSTRALIAN.

In order to appraise the amount of variation in the pure, mixed, and doubtful races selected for the comparison now to be established, we have

selected the Tasmanian as the basis. In fig. 10 the maximum and minimum range of variation of this race is represented by two horizontal lines which are supposed to pass through the extremes of variation uniformly regarded as being equal to 100 per cent. If, for example, the average maximum breadth of a series of Tasmanian skulls be found to be 135 with a minimum breadth of 120, the range of variation on the minimum side would be 15. If now the average maximum breadth of a series of Australian crania be found to be 131 with a minimum of 120, the range of variation would be 11 and the relative values of the Tasmanian and Australian ranges of variation would be expressed by the formula as 15 (Tasmanian) is to 100 so is 11 (Australian) to the answer, namely 73 in round numbers. As the data from which our fig. 10 is compiled include other skulls besides those specifically dealt with here and taken as stated from the Royal College of Surgeons Catalogue, we can only indicate generally the process by which the results are attained. We may, however, add that the very greatest care has been taken in the calculations, which have been made throughout by mechanical appliances.

The most cursory glance at the graph shows that the range of variation is in the Australian greater than in the Tasmanian. It further shows that certain individual Australians are at a much lower position in the evolutionary scale than are the most lowly of the Tasmanians; that certain individual Australians have, on the other hand, attained a higher position than have the most highly evolved Tasmanians; whilst lastly, the application of Cross's formula demonstrates that the average Australian remains at a slightly lower level in the evolutionary scale than does the average Tasmanian.

Concerning the Andamanese the same graph, fig. 10, demonstrates that, as judged by the range of variation, the Andamanese are an even purer race than are the Tasmanians, but that, with one or two exceptions, the most advanced Andamanese has not attained so high a position in the evolutionary scale as have either the Tasmanian or the Australian.

Regarding the primary object of the graph, it will be evident that the Australian, in his maximum and minimum ranges of variation, is more closely related to the admittedly mixed race—the Italian—than to the two supposed pure races. To test the point still further, we have submitted the range of variation in all the compared races to a numerical proof. The race taken as the basis, the Tasmanian, is regarded as possessing an amount of variation equal to 100 per cent. The variations of the other races are calculated therefrom in percentages, added together and divided by the number of observations—14. As both the maximum and minimum series

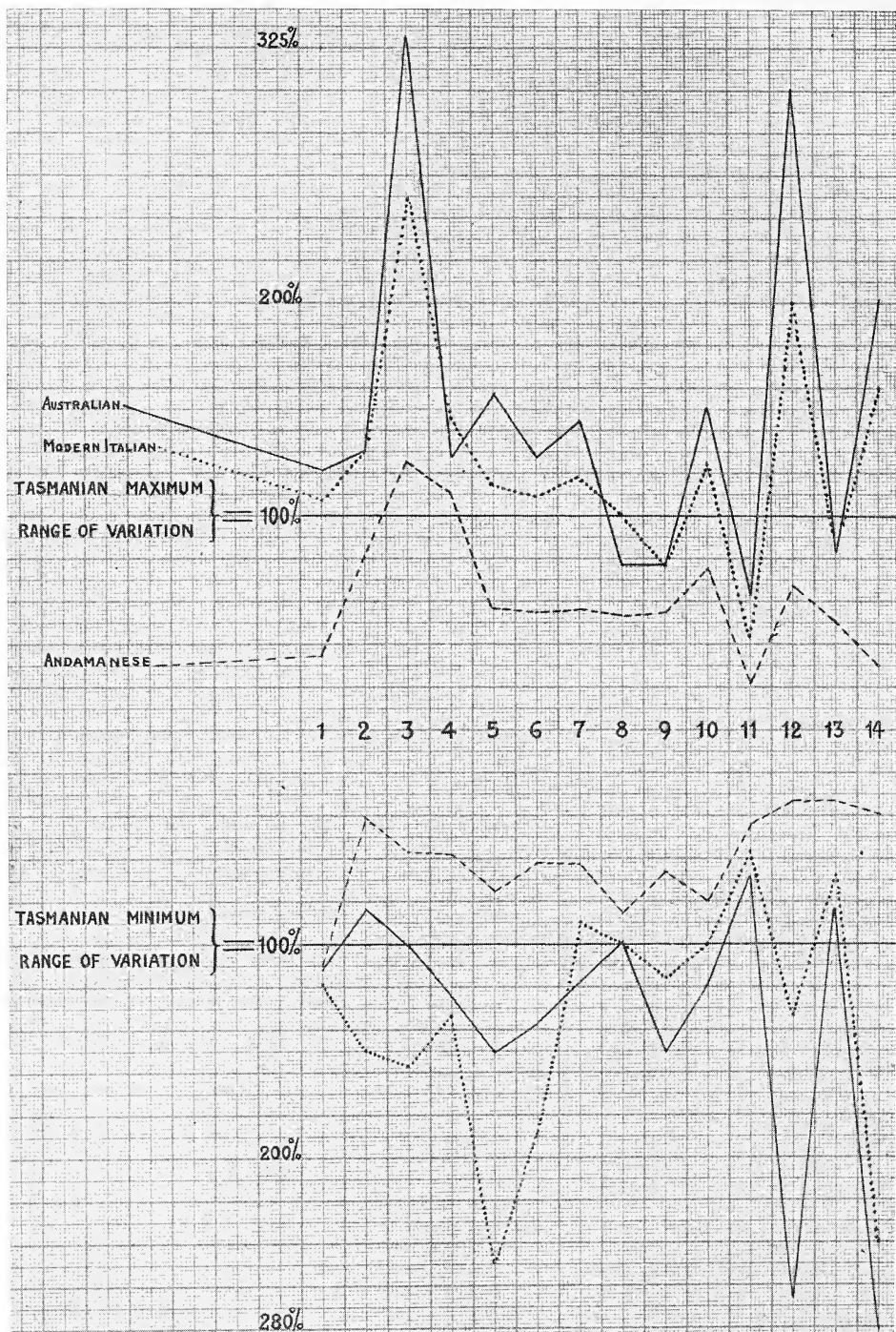


FIG. 10.—The Ranges of Variation of the Australian, Modern Italian, and Andamanese for 14 general craniological observations plotted out upon the Tasmanian as the basis, and expressed in percentages of the latter.

have to be taken into consideration, the divisor is naturally twice 14. The results are as follows:—

Supposed homogeneous races—

Andamanese . . . . .	62·7 per cent.
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Tasmanians . . . . .	100·0 „ „
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Admitted heterogeneous race—

Modern Italians . . . . .	130 per cent.
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Race of doubtful origin—

Australians . . . . .	141·9 per cent.
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#### SUMMARY OF THE OBSERVED FACTS.

In view of the complicity of the problem now under consideration, it would seem advisable here to recapitulate the new facts brought out in the present paper before we pass to their interpretation.

1. The present work contains the detailed measurements of 32 form analysis measurements of 100 Australian aboriginal crania not previously examined. The measurements so recorded are those introduced by Schwalbe and Klaatsch, and have been previously utilised by us for some 52 Tasmanian crania.

2. The present work also incorporates, but does not give the detailed measurements of, 14 observations of a general craniological character, but more particularly on the face, of the above-mentioned 100 Australian and 52 Tasmanian crania.

3. For purposes of comparison the corresponding series of measurements, wherever available, have been worked out from other sources for 3 Spy-Neandertal crania, 19 Andamanese Islanders, and 90 modern Italians.

4. The data resulting have been utilised in order to see what evidence they afford as to the purity of stock or otherwise of the Australian; hence the races selected for comparison have been specially chosen as representatives of either admittedly pure races or of equally undoubted impure races.

5. The use of Mollison's variation index shows that Tasmanians and Australians are a common stock, and that both these races are very much more closely related to each other than they are to either Spy-Neandertal or Andamanese.

6. Mollison's index is shown to be an unreliable guide to the differentiation of race once the range of variation in one or both of the compared stocks exceeds a certain limit. What this limit is requires further proof, but is certainly exceeded by Australians and modern Italians. A study of the range of variation shows that the Australian agrees much more closely

with an admittedly impure race like the modern Italian than with supposed pure stocks like the Andamanese or the Tasmanians.

7. A study of the mean values of all the observations recorded as shown by Cross's method of dealing with such calculations proves that the average Australian is not such a highly evolved type as was the average Tasmanian.

#### INTERPRETATION OF THE FACTS.

Biasutti (25), in a recently published paper the original of which is not, unfortunately, available to us in Melbourne, has apparently devoted some attention to Tasmanian and Australian literature, and assumes therefrom that the Tasmanian is the older of the two types, was developed on the Australian mainland, and migrated thence to Tasmania. The Australian race has been developed from this primitive type and has preserved itself as a mixture, and by insular isolation.

It is impossible for us, under the circumstances, to say whether Biasutti regards his theory as either new or original; but it is hardly necessary to add that it is neither one nor the other, and we only notice the work at all for the simple reason that its author revives the opinion that the Australian aboriginal is a mixed type resulting from a cross, presumably with the older and more primitive Tasmanian type; and this, be it noted, is the only theory which fits the facts adduced in this, and our other papers, as also certain other well-known ethnological, cultural, and linguistic data.

Professor Sergi of Rome has recently published a most important monograph on the racial affinities of the Tasmanian and Australian, under the title "Tasmanier und Australier (*Hesperanthropus tasmanianus* spec.)" (26). We consider ourselves indeed fortunate that the amount of detailed work in our present paper has so far delayed publication as to enable Professor Sergi to be first in the field, and this for reasons which will shortly be apparent.

In this work Sergi has availed himself of the Tasmanian material recently made available by us in our "Dioptrographic Tracings in Four Normæ of Tasmanian Crania" (5), as also of certain crania in Cambridge and elsewhere, and of the recent valuable contributions of Turner (11) and others. Of our own work Sergi states that he prefers to use the skull tracings delineated in the publication just referred to, because they are exact dioptrographic outlines in four normæ of the skulls recorded, and adds that when a sufficient number of crania are so studied, their outlines can easily replace direct observation on the skulls themselves.

From a study of these crania Sergi deduces the fact that they are

lophocephalic, as are also many Australian crania. He states, therefore, that lophocephaly is an absolutely characteristic feature of a certain type of skull widely spread throughout the islands of the Pacific Ocean and which he recognises as the Tasmanian-Australian skull type. The geographical distribution of the lophocephalic Tasmanian-Australian skull is given by Sergi as extending from the Hawaii Islands in the north to New Zealand in the south, and from Australia in the west to Easter Island in the east, all inclusive. For the primeval home of this skull type Sergi, for reasons with which we are not especially concerned, but which seem sound, instances the American continent. For the primitive parent of this skull type he further proposes the name *Homo tasmanianus*, and adds that, whilst it is difficult to state exactly when he wandered into the Pacific Ocean from America, it is not improbable that the migration took place in late Pliocene or early Quaternary times and that he took with him no domestic animals of any kind. *Homo tasmanianus* wandered over the Australian continent into Tasmania, and, becoming isolated there, eventually developed into the Tasmanian aboriginal of recent times, and for him Sergi proposes the name *Hesperanthropus tasmanianus* spec.

The many difficulties attending the Australian aboriginal are solved, says Sergi, by assuming that on the Australian continent there subsequently entered a Polynesian element, and in Sergi's own words: "die Kreuzung der Polynesier mit den ursprünglichen Australieneinwohnern tasmanischen Ursprungs erzeugte eine Bastardvarietät, welche die heutigen Australier sind . . . die Australier hybride Tasmanier waren." For us this quotation is of such vital import that we may, perhaps, be pardoned for translating it as follows:—

"The crossing of the Polynesian with the original inhabitants of Australia of Tasmanian origin begot a bastard variety—the Australian aboriginal of to-day . . . the Australian aboriginal is a hybrid Tasmanian."

For the hybrid Australian Sergi proposes the name *Hesperanthropus tasmanianus polynesianus*, var. *hybrida*.

The thesis here briefly set forth Sergi ably supports by many facts and different lines of evidence. With these lines of evidence we are not here specially concerned, nor are we vitally interested at the moment with the Polynesian character of the cross assumed by the distinguished Italian anthropologist. For ourselves we should have assumed an earlier cross than that emanating from the Polynesian element. This, however, is a minor point contrasted with the more important fact that Sergi, working by different methods and with additional material from other sources, comes to precisely the same results as ourselves—namely, commonality of origin

and race of Tasmanians and Australians with the latter subsequently resulting from a racial cross with the primitive stock.

To Sergi's theory, based, be it remembered, on facts which he has firmly established, let us now apply our own results.

Commonality of origin of Australians and Tasmanians is shown in the present work by the results of Mollison's variation index for Australian and Tasmanian. On no other grounds can the remarkably uniform results attained by us with this index be explained. Read in conjunction with Sergi's study of lophocephaly we regard *Homo tasmanianus* as proved.

That the Tasmanian aboriginal was as nearly as possible the pure descendant of *Homo tasmanianus* we regard as certain on account of (1) the results attained by Sergi himself; (2) the remarkably small range of variation found in the Tasmanian for the observations of the present work; (3) the equally small amount of variation recorded by Büchner (27) in his recent works on Tasmanian prognathism, craniotrigonometry, and curvature indices; and (4) the close approach to unity for the coefficients of correlation already recorded by us in our biometrical study of the Tasmanian (2). It is thus clear that the morphological studies of Sergi, the craniological researches of Büchner and ourselves, and the biometrical work of Dr Cross and ourselves all alike testify to the purity of type of the Tasmanian and the truth of Sergi's hypothesis.

That the Australian aboriginal is a hybrid is, we believe, proved by (1) the results recently recorded by Sergi; (2) the study in the range of variation adduced in the present work—a study which clearly proves that the Australian is more variable than an admittedly crossed race like the modern Italian; (3) our own previously recorded study in the Australian coefficients of correlation; and (4) Broca's statement, quoted by Topinard (28), that it is only when the variations reach 15 or 18 per cent. that we can say with certainty that they are due to mixture of race. In the present work the Australian range of variation has been proved to be 40 per cent. more than in the Tasmanian.

If it be argued that we deduce too much from this range of variation—a study as yet largely in its infancy as regards the precise meaning to be attached to it—we reply in the words of Cossar Ewart, than whom there is no greater living authority on the subject of variation by crossing of types (29):—

“Domestic animals reproduce themselves with great uniformity if kept apart; but the moment one mixed up two different races, strains, or breeds, one did something that was difficult to put in words, but the result was what has been best described as an epidemic of variations.”

Were we to interpolate in the above quotation the words "such as *Homo tasmanianus* and another primitive stock" after the words "strains or breeds," and add at the end "resulting in the Australian aboriginal," it might stand as a perfect exposition of our views on the hybridity of the Australian.

The hybrid character of the Australian is still further supported by Lapicque (30), Baudouin (30), and Krüger-Kelmar (31), and by such well-known ethnological facts as the use of the boomerang and throwing-stick in Australia and their absence in Tasmania; the presence of a domesticated animal, the dingo, in Australia and its complete absence in Tasmania; and the more evolved cultural character of the Australian flints as opposed to the more primitive Tasmanian type of instrument. The total absence of domesticated animals amongst the Tasmanians is further proof of their great antiquity.

The study of language, too, indicates—some would say proves—the hybrid character of the Australian. No physical anthropologist would rely solely on linguistics as a proof of origin of race. Taken, however, in conjunction with somatology and ethnology, it is a valuable line of research. Mathew (32) has pretty well established the hybrid element in the Australian aboriginal, and here we find physical anthropology, range of variations, ethnology, and linguistics all alike pointing distinctly to the impurity of stock of the Australian.

But what of the other side? So far as our study of the literature enables us to judge, there are only some—to use his own word—"superficial" observations of Klaatsch, various unsupported theories of Schoetensack, Stratz, and others, the recent work of Basedow, and the difficulties—prior to the publication of this and the recently issued memoirs of Turner and Sergi—experienced by all in adequately explaining the position of the Australian. Basedow, it is true, boldly declared the Tasmanian to be but an insular type of Australian; but as von Luschan (33) has dealt pretty trenchantly with this observer, we need not further consider him. There is nothing whatsoever in the environment, climate, animal or plant food of Australia and Tasmania, to account for the differences in types of Australians and Tasmanians, and for the enormous range of variation of the former. As regards the last-mentioned point, it is indeed rather the reverse. Dr Cherry, Professor of Agriculture in this University, assures us, as the results of his own observations and experiments, that the soil of the Australian continent is peculiarly deficient in phosphorus and that, relative to the soils of the European and other continents, those of Australia are in a miocene or pliocene condition.

Sofer (34), too, distinctly states that external influences do not affect races, whilst Doncaster (35) stresses heredity rather than environment. Here, then, is nothing to account for the excessive range of variation of the Australian as compared with the Tasmanian, and we are thrown back on the hypothesis already furnished—namely, hybridity, and the explanation of same as given by Ewart.

From a study of the question in all its phases we are, therefore, forced to the conclusion that the Australian is a hybrid.

One other interesting fact results from our study of the hybridity of the Australian aboriginal as deduced from his range of variation, and that is that the result of the cross has not benefited the race from the evolutionary standpoint. The modern-day Australian aboriginal stands rather nearer the anthropoid ape, or the common ancestor, than did the Tasmanian. Isolated individuals of the Australian race have, on the other hand, surpassed the Tasmanian. These facts are evidenced in the present work by the application of Cross's formula to all the evolutionary objects under comparison and by the study of maximum and minimum ranges of variation for Australian and Tasmanian.

#### RESULTS.

In conclusion we may say that, as a result of our prolonged study of Australian and Tasmanian craniology—a study which has now occupied us over five years and is still in progress,—we are led to the following conclusions :—

1. The Australians and Tasmanians are the descendants of a common late Pliocene or early Quaternary stock which, for want of a better term, may be called with Sergi, *Homo tasmanianus*. *H. tasmanianus* had a wide range of distribution within the islands of the Pacific Ocean (Sergi).

2. The Tasmanian aboriginal was the almost unchanged offspring of this type, but evolved on his own lines and in his own way.

3. The Australian aboriginal is the result of a cross between the primitive *Homo tasmanianus* and some other unknown race—Polynesian, according to Sergi; Dravidian, according to Mathew—and is, therefore, a hybrid. From the evolutionary standpoint the result of the cross, whilst it has not been favourable to the race as a whole, has benefited individual members of it. Once evolved, the Australian has, like the Tasmanian, progressed on his own lines and in his own way.

4. Both Australian and Tasmanian have attained, morphologically, to a higher stage in the evolutionary scale than is usually supposed.

5. Neither Australian nor Tasmanian have any direct relationship with *Homo primigenius* as represented by the crania of the Spy-Neandertal men. The superficial points of cranial resemblance are explicable solely on the grounds of the remoteness of the ancestry. In the Spy-Neandertal crania we see them as they were; in the modern Australian-Tasmanian type as they have evolved.

6. The range of variability of structure is, in the Australian, as great as in any other known race of impure origin; in the Tasmanian, on the other hand, it is as small as in any other known or supposed pure race.

7. Mollison's variation index as a test of type must be read in conjunction with the range of variation.

#### LITERATURE.

- (1) ROBERTSON, A. W. D., "Craniological Observations on the Lengths, Breadths, and Heights of a Hundred Australian Aboriginal Crania," *Proc. Roy. Soc. Edin.*, vol. xxxi., 1910-11, pp. 1-16.
- (2) BERRY, R. J. A., A. W. D. ROBERTSON, and K. S. CROSS, "A Biometrical Study of the Relative Degree of Purity of Race of the Tasmanian, Australian, and Papuan," *Proc. Roy. Soc. Edin.*, vol. xxxi., 1910-11, pp. 17-40.
- (3) BERRY, R. J. A., and A. W. D. ROBERTSON, "The Place in Nature of the Tasmanian Aboriginal as deduced from a Study of his Calvaria; Part i., His Relations to the Anthropoid Apes, etc." *Proc. Roy. Soc. Edin.*, vol. xxxi., 1910-11, pp. 41-69.
- (4) CROSS, K. S., "On a Numerical Determination of the Relative Positions of certain Biological Types in the Evolutionary Scale, and of the Relative Values of various Cranial Measurements and Indices as Criteria," *Proc. Roy. Soc. Edin.*, vol. xxxi., 1910-11, pp. 70-84.
- (5) BERRY, R. J. A., and A. W. D. ROBERTSON, "Dioptrographic Tracings in Four Normæ of Fifty-two Tasmanian Crania," *Trans. Roy. Soc. Vict.*, vol. v. pt. i., 1910.
- (6) BERRY, R. J. A., "A Living Descendant of an Extinct (Tasmanian) Race," *Proc. Roy. Soc. Vict.*, vol. xx., N.S., 1907.
- (7) KLAATSCH, H., "The Skull of the Australian Aboriginal," *Reports from Path. Lab. of Lunacy Dept., N.S.W.*, vol. i., pt. iii., 1908, pp. 43-167.
- (8) FRÉDÉRIC, J., "Das Schädelfragment von Stängenäs in Schweden," *Zeit. f. Morph. u. Anthropol.*, vol. xi., 1908, p. 317.
- (9) MOLLISON, TH., "Beitrag zur Kranieologie und Osteologie der Maori," *Zeit. f. Morph. u. Anthropol.*, vol. xi., 1908, p. 529.
- (10) TURNER, Sir WILLIAM, "The General Characters of the Crania of the People of Scotland," *Journ. Anat. and Phys.*, vol. xxxvii., 1903, pp. 392-408.

(11) TURNER, Sir WILLIAM, "The Craniology, Racial Affinities, and Descent of the Aborigines of Tasmania," and "The Aborigines of Tasmania," *Trans. Roy. Soc. Edin.*, vol. xlvi., 1908, and vol. xlvii., 1910.

(12) BASEDOW, H., "Der Tasmanierschädel, ein Insulartypus," *Zeit. f. Ethnol.*, vol. xlvi., 1910.

(13) ALSBERG, C. L., "Neuere Probleme der menschlichen Stammesentwicklung," *Arch. Rassen- u. Ges.-Biol.*, Band iii., p. 28.

(14) HADDON, A. C., *Races of Man and their Distribution*, Miller & Co., London, 1912.

(15) CZEKANOWSKI, H., "Verwandschaftsbeziehungen der zentralafrikanischen Pygmäen," *Korr.-Blatt. d. deutsch. Gesell. Anthropol., Ethnol. u. Urgeschichte*, vol. xlvi., 1910, p. 101.

(16) OPPENHEIM, S., "Zur Typologie des Primatocraniums," *Zeit. f. Morph. u. Anthropol.*, vol. xiv., 1911, pp. 1-204.

(17) RADLAUER, C., "Beiträge zur Anthropologie des Kreuzbeines," *Gegenbaur's morph. Jahrbuch*, Band xxxviii., 1908, pp. 322-447.

(18) KLAATSCH, "Das Gesichtsskelett der Neandertalrasse und der Australier," *Verh. anat. Ges.*, 22. Vers., Berlin, 1908, pp. 223-273.

(19) VON LUSCHAN, "Neuholländische Typen," *Korr.-Blatt. d. deutsch. Gesell. Anthropol., Ethnol. u. Urgeschichte*, vol. xl., 1909.

(20) STRATZ, C. H., "Das Problem der Rasseneinteilung der Menschheit," *Arch. f. Anthropol.*, vol. xxix., 1903, p. 189.

(21) SCHOETENSACK, O., "Die Bedeutung Australiens für die Heranbildung des Menschen aus einer niederen Form," *Zeit. f. Ethnol.*, Jahrg. xxxiii., 1901, p. 127.

(22) SCHWALBE, G., "Über das Schädelfragment von Brüx und seine Bedeutung für die Urgeschichte des Menschen," *Verhand. des anthrop. Congr. zu Salzburg*, August 1905.

(23) KLAATSCH, H., "Ergebnisse meiner australischen Reise," *Korr.-Blatt. d. deutsch. Gesell. Anthropol., Ethnol. u. Urgeschichte*, vol. xxxviii., 1907.

(24) WETZEL, G., "Die Wirbelsäule der Australier. Erste Mitteilung: Das Volumen der knöchernen Wirbelsäule und ihre Abschnitte," *Zeit. f. Morph. u. Anthropol.*, vol. xii., 1909.

(25) BIASUTTI, R., "I Tasmaniani come forma d' Isolamento Geografico," *Arch. l'Antropol.*, vol. xl. p. 108.

(26) SERGI, G., "Tasmanier und Australier. (*Hesperanthropus tasmanianus, spec.*)," *Archiv f. Anthropol.*, Neue Folge, Band xi., 1912, p. 201.

(27) BÜCHNER, L. W. G., "An Investigation of Fifty-two Tasmanian Crania by Klaatsch's Craniotrigonometrical Methods," and "A Study of the Prognathism of the Tasmanian Aboriginal," *Proc. Roy. Soc. Vict.*, vol. xxv., 1912; also the "Curvature Indices of the Tasmanian Aboriginal Cranium," ready for publication.

(28) TOPINARD, P., *Anthropology*, Chapman & Hall, London, 1890.

(29) EWART, J. COSSAR, "Discussion on Heredity in Disease," *Scot. Med. and Surg. Journ.*, vol. vi., 1900, p. 308.

(30) LAPICQUE and BAUDOUIN, Discussion on Zaborowki's "Métis d'Australien et d'Anglais," *Bull. Soc. d'Anthrop.*, 5th series, vol. viii. p. 384.

- (31) KRÜGER-KELMAR, P., *Beiträge zur vergleichenden Ethnologie und Anthropologie der Neuholländer, Polynesier und Melanesier*, Inaug. Diss. Gottingen, 1905.
- (32) MATHEW, *Eaglehawk and Crow*, David Nutt, London, 1899.
- (33) VON LUSCHAN, "Zur Stellung der Tasmanier in anthropologischen System," *Zeit. f. Ethnol.*, Jahrg. xliii., 1911, p. 287.
- (34) SOFER, L., "Über die Plastizität der menschlichen Rasse," *Arch. Rassen- u. Ges.-Biol.*, Jahrg. v., p. 660.
- (35) DONCASTER, *Heredity*, Cambridge University Press, 1911.

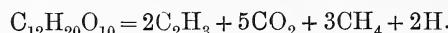
(Issued separately April 29, 1914.)

XIII.—A Chemical Examination of the Organic Matter in Oil-Shales. By John B. Robertson, M.A., B.Sc., Carnegie Scholar.  
*Communicated by Dr J. S. FLETT, F.R.S.*

(MS. received February 28, 1914. Read March 16, 1914.)

HISTORICAL.

INVESTIGATIONS into the nature of the organic matter in oil-shales began at the time of the famous Torbanehill case in 1854, when experts attempted to settle the question as to whether the substance known as "Torbanite" or "Boghead Mineral" was a coal or an oil-shale. Several witnesses at the trial (*Gillespie v. Russel, Session Papers*, 1854) maintained that the oil-producing material in the Mineral was of organic origin, while others pronounced it to be bituminous and produced by subaqueous eruptions. T. S. Traill, M.D., proposed for the Boghead Mineral the name "Bitumenite," as it seemed to him to "consist of much bitumen, mingled with earthy matter" (*Trans. Roy. Soc. Edin.*, 1857, xxi. p. 7). Dr Redfern (*Quart. Journ. Microsc. Soc.*, 1855, x. pp. 118–119), on the other hand, supposed the round orange-yellow bodies which occur in torbanite to have had their origin in "a mass of vegetable cells and tissues which have been disintegrated and otherwise changed by maceration, pressure, and chemical action, and subsequently solidified." C. E. Bertrand and B. Renault (*Bull. Soc. Hist. Nat. Autun*, 1892–3) on microscopic examination have classed these bodies as the remains of gelatinous algae which have been altered by bacterial action. They ascribe the genus "Pila" to those occurring in the northern and "Reinschia" to those found in the southern hemisphere, class the bacteria as "micrococci," and represent the transformation of the vegetable matter by the equation



This equation is purely empirical, although no doubt carbon dioxide, methane, and hydrogen are products of bacterial action upon organic matter. Recently this view has been disputed by E. C. Jeffrey (*Rhodora*, vol. xi. p. 61), who has subjected bogheads to a chemical treatment with nitric and hydrofluoric acids before making sections, and has affirmed that "the so-called algae" are in reality "the strongly sculptured megasporites of vascular cryptogams." Dr W. Scheithauer (*Oil-Shales and Tars*, 1913)

expresses the opinion that the remains of dead animals form part of the organic material. This view is discussed later.

Lastly, J. Schuster (*Neues Jahrbuch f. Mineralogie*, 1912, Bd. ii. p. 33) objects to the algal theory, and states that the yellow bodies are in some cases concretions of resin, and in others spherulites of silica, calc spar, or siderite.

Little chemical work has been done on the subject beyond a few ultimate analyses for purposes of the above-mentioned trial. D. R. Steuart (*Oil-Shales of the Lothians*, 1912, p. 164) prepared artificial shale from lycopodium dust and Florida fuller's earth, and obtained from it on distillation oil and ammonia in quantities similar to those obtained from torbanite. He also pointed out that there is very little in torbanite or in ordinary oil-shales which can be extracted by petroleum spirit, benzene, carbon disulphide, or ether, and that therefore the substance cannot be of the nature of petroleum, bitumen, or resin.

#### EXPERIMENTAL.

In order to determine whether the organic matter varied in composition, and, if so, what was the nature and extent of the variation, ultimate analyses were made of thirteen samples of oil-shales. The methods of analysis used were the same as those employed by Strahan and Pollard in their analyses of coals (*The Coals of South Wales*, 1908, p. 6), except in the case of carbon and hydrogen determinations, where Walker and Blackadder's modification of the Dennstedt combustion furnace was used. The samples were powdered in an agate mortar until sufficiently fine to pass through a 90-mesh sieve, then dried in a toluene-bath at 105°-107° C. for an hour. No boat was used in the combustions, the powdered shale being mixed with the copper oxide. The ash was determined separately by igniting the shale in a muffle at bright red heat.

The Kjeldahl method was employed for the nitrogen estimations, a gram of shale being used for each determination. The sulphur was estimated by heating one gram of the shale with five grams of sodium carbonate in a muffle till all the carbon was burned, digesting with warm water, filtering, acidifying the filtrate with hydrochloric acid (after the addition of 10 c.c. bromine solution to secure complete oxidation), and precipitating the sulphate formed with barium chloride.

It was found that a slight correction was necessary in the hydrogen and ash determinations, there being present in the shales hydrous minerals from which the water was not expelled at 105° C. The amount of this water was estimated by heating a weighed quantity of the shale to a temperature

between 200° and 250° C. in a current of dry nitrogen, and collecting the water evolved in a calcium chloride tube. Little or no oil was given off at this temperature, but to secure that none was retained in the calcium chloride tube a current of dry air was passed through it for twenty minutes after each experiment. In a distillation of torbanite under a pressure of 1 mm., no perceptible quantity of oil was produced until a temperature of 380° C. had been attained by means of an electric furnace. The correction thus applied in no case exceeded 0·15 deduction from the hydrogen percentage, or 1·35 addition to the ash percentage. The detailed analyses are given below along with the works' yield of oil and ammonium sulphate where these are known. The samples of Dunnet shale are from a bore at Broxburn, each one representing one foot of the seam in vertical thickness from the top downwards. The last two analyses are those of specimens of "burnt" shales occupying the positions of the "Maybrick" and "Curly" seams of the Pumpherston shales in a bore at Knightsbridge, and described by R. G. Carruthers (*Oil-Shales of the Lothians*, 1912, p. 88). He puts forward the theory that these shales have been "burnt" or rendered useless not through the proximity of any igneous rock, but through the oil having been distilled off from them by the heat evolved from the oxidation of pyrites dust in the underlying tuff beds.

The relatively high percentage of oxygen in sample 12 might be considered to lend support to this theory, but this is somewhat negatived by the much lower oxygen percentage in sample 13. The high nitrogen percentages in each show, however, that whatever changes the shale has undergone, the nitrogenous matter has been the last to be affected.

#### LIST OF SHALES ANALYSED.

1. Dunnet—Top foot (Broxburn).
2. „ 2nd „ „
3. „ 3rd „ „
4. „ 4th „ „
5. „ 5th „ „
6. „ 6th „ „
7. Camps—Flat portion (Pumpherston).
8. Camps—Inclined portion (Pumpherston).
9. Broxburn (Pumpherston).
10. Torbanite (Armadale).
11. Australian Commonwealth.
12. "Burnt" Shale—"Maybrick" (Knightsbridge).
13. "Burnt" Shale—"Curly" (Knightsbridge).

## DATA OF ANALYSES.

## I.—DETERMINATION OF CARBON AND HYDROGEN.

No. of Shale.	Weight taken.	CO <sub>2</sub> found.	H <sub>2</sub> O found.	Carbon per cent.	Hydrogen per cent.	Carbon per cent. (Average.)	Hydrogen per cent. (Average.)
1 (a)	.6309	.2347	.0924	10·15	1·63	...	...
(b)	.5657	.2134	.0838	10·29	1·65	10·22	1·64
2 (a)	.6226	.2869	.0953	12·57	1·70	...	...
(b)	.4704	...	.0720	...	1·70	...	...
(c)	.4052	.1873	.0643	12·61	1·76	12·59	1·72
3 (a)	.3415	.2024	.0689	16·16	2·24	...	...
(b)	.3374	.1983	.0681	16·03	2·24	16·09	2·24
4 (a)	.2938	.1999	.0660	18·52	2·50	...	...
(b)	.3521	...	.0786	...	2·48	...	...
(c)	.3303	.2933	.0729	18·44	2·45	18·48	2·48
5 (a)	.3386	.2422	.0769	19·51	2·52	...	...
(b)	.3208	.2264	.0750	19·25	2·60	...	...
(c)	.3464	.2512	.0780	19·77	2·50	19·51	2·54
6 (a)	.5936	.3546	.1171	16·29	2·19	...	...
(b)	.3781	...	.0781	...	2·30	...	...
(c)	.3575	...	.0744	...	2·31	...	...
(d)	.3590	.2165	.0729	16·44	2·26	16·36	2·26
7 (a)	.2450	.1979	...	22·03	...	...	...
(b)	.2474	.1986	.0653	21·90	2·93	21·96	2·93
8 (a)	.3558	.1318	.0567	10·10	1·77	...	...
(b)	.4118	.1534	.0655	10·16	1·77	10·13	1·77
9 (a)	.3694	.3382	.1260	24·97	3·79	...	...
(b)	.2831	.2576	.0967	24·80	3·79	24·88	3·79
10 (a)	.1160	.2780	.0861	65·34	8·25	...	...
(b)	.1181	.2864	...	66·13	...	...	...
(c)	.1016	.2447	...	65·69	...	...	...
(d)	.0970	...	.0735	...	8·42	...	...
(e)	.0988	...	.0746	...	8·39	65·72	8·35
11 (a)	.0885	...	.0626	...	7·86	...	...
(b)	.0900	.2093	...	63·60	...	...	...
(c)	.0851	...	.0601	...	7·85	...	...
(d)	.0856	.1994	...	63·56	...	63·58	7·86
12 (a)	.5025	.1085	.0451	5·89	1·00	...	...
(b)	.9977	.2193	.0843	5·99	0·94	...	...
(c)	.5985	.1333	.0525	6·07	0·97	5·98	0·97
13 (a)	.8383	.2459	.0921	8·00	1·22	...	...
(b)	.6971	.2123	.0830	8·30	1·32	8·15	1·27

## II.—DETERMINATION OF NITROGEN.

No. of Shale.	Weight taken.	Nitrogen found.	Nitrogen per cent.	Nitrogen per cent. (Average.)
1 (a)	.9984	.004766	0·48	...
(b)	1·0008	.005057	0·51	0·49
2 (a)	1·0025	.005538	0·55	...
(b)	.9969	.005104	0·51	0·53
3 (a)	1·0046	.006161	0·61	...
(b)	.9999	.005989	0·60	0·61
4 (a)	.9976	.006810	0·68	...
(b)	1·0015	.006569	0·66	0·67
5 (a)	1·0019	.006746	0·67	...
(b)	1·0020	.007100	0·71	0·69
6 (a)	1·0017	.005925	0·59	...
(b)	1·0020	.006166	0·62	0·60
7 (a)	1·0031	.006182	0·62	...
(b)	1·0033	.006907	0·69	0·65
8 (a)	1·0011	.005361	0·54	...
(b)	.9971	.005313	0·53	0·53
9 (a)	.9971	.006585	0·66	...
(b)	.7123	.005071	0·71	0·68
10 (a)	.9985	.006311	0·63	...
(b)	1·0039	.006327	0·63	0·63
11 (a)	.9984	.007986	0·80	...
(b)	1·0009	.008147	0·81	0·81
12 (a)	1·0004	.007736	0·77	...
(b)	.9946	.007223	0·73	0·75
13	1·0001	.01233	1·23	...

## III.—DETERMINATION OF SULPHUR.

No. of Shale.	Weight taken.	Barium Sulphate found.	Sulphur per cent.	Sulphur per cent. (Average.)
1 (a)	.997	.0682	0·94	...
(b)	1·005	.0659	0·90	0·92
2 (a)	1·004	.0871	1·19	...
(b)	.998	.0860	1·18	1·19
3 (a)	1·005	.1277	1·75	...
(b)	1·005	.1262	1·73	1·74
4 (a)	1·003	.0953	1·31	...
(b)	1·003	.0953	1·31	1·31
5 (a)	1·006	.0992	1·36	...
(b)	1·006	.0990	1·36	1·36
6 (a)	.998	.1982	2·73	...
(b)	1·000	.1970	2·71	2·72
7 (a)	.998	.0914	1·26	...
(b)	.997	.0912	1·26	1·26
8 (a)	1·002	.0382	0·52	...
(b)	1·004	.0395	0·54	0·53
9 (a)	1·005	.0589	0·81	...
(b)	1·004	.0583	0·80	0·80
10 (a)	1·004	.0196	0·27	...
(b)	.999	.0213	0·29	0·28
11 (a)	1·002	.0285	0·39	...
(b)	.997	.0336	0·46	0·43
12 (a)	.997	.0511	0·70	...
(b)	1·002	.0483	0·66	0·68
13	1·002	.1716	2·35	...

## IV.—DETERMINATION OF ASH.

No. of Shale.	Weight taken.	Ash.	Ash per cent.	Ash per cent. (Average.)
1 (a)	.2025	.1655	81·73	...
(b)	.1995	.1629	81·65	81·69
2 (a)	.2054	.1629	79·30	...
(b)	.1965	.1553	79·02	79·16
3 (a)	.2013	.1514	75·21	...
(b)	.1998	.1496	74·87	75·04
4 (a)	.2032	.1506	74·11	...
(b)	.1982	.1469	74·13	74·12
5 (a)	.2027	.1455	71·78	...
(b)	.1974	.1430	72·43	72·11
6 (a)	.2002	.1493	74·58	...
(b)	.2009	.1499	74·64	74·61
7 (a)	.2050	.1413	68·92	...
(b)	.2070	.1418	68·48	68·70
8 (a)	.2023	.1674	82·75	...
(b)	.1962	.1612	82·15	82·45
9 (a)	.1997	.1215	60·81	...
(b)	.1997	.1216	60·87	60·84
10 (a)	.6994	.1237	17·68	...
(b)	.6998	.1234	17·63	17·66
11 (a)	.7036	.1581	22·47	...
(b)	.7063	.1586	22·46	22·47
12 (a)	.2009	.1669	83·10	...
(b)	.2007	.1659	82·68	82·89
13 (a)	.1961	.1653	84·29	...
(b)	.1963	.1656	84·35	84·32

## V.—CORRECTION FOR HYDROGEN MINERALS.

No. of Shale.	Weight taken.	Weight of Water.	Hydrogen per cent. to be deducted.	Ash per cent. to be deducted.
1	1·1199	.0094	0·09	0·84
2	.9110	.0034	0·04	0·37
3	1·0799	.0054	0·06	0·50
4	.9638	.0038	0·04	0·39
5	.8944	.0048	0·06	0·54
6	1·0090	.0046	0·05	0·46
7	.6948	.0042	0·07	0·60
8	1·0027	.0065	0·07	0·63
9	.5275	.0056	0·12	1·06
10	.5310	.0038	0·08	0·72
11	.6088	.0030	0·05	0·49
12	1·0041	.0135	0·15	1·35
13	1·0303	.0107	0·12	1·04

## GENERAL RESULTS OF ANALYSES.

No.	Shale.	C.	H.	N.	S.	O (by differ- ence).	Ash.	C/H.	Oil (gals. per Ton).	$(\text{NH}_4)_2\text{SO}_4$ (lbs. per Ton).
1	Dunnet—Top foot (Broxburn)	.	10.22	1.55	0.49	0.92	4.29	82.53	6.59	9.6
2	," 2nd ,,	,	12.59	1.68	0.53	1.19	4.48	79.53	7.49	12.2
3	," 3rd ,,	,	16.09	2.18	0.61	1.74	3.84	75.54	7.38	20.1
4	," 4th ,,	,	18.48	2.44	0.67	1.31	2.59	74.51	7.57	20.1
5	," 5th ,,	,	19.51	2.48	0.69	1.36	3.31	72.65	7.86	23.3
6	," 6th ,,	,	16.36	2.21	0.60	2.72	3.04	75.07	7.40	13.4
7	Camps—Flat portion (Pumpherston)	.	21.96	2.86	0.65	1.26	3.97	69.30	7.68	17.0
8	Camps—Inclined portion	,	10.13	1.70	0.53	0.53	4.13	83.08	5.96	18.5
9	Broxburn (Pumpherston)	.	24.88	3.67	0.68	0.80	8.07	61.90	6.78	31.0
10	Torbanite (Armadale)	.	65.72	8.27	0.63	0.28	6.72	18.38	7.95	90-130
11	Australian Commonwealth	.	63.58	7.81	0.81	0.43	4.41	22.96	8.14	10.8
12	" Burrit" Shale ("Maybrick")	.	5.98	0.82	0.75	0.68	7.53	84.24	7.29	..
13	" Burnt" Shale ("Curly")	.	8.15	1.15	1.23	2.35	1.76	85.36	7.09	..

## DISCUSSION OF ANALYTICAL DATA.

Strahan and Pollard have adopted the "carbon-hydrogen ratio" (C/H) as a basis for the classification of coals, and have found it to vary from 12·9 in "per-bituminous" coals to upwards of 30 in anthracites. A variation in a smaller degree can be seen in the shales, the limits in the above analyses being 5·96 and 8·14. In all cases the ratio is lower than in the coals. The analyses show that the organic matter varies considerably in constitution in different shales. The most interesting fact revealed, however, is the connection between the carbon-hydrogen ratio and the yield of oil. The law would seem to be established that *the yield of oil varies directly as the percentage of organic matter, and inversely as a function of the carbon-hydrogen ratio.* This is strikingly shown by comparing analyses 3 and 4, where the same oil yield is obtained from the two samples, the excess of 2·4 per cent. of carbon in 4 being neutralised by its higher carbon-hydrogen ratio of 7·57 as compared with 7·38 in 3. It is still more evident on a comparison of Nos. 7 and 8, where the oil yield in the latter is actually 1·5 gallons more than in the former, although the carbon percentages are 10·13 and 21·96 respectively, the explanation being that in 8 the carbon-hydrogen ratio is only 5·96, whereas in 7 it is comparatively high, viz. 7·68. It is thus shown that the all-important factor in shale analysis is the determination of the relative percentage of hydrogen present, and that an approximate analysis of a sample into volatile matter, coke, and ash may not shed so much light on its oil-producing properties as an ultimate carbon and hydrogen analysis.

## ACTION OF SOLVENTS ON SHALE.

Mention has already been made of the fact that very little is extracted from shale by the common organic solvents. Dichlorhydrin, a high boiling solvent (b.p. 174° C.) was tried without success, but pyridine (b.p. 117° C.) was found to be an effective solvent.

The Committee on Explosions in Mines (2nd Report, 1912) has investigated the solubility of coals in pyridine and shown that quantities of extract may be obtained varying from 3·7 to 38·8 per cent. on the ash-free dry coal. Torbanite and Broxburn shale were both completely extracted with pyridine, and it was found that 4·92 and 3·29 per cent. respectively of the ash-free dry shale was dissolved. If then, as is suggested in the report of the above committee, the extracted material represents the resinous part of the coal (the small percentages are from semi-bituminous, and the high from bituminous, coals), it is evident that only a very small portion of the organic matter in shale is of a resinous character.

The extracts were obtained by treating the finely divided shale with pyridine in a Soxhlet extraction apparatus until the solvent siphoning over was no longer coloured, distilling off the pyridine at 60° C. under reduced pressure, transferring the semi-solid residue to a watch-glass and drying it in a vacuum desiccator over sulphuric acid. The extracts were dark brown in colour and showed a tendency to crystallise in radiating needles. They gave the following results on analysis:—

#### EXTRACT FROM TORBANITE.

(1) .0864	gave .0694 H <sub>2</sub> O and .2590 CO <sub>2</sub>	C=81·75	H=8·93
(2) .0808	" .0653 " .2467 "	C=83·26	H=8·98
(3) .0679	" .0560 " .2042 "	C=82·01	H=9·16
(4) .0684	" .2088 "	C=83·26	
Average . . .		C=82·57 per cent.	H=9·02 per cent.

.5031 gave .005023 N<sub>2</sub>      N=1·00 per cent.  
.502    " .0138 BaSO<sub>4</sub>      S=.38    ,

#### EXTRACT FROM BROXBURN SHALE.

.0726 gave .0768 H<sub>2</sub>O and .2196 CO<sub>2</sub>      C=82·49 per cent.      H=11·75 per cent.

From Torbanite.	From Broxburn Shale.
C 82·57	C 82·49
H 9·02	H 11·75
N 1·00	N
S .38	S } not estimated.
(by difference) O 7·03	O }
<hr/> 100·00	

These extracts are in all probability mixtures, as small proportions of them are dissolved by alcohol, benzene, etc.

#### ACTION OF NITRIC ACID ON SHALES.

Carrick Anderson (*Jour. Soc. Chem. Ind.*, 1898, vol. xvii. p. 1018) described the action of nitric acid on coals, and gave analyses of the products obtained from seven different samples. These products are acids, and the product from any coal is always of constant composition provided an excess of acid is used. As it seemed possible that a comparison of these acids with any similar compounds which might be obtained from shales and other substances might throw some light on the origin and nature of the organic matter in shale, an endeavour was made to obtain similar derivatives from the following:—(1) Torbanite, (2) Broxburn shale, (3) Newbattle cannel coal, (4) peat from Glenfalloch, (5) lycopodium spore dust, and (6) an organic sludge consisting mainly of decomposed leaf and root remains, microscopic algae, diatoms, and bacteria. From all of these, except

the last, derivatives were obtained. In the case of the organic sludge, oxidation, even although moderated by careful cooling, was sufficiently vigorous to change most of the oxidisable material into oxalic acid and carbon dioxide. The finely divided material was evaporated to dryness with excess of concentrated nitric acid, the solid residue treated with ammonia solution, filtered, and the acid precipitated from the filtrate by the addition of dilute hydrochloric acid. This precipitate was filtered, washed with water till free from chloride, and dried in a vacuum over sulphuric acid. In the experiments with lycopodium and with peat the first action had to be moderated by cooling. The substance obtained from lycopodium was light brown in colour and gummy. All the other preparations were more or less dark brown in colour, hard, and brittle. They all contained traces of sulphur and a small amount of ash. In no case was the whole of the organic matter converted into acid, there being formed in all the preparations a larger or smaller quantity of oxalic acid. A small quantity of powdered torbanite, after being repeatedly treated with nitric acid and ammonia as above, was examined under the microscope. The residue was found to consist of inorganic materials, with here and there particles of organic matter which had been prevented from going into solution through being enveloped in inorganic materials. These acids form insoluble salts with some metallic radicals such as silver, lead, copper, iron, cobalt, and barium. They gave the following figures on analysis (neglecting traces of sulphur and ash):—

#### ACID FROM TORBANITE.

(1) ·0784 gave	·0533 H <sub>2</sub> O and	·1765 CO <sub>2</sub>	C=61·39	H=7·55
(2) ·0964 „	·0632 „ „	·2162 „	C=61·13	H=7·29
		Average . . .	C=61·26 per cent.	H=7·42 per cent.
(1) ·5993 gave	·02189 N <sub>2</sub>	N=4·38		
(2) ·4993 „	·02181 N <sub>2</sub>	N=4·37		
		Average . . .	N=4·37 per cent.	

#### ACID FROM BROXBURN SHALE.

(1) ·0937 gave	·0576 H <sub>2</sub> O and	·2040 CO <sub>2</sub>	C=59·38	H=6·83
(2) ·0897 „	·0533 „			H=6·60
(3) ·0947 „		·2079 „	C=59·87	
		Average . . .	C=59·62 per cent.	H=6·72 per cent.
·5065 gave	·02175 N <sub>2</sub>	N=4·29 per cent.		

#### ACID FROM CANNEL COAL.

(1) ·0991 gave	·0474 H <sub>2</sub> O and	·1934 CO <sub>2</sub>	C=53·22	H=5·31
(2) ·1146 „	·0576 „ „	·2270 „	C=54·01	H=5·58
		Average . . .	C=53·61 per cent.	H=5·44 per cent.
·5008 gave	·01963 N <sub>2</sub>	N=3·92 per cent.		

## ACID FROM PEAT.

·0557 gave ·0274 H<sub>2</sub>O and ·1081 CO<sub>2</sub>      C=52·93 per cent.      H=5·47 per cent.  
 ·0976    „    ·004477 N<sub>2</sub>      N=4·59 per cent.

## ACID FROM LYCOPODIUM.

(1) ·1051 gave ·0891 H <sub>2</sub> O and ·2334 CO <sub>2</sub>	C=60·56	H=9·42
(2) ·1006    „    ·0848    „    ·2204    „	C=59·75	H=9·37
	Average . . . C=60·15	per cent. H=9·39 per cent.
(1) ·4966 gave ·01904 N <sub>2</sub>	N=3·83	
(2) ·5106    „    ·01916    „	N=3·77	
	Average . . . N=3·80	per cent.

## ACIDS OBTAINED FROM SHALES, ETC.

	Torbanite.	Broxburn Shale.	Cannel Coal.	Peat.	Lycopodium.
C	61·26	59·62	53·61	52·93	60·15
H	7·42	6·72	5·44	5·47	9·39
N	4·37	4·29	3·92	4·59	3·80
O	26·95	29·37	37·03	37·01	26·66
(by difference)	100·00	100·00	100·00	100·00	100·00

The estimation of the metallic radicals in the silver and ammonia salts of the acids from torbanite and Broxburn shale confirm these analyses, and point to the empirical formulae:—

From :—

Torbanite.	Broxburn Shale.	Cannel Coal.	Peat.	Lycopodium.
C <sub>16</sub> H <sub>24</sub> NO <sub>5</sub>	C <sub>16</sub> H <sub>22</sub> NO <sub>6</sub>	C <sub>16</sub> H <sub>19</sub> NO <sub>8</sub>	C <sub>13</sub> H <sub>17</sub> NO <sub>7</sub>	C <sub>19</sub> H <sub>35</sub> NO <sub>6</sub>

The empirical formulae calculated from Carrick Anderson's coal-acid analyses are:—C<sub>14</sub>H<sub>9</sub>NO<sub>6</sub> (Ell); C<sub>15</sub>H<sub>9</sub>NO<sub>5</sub> (Splint); C<sub>15</sub>H<sub>9</sub>NO<sub>5</sub> (Gas); C<sub>17</sub>H<sub>10</sub>NO<sub>7</sub> (Virgin); C<sub>17</sub>H<sub>10</sub>NO<sub>6</sub> (Lower Drumgray); C<sub>16</sub>H<sub>10</sub>NO<sub>6</sub> (Bannockburn Main); and C<sub>21</sub>H<sub>13</sub>NO<sub>8</sub> (Kilsyth Coking).

These substances, which are evidently all of the same nature, can be arranged into a series commencing with lycopodium acid where the hydrogen is relatively highest, and passing through torbanite, Broxburn shale, peat, and cannel coal-acids to ordinary coal-acids where the hydrogen is relatively lowest. It being recognised that the first and last terms of the series represent derivatives of pure vegetable matter and of highly metamorphosed vegetable matter respectively, the probable conclusion is that the intermediate terms represent different stages in the alteration of vegetable matter. This does not, of course, infer that the process of change

has been through the above steps, as no doubt the different substances experimented on were produced under different conditions of moisture, temperature, bacterial action, etc. Each product may, however, represent the end point of a definite series of reactions produced by definite conditions. The close relationship between the formulae for the shale acids, peat acid, and cannel coal-acid may signify a definite stopping-place in the process of decomposition of vegetable matter, the carbon having increased at the expense of the hydrogen and highly complex substances having been formed.

There would seem to be no experimental ground for concluding that animal remains are mingled with this vegetable product, as on careful examination no trace of phosphates could be found in samples of torbanite, Broxburn, Camps, or Dunnet shales, and as the lime in the ash of shales is low, varying from a "trace" to 1·55 per cent. (*Oil-Shales of the Lothians*, 1912, pp. 159, 161).

#### SUMMARY.

1. The carbon-hydrogen ratio varies in the oil-shales from 6 to 8 and over. The lower this ratio the larger is the amount of oil produced from a definite percentage of organic matter. The carbon-hydrogen ratio is, in all the shales examined, lower than that of ordinary bituminous coals. The oil-shales are thus distinct from coals, although the richer varieties may approach cannel coals in properties.

2. There is but little resinous substance in oil-shales, the main bulk of the organic material being insoluble in organic solvents.

3. The organic substance in oil-shale is a decomposition product of vegetable matter (originally algae, spores, or simply concretions of macerated organic material) similar in nature to that found in peat and in cannel coal, and produced by a definite combination of external conditions.

In conclusion, I desire to thank Dr Flett for suggesting the lines of this research; Mr R. G. Carruthers, Mr D. Tait, Mr D. R. Steuart, F.I.C., Mr Wm. Caldwell, and Messrs Muir & Co. for assistance in securing samples; and Professor Walker, Dr J. E. Mackenzie, Dr Gordon, and Dr Campbell for their advice, assistance, and criticism throughout the research.

(Issued separately July 15, 1914.)

XIV.—Notes on the Atmospheric Electrical Potential Gradient in the Industrial Districts around Leeds. By Dan. W. Steuart and Ingvar Jörgensen. *Communicated by* JAMES A. S. WATSON, B.Sc.

(MS. received February 13, 1914. Read March 16, 1914.)

THE atmosphere of industrial districts is characterised by the pollution which it receives from smoke, comprising solid matters like carbon, tar, and mineral ash, and gaseous constituents such as  $\text{SO}_2$  and  $\text{CO}_2$ .

Much work has now been done with regard to the ionisation of gases by various means.\* Small ions, with a velocity of 1·6 cms. per second in an electric field of 1 volt per cm., have long been known to exist in the atmosphere. About ten years ago Langevin,† working in Paris, demonstrated the presence of large ions in addition, velocity 1/3000 cm. per second. McClelland and Kennedy‡ described the formation of large ions in the products of combustion, and later Kennedy,§ comparing town and country air, found in town air (Dublin) a larger number of ions, due to combustion processes; the large ions being increased, and to some extent at the expense of the small ones. Aitken|| has shown that the various products of combustion include nuclei of condensation and of spontaneous condensation, due largely to the presence of sulphur in the fuel. As the envelope which transforms small into large ions often consists of water, these two sets of results may be correlated to some extent. Eve,¶ for example, concluded that dust, smoke, or mist in air causes a transformation of small into large ions. Several sizes of ions are now known to exist in air, commencing with the small ions and with decreasing velocities as the size increases.\*\*

The foregoing researches indicate that ionisation by combustion and the presence of combustion products in the air may be essential factors in the phenomena of atmospheric electricity in industrial districts.

The following notes deal with measurements of potential gradients

\* J. J. Thomson, *Conduction of Electricity through Gases*. H. A. Wilson, *Electrical Properties of Flames and Incandescent Solids*, 1912.

† *Comptes Rendus*, 1905, p. 233.

‡ *Proc. Roy. Irish Acad.*, 1912, xxx., A, No. 5.

§ *Proc. Roy. Irish Acad.*, 1913, xxxii., A, No. 1.

|| These *Proceedings*, 1912, xxxii., Part 2, No. 16, and earlier.

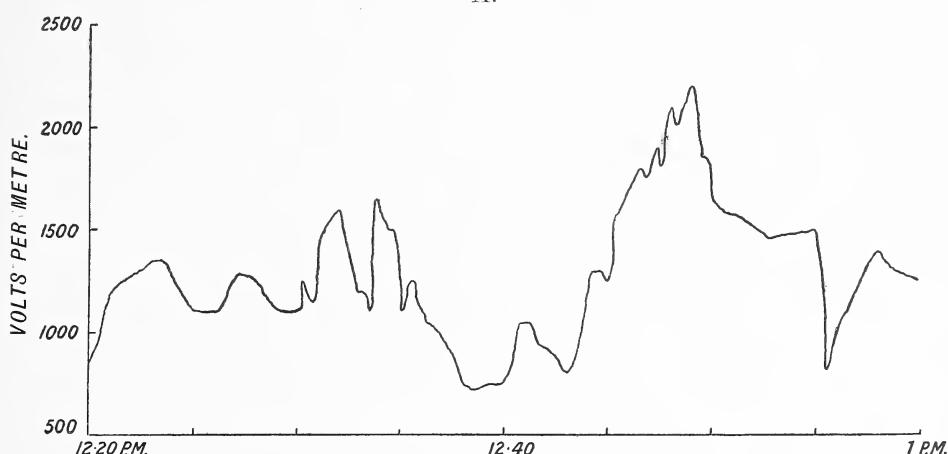
¶ *Phil. Mag.*, ccxxxv. p. 257.

\*\* Cf. Sutherland, *Phil. Mag.*, 1909, p. 341.

in such districts in the neighbourhood of Leeds, and with a few experiments designed to suggest an explanation of certain abnormalities, as compared with previous records. Our apparatus\* comprised a Lutz flame collector on an ebonite rod, an Exner electrometer for measuring potential differences to 800 volts, and a Braun electrometer for readings over 800. All measurements were made at a height of 1 metre from earth.

*Curve A.*—These measurements were made on a grass field near Kirkstall Forge, Leeds. The wind was blowing from the forge chimneys about 150 yards away; the tops of the chimneys being a little below the

A.



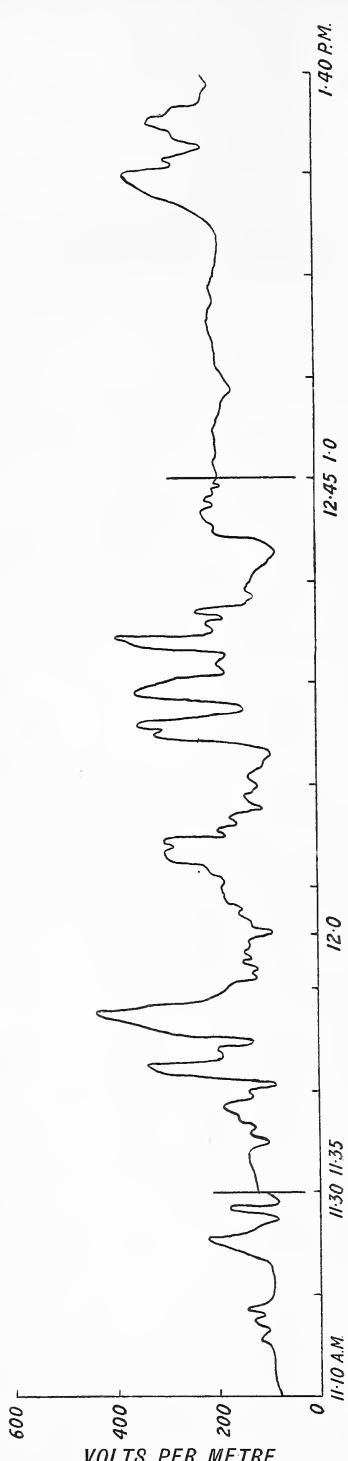
level of the instruments. During forty minutes the potential gradient varied rapidly between 720 and 2200 volts per metre.

On another day the instruments were 350 yards from these chimneys in the same direction, and the smoke caused a variation between 300 and 2250 during thirty-five minutes. A few hours later, owing to a change in the wind, most of the smoke blew somewhat to our right, and the variations were in consequence only from 120 to 300 during twenty minutes.

On another occasion we were half a mile from the forge in the same direction, and during fifty minutes the reading was never below 630. At this same spot, with the same wind direction and on two different days, the minimal readings were 390 during ninety minutes and 120 in twenty minutes, depending on the amount of smoke which reached the collector.

\* For the loan of the apparatus we are indebted to Prof. J. H. Priestley.

B.

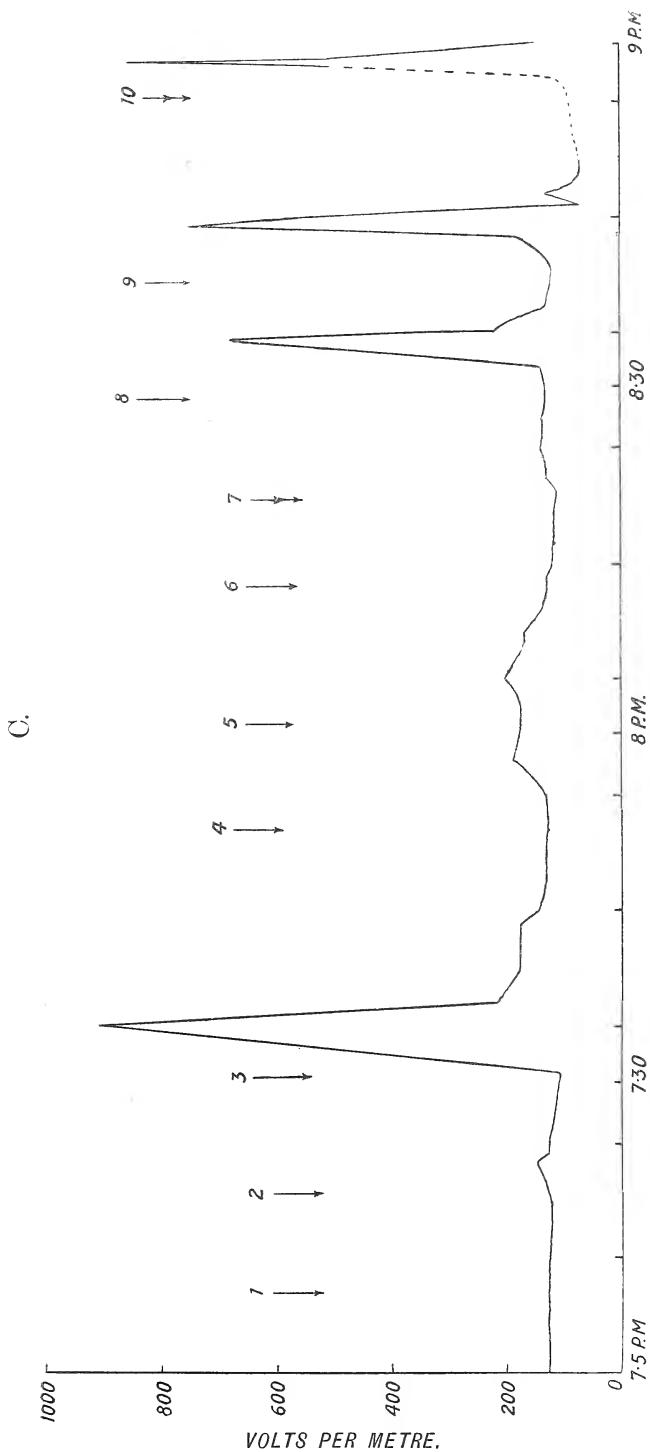


With a different wind direction the smoke from the forge was rising rapidly out of the valley and drifting over a hill. At the top of the hill the smoke was still mostly going right over our heads, and the readings in forty minutes were from 140 to 220, while 50 yards down the slope during thirty minutes the variations were from 75 to 130.

*Curve B* was taken near Garforth colliery, seven miles east of Leeds. It shows readings taken at distances of 100, 350, and 880 yards from a tall chimney when the wind was blowing smoke towards the instruments. About two hours after the last reading was made the instruments were taken to a position about half a mile to the windward of the chimney, and during twenty minutes the reading was never above 135.

It will be seen that fresh smoke reaching the collector caused an increase in the positive potential gradient. We commonly got readings of over 800 volts at distances over a mile from large chimneys. The interpretation of these measurements is slightly complicated owing to the ordinary variations in the potential gradient due to other causes. It is consequently more convenient to study the effect of smoke by means of passing trains, as in that case the smoke effect is limited to a definite interval of time, as will be seen from the following curve.

*Curve C.*—A slight wind was blowing from a railway 300 to 400 yards away (wind direction roughly at right angles to the railway). The ground level was below the level of the railway, and smoke from trains was wafted slowly down to



the instruments. The passage of a train is marked, and the effect of its smoke came several minutes later.

No effect was recorded, probably owing to some slight variation in the wind.

Train 8. White smoke.  
 „ 9. White smoke.  
 „ 10. Two trains { (a) Dense black smoke.  
                          (b) No visible smoke.

In the case of the white smoke the colour is due to moisture, and all whiteness had generally disappeared long before the smoke reached the instruments. Wilson concluded that the positive ions in a bunsen flame consist of charged molecules of the gases present. Similarly solid particles do not seem to be necessary for the carriage of the positive charge in smoke.

These potential gradient measurements confirm the conclusions of others that by combustion a considerable amount of ionisation is produced; but as the effect is always to produce an increase of the positive potential gradient, more positive than negative ions may be formed.

By burning considerable quantities of benzene, methylated spirits, and sulphur, separately and simultaneously in the open, at distances up to 25 yards from the collector, and under various meteorological conditions we were unable to reproduce the smoke effect. On burning these substances in the laboratory we found that the cooled combustion gases, in each case, contained both positive and negative ions,\* and in approximately equal numbers as far as we could gauge with the apparatus at our disposal. The mixed products of combustion would contain  $\text{CO}_2$ ,  $\text{SO}_2$ ,  $\text{SO}_3$ , carbon particles, water vapour, and nuclei both of condensation and of charge, as in the case of coal smoke.

We consider, therefore, that the ionisation giving rise to the largely

\* L. Bloch, *Annales de Chemie et de Physique*, xxii. and xxiii. Reoglie and Brizard, *Comptes Rendus*, 1909, p. 146.

increased potential gradient must produce many more positive than negative ions, due to some characteristic of the mechanism of the combustions investigated. Such is the case, for example, with ionisation by certain incandescent particles at moderately high temperatures.\* In the cases cited the effluent gases would have been subjected to a temperature of perhaps from 600 to over 1000° C.†

This work was done during the summers of 1912 and 1913 in connection with other smoke experiments being conducted at Leeds University. In studying the effects of smoke on plant growth it is very desirable to have some means of measuring the concentration of noxious smoke gases in the atmosphere, and we hoped that this object might be attained by measurements of the air potential gradient. It does not seem, however, as if these would give much guidance.

#### SUMMARY.

The general effect of products of combustion would be to cause a transformation of the small ions of the air into large ions, which, acting alone, would tend to decrease the air conductivity. Ionisation by flames, however, adds to the number of ions in the air, so that the size of the ions might be increased without the conductivity of the air diminishing. In the case of the fresh smoke direct from the forge or colliery chimney-stalks or railway engines of our experiments, it is suggested that combustion in the furnaces would result in an ionisation producing more positive than negative ions. It is only where similar conditions obtain that we should expect such large increases in the positive potential gradient, due to smoke, as we have recorded.

\* H. A. Wilson, *loc. cit.*

† Rusby, *Journal of Franklin Inst.*, July 1913.

XV.—On the Hall and the Transverse Thermomagnetic Effects and their Temperature Coefficients. By F. Unwin, M.Sc., Assistant Lecturer in Physics, Heriot-Watt College, Edinburgh. *Communicated by Professor F. G. BAILY.*

(MS. received May 5, 1914. Read June 15, 1914.)

#### INTRODUCTION.

Of the more recent researches on the subject of this paper, mention may be made of the work of H. Zahn \* on the galvanomagnetic and thermomagnetic effects in various metals. Zahn has measured these effects in many different metals, and has used his results to test the electron theory of the properties of metals as developed by P. Drude. He has also determined in some cases the temperature variation of the effects.

The author of the present paper has confined his attention to the thermomagnetic transverse effects and the Hall effect. These have been measured in magnetic fields of various strengths and at temperatures varying over a range of about 100 Centigrade degrees.

The experiments were carried out with a view to obtaining some light on the electron theory, and the ratios of the effects are discussed in relation to this theory.

#### DEFINITION OF THE COEFFICIENTS OF THE EFFECTS AND THE CONVENTION WITH RESPECT TO THE SIGNS.

In accordance with the custom of other workers, the Hall coefficient is denoted by R, the Thermomagnetic Temperature Effect by S, and the Thermomagnetic Potential Effect by Q.

The directions of the effects corresponding to positive values of these coefficients are indicated by the diagrams (fig. 1) given below; it being understood that the magnetic field is in each case directed downwards at right angles to the plane of the diagram.

The value of S is found as usual by calculating the transverse temperature difference in a plate 1 cm. broad placed in unit magnetic field, when the temperature gradient along the axis is 1° C. per cm.

The value of Q is found by calculating the transverse E.M.F. (in electromagnetic units) under the same conditions.

\* *Ann. d. Phys.*, xiv. p. 886, 1904.

## INSTRUCTIONS TO AUTHORS.

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In the case of the Hall effect, the value of  $R$  is calculated for unit potential gradient instead of for unit current density, as it would seem that the coefficient calculated in this way should be more directly comparable with the coefficients  $S$  and  $Q$ . The value of  $R$ , therefore, is the transverse

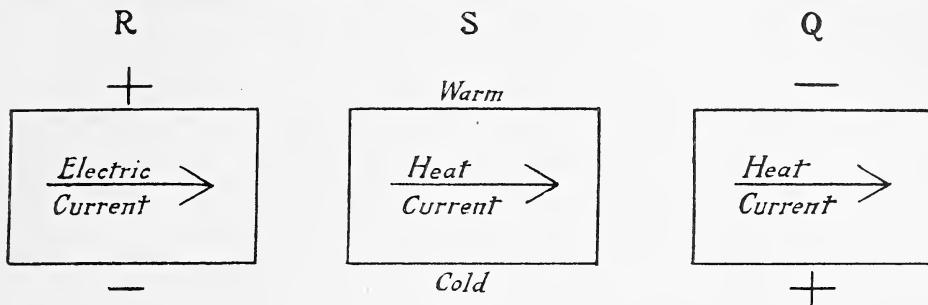


FIG. 1.

E.M.F. (electromagnetic units) in a plate of unit breadth when placed in unit magnetic field, the potential gradient along the axis being 1 electromagnetic unit per cm.

#### APPARATUS.

The specimen plate, *abcd* (fig. 2), to be tested was soldered to two copper lugs *L*, *L*, which were themselves soldered to two brass tubes *T*, *T*. These tubes were fixed to a wooden frame, which served as a support.

The breadth of the specimen plate was in all cases about 2 cm., and the distance between the lugs about 5 cm. The thickness differed for the different specimens, but was in all cases less than a millimetre. Wires were soldered to the brass tubes to enable an electric current to be sent through the plate for the Hall effect measurement.

Five copper-constantan thermocouples were soldered to the plate at five points *A*, *B*, *C*, *D*, *E*. The three points *E*, *B*, and *D* were on the axis of the plate, while *A* and *C* were on a line perpendicular to this axis and passing through *B*. The distances between *B* and the other points were approximately 1 cm., but the actual distances were measured for each plate.

The thermocouples were made of fine wire (No. 42 S.W.G.), so as to avoid as far as possible any cooling effect due to conduction of heat along the wires.

Two water jackets *J*, *J*, 8 cm. by 5 cm. and about 1 mm. thick, were placed one on each side of the plate, and were kept at a distance of 0·84 cm. apart by brass distance-pieces, thus forming a kind of box enclosing the specimen plate. The space between the plate and these jackets was filled

with loosely packed cotton-wool. These jackets were found to be necessary in order to bring the plate to a stationary condition as regards temperature. In some cases they were supplied with cold water, and in others with steam.

The electromagnet used was of the "ironclad" type, and was designed by Professor Baily and built up in the workshops of the Heriot-Watt College. The pole-pieces had a maximum diameter of 20 cm. and were coned down to a pole face of 5 cm. diameter. Each pole carried an exciting coil

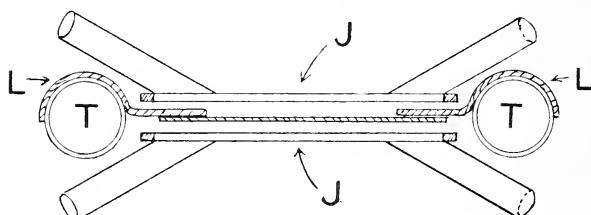
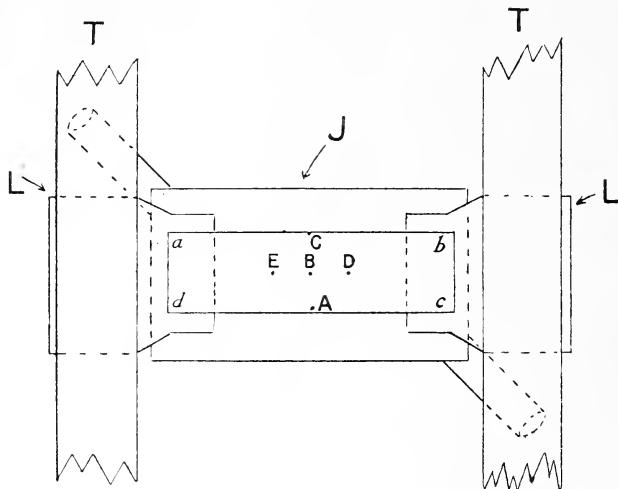


FIG. 2.

of 880 turns, so that with a current of 20 amperes a magneto-motive force of 35,200 ampere-turns was obtained. This was found to produce a field strength of about 20,000 C.G.S. units.

#### MEASURING ARRANGEMENTS.

The free ends of the wires of the five thermocouples were soldered to stouter copper wires, and these junctions were kept in a Thermos flask, by which means their temperature was maintained constant for the length of time required for the observations.

The ten copper wires from this junction box were led to a small distributing board (fig. 3). Two similar moving coil galvanometers were used, and these were connected to the distributing board through the keys  $P_1$  and  $P_2$ . The measured low resistances  $S_1$  and  $S_2$  formed portions of the galvanometer circuits and served as potentiometer wires. They carried currents supplied by the accumulators  $V_1$  and  $V_2$ , and regulated by the resistance boxes  $R_1$  and  $R_2$ .

One of the measuring circuits was connected to the contacts H and I

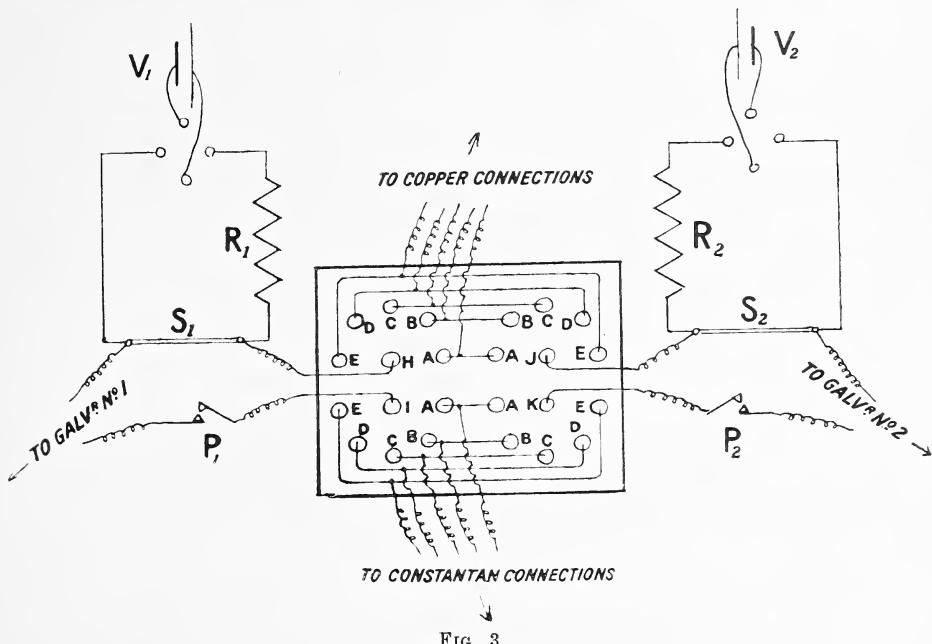


FIG. 3.

on the distributing board, and the other to the contacts J and K; the galvanometers could thus be easily connected to any of the couples or to the separate wires of different couples.

#### METHOD OF OBSERVATION.

##### *Thermomagnetic Effects.*

The brass tubes were supplied respectively with aniline vapour and steam, or aniline vapour and cold water, or steam and cold water, as the case might be, so as to produce the required temperature gradient, and the jackets were supplied with cold water or steam according to circumstances. When the plate had attained a steady state, the temperatures at E, B, and D were determined by means of the corresponding thermocouples.

The copper wires belonging to the junctions at A and C were then connected to one of the galvanometer circuits, and the constantan wires from A and C to the other galvanometer circuit. Simultaneous readings of the E.M.F.'s in these two circuits were taken, first when the magnetic field was zero, and afterwards with magnetic fields of known strength in each direction. The thermoelectric force between copper and constantan at various temperatures being known from the results of a special experiment, the transverse temperature difference and the transverse potential difference due to the magnetic field could then be calculated, and hence the values of S and Q.

#### CALCULATION OF THE TRANSVERSE TEMPERATURE DIFFERENCE AND POTENTIAL DIFFERENCE.

Before the magnet is excited there will be a certain difference of temperature between A and C. Let  $\theta$  be this difference, A being at a higher temperature than C. On account of this there will be an E.M.F.,  $E$ , acting from A to C in the copper circuit, and an E.M.F.,  $e$ , acting from A to C in the constantan circuit. If M and N are the thermoelectric powers of the metal of the plate with respect to copper and constantan respectively, there will be the relations:—

$$E = -M\theta \quad . . . . . \quad (1)$$

$$e = -N\theta \quad . . . . . \quad (2)$$

When the magnetic field is excited, the temperature difference  $\theta$  is altered to some value  $\theta + \delta\theta$ , and at the same time a transverse potential difference is set up. Let this potential difference, measured from A to C, be denoted by V.

Let the new values of  $E$  and  $e$  be  $E + \delta E$  and  $e + \delta e$ .

There will be the relations:—

$$E + \delta E = V - M(\theta + \delta\theta) \quad . . . . . \quad (3)$$

$$e + \delta e = V - N(\theta + \delta\theta) \quad . . . . . \quad (4)$$

Now, by subtraction of (4) from (3),

$$(E - e) + (\delta E - \delta e) = (N - M)\theta + (N - M)\delta\theta$$

and from (1) and (2),

$$E - e = (N - M)\theta;$$

therefore

$$\delta E - \delta e = (N - M)\delta\theta,$$

or

$$\delta\theta = \frac{\delta E - \delta e}{N - M}.$$

Now  $(N - M)$  is the thermoelectric power of copper with respect to constantan, and is known from the results of a special experiment. The value of  $\delta\theta$  can therefore be calculated from the observations  $\delta E$  and  $\delta e$ .

Again, by (1) and (2),

$$\frac{E}{e} = \frac{M}{N},$$

so that (3) and (4) may be written

$$E + \delta E = V - M(\theta + \delta\theta)$$

$$(e + \delta e) \times \frac{E}{e} = V \times \frac{E}{e} - M(\theta + \delta\theta),$$

from which

$$\delta E - \delta e \times \frac{E}{e} = V \left(1 - \frac{E}{e}\right),$$

and hence

$$V = \frac{e\delta E - E\delta e}{e - E}.$$

The transverse potential difference  $V$  can thus be determined from the observations  $E$ ,  $e$ ,  $\delta E$ , and  $\delta e$ .

#### HALL EFFECT.

The tubes and jackets were supplied with steam or cold water in order to bring the plate to the required temperature. A current of 10 amperes was passed through the plate, and the temperatures at E, B, and D were measured; observations being taken also with the current reversed, in order to correct for any direct action of the current on the E.M.F.'s at the junctions.

The copper wires at A and C were then connected to one galvanometer circuit and the constantan wires at A and C to the other, and readings were taken with magnetic field zero and also with known magnetic fields in each direction. From these readings the transverse E.M.F. due to the magnetic field could be calculated.

The potential gradient along the axis of the plate was determined by connecting the copper wires at E and D to one galvanometer and the constantan wires at E and D to the other, and taking readings with both directions of the current.

The value of R was then calculated in accordance with the definition given.

No correction was made for the influence of the Transverse Galvano-magnetic Temperature Effect, as, in the case of the metals tested, this effect is very small.

## RESULTS.

*Nickel*.—Thickness of plate = 0·275 mm.

The thermomagnetic temperature and potential effects were measured in magnetic fields of various strengths and at three temperatures, namely, 43° C., 61·5° C., and 126° C.

Temperature of Plate = 43° C.			Temperature of Plate = 61·5° C.			Temperature of Plate = 126° C.		
Magnetic Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Magnetic Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Magnetic Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .
2,000	- 7·10	- 31·0	...	...	...	2,900	- 5·55	- 78·0
4,200	- 6·80	- 29·5	...	...	...	4,800	- 5·15	- 76·5
6,500	- 6·62	- 27·5	...	...	...	6,800	- 4·70	- 68·5
8,300	- 5·78	- 23·5	8,500	- 5·30	- 34·5	8,300	- 4·17	- 58·6
14,500	- 3·50	- 14·5	...	...	...	12,000	- 3·01	- 41·4
19,100	- 2·78	- 11·5	...	...	...	17,350	- 2·12	- 29·0
21,900	- 2·42	- 10·0	22,600	- 2·25	- 14·5	22,100	- 1·78	- 23·5
...	...	...	...	...	...	22,800	- 1·74	- 22·2

The coefficients S and Q are both negative, and vary greatly with the strength of the magnetic field. The numerical value of the coefficients decreases as the field strength increases.

An increase of temperature causes a decrease in the numerical value of S, but a very large increase in the numerical value of Q.

Magnetic Field.	Range of Temperature.	Temperature Coefficient of S.	Temperature Coefficient of Q.
8,500 {	43° C.-61·5° C.	- 0·0039	+ 0·027
	61·5° C.-126° C.	- 0·0033	+ 0·011
22,000 {	43° C.-61·5° C.	- 0·0028	+ 0·026
	61·5° C.-126° C.	- 0·0035	+ 0·009

*Hall Effect*.—This was measured in magnetic fields of strength varying from 2000 to 22,000 units, and at temperatures of 14° C. and 101° C.

Temperature of Plate = 14° C.		Temperature of Plate = 101° C.	
Field.	R × 10 <sup>7</sup> .	Field.	R × 10 <sup>7</sup> .
2,050	- 9·10	2,100	- 12·0
3,800	- 8·95	4,060	- 10·9
6,300	- 8·10	6,400	- 10·0
8,200	- 7·10	9,500	- 7·27
14,300	- 4·50	...	...
18,800	- 3·59	...	...
21,750	- 3·21	21,900	- 3·48

The coefficient R is negative and varies with the field strength in practically the same way as S and Q.

Increase of temperature increases the numerical value of R.

Temperature coefficient of R in field of 6,400 units = +0·0025.

" " " 22,000 " = +0·0010.

*Iron.* Thickness of plate = 0·51 mm.

*Thermomagnetic Temperature and Potential Effects.*—These were measured in magnetic fields varying from 2000 to 22,000 units, and at temperatures of 48·6° C., 71·5° C., 97·9° C., and 129·2° C.

Temperature of Plate = 48·6° C.			Temperature of Plate = 71·5° C.			Temperature of Plate = 97·9° C.			Temperature of Plate = 129·2° C.		
Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .
2,100	+5·75	+10·0	2,000	+6·45	+10·8	2,100	+7·16	+12·8	2,000	+6·05	+10·9
4,000	+5·44	+10·2	4,000	+6·27	+9·7	4,300	+6·24	+11·7	4,500	+6·98	+10·1
6,500	+5·20	+10·1	6,000	+6·00	+10·5	6,300	+6·10	+11·5	6,650	+6·31	+10·4
8,550	+5·17	+10·6	8,300	+5·82	+10·4	9,250	+6·26	+10·6	8,200	+6·25	+10·5
11,300	+5·54	+10·9	11,400	+6·08	+10·3	...	...	...	11,500	+6·21	+10·3
14,800	+5·43	+10·6	14,900	+5·92	+10·8	16,400	+5·85	+11·6	14,250	+6·12	+9·9
17,600	+5·26	+9·9	17,550	+5·72	+9·7	20,300	+5·65	+10·5	19,300	+5·70	+9·2
22,650	+4·88	+9·1	22,500	+5·40	+8·7	23,300	+5·30	+9·5	22,100	+5·39	+8·6

The coefficients S and Q are both positive; they decrease slightly as the magnetic field is increased. As the field approaches the value 20,000 units, a more rapid decrease in the values of S and Q is observed.

Field.	Range of Temperature.	Temperature Coefficient of S.	Temperature Coefficient of Q.
22,500 {	48·6° C.-71·5° C. 71·5° C.-97·9° C. 97·9° C.-129·2° C.	+0·0045 zero zero	-0·002 +0·005 -0·002

Up to about 70° C. the value of S increases considerably with rising temperature, but between 70° C. and 130° C. it remains nearly constant.

The variation of Q is not so simple. As the temperature is increased the value of Q at first decreases, then increases, and finally decreases again.

*Hall Effect.*—This was measured in magnetic fields of strengths varying from 2000 to 22,000 units, at temperatures of  $13.3^{\circ}\text{ C}$ . and  $99.8^{\circ}\text{ C}$ .

Temperature of Plate = 13.3° C.		Temperature of Plate = 99.8° C.	
Field.	R × 10 <sup>7</sup> .	Field.	R × 10 <sup>7</sup> .
2,250	+6.60	1,950	+9.65
...	...	4,050	+8.75
5,500	+6.10	6,400	+8.83
8,450	+6.17	9,500	+9.00
15,000	+6.26	14,850	+8.95
19,000	+6.06	17,300	+8.70
22,600	+5.73	22,600	+8.01

The coefficient R is positive; it decreases with increasing field, very slightly at first, but more rapidly as the field approaches 20,000 units.

Increase of temperature produces a very marked increase in the value of R.

Temperature coefficient of R in field of 6,400 units = +0.0052  
 " " " 22,600 " = +0.0046.

*Copper.* Thickness of plate = 0.063 mm.

*Thermomagnetic Temperature and Potential Effects.*—These were measured in magnetic fields of three different strengths, and at three temperatures, viz.  $43.9^{\circ}$  C.,  $70.7^{\circ}$  C., and  $125.8^{\circ}$  C.

Temperature of Plate = 43.9° C.			Temperature of Plate = 70.7° C.			Temperature of Plate = 125.8° C.		
Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .	Field.	S × 10 <sup>7</sup> .	Q × 10 <sup>4</sup> .
7,850	-2.04	+1.61	7,900	-1.75	+1.53	7,700	-1.74	+1.48
13,550	-2.20	+1.67	13,650	-1.84	+1.53	13,000	-1.80	+1.47
21,400	-2.27	+1.69	21,400	-1.86	+1.55	21,150	-1.74	+1.47

The effects are of opposite signs, S being negative and Q positive. The values of S and Q are only slightly affected by the strength of the magnetic field.

Increase of temperature causes a diminution in the numerical values of both S and Q, and the rate of diminution decreases as the temperature rises.

Temperature coefficient of S in field of 21,000 units for range—

				$43.9^{\circ}\text{ C.}$ to $70.7^{\circ}\text{ C.}$ = -0.0068.
"	"	"	"	$70.7^{\circ}\text{ C.}$ to $125.8^{\circ}\text{ C.}$ = -0.0012.
"	"	Q	"	$43.9^{\circ}\text{ C.}$ to $70.7^{\circ}\text{ C.}$ = -0.0031.
"	"	"	"	$70.7^{\circ}\text{ C.}$ to $125.8^{\circ}\text{ C.}$ = -0.0009.

*Hall Effect.*—This was measured in magnetic fields of strengths of about 8000 and 22,000 units, and at temperatures of 15·6° C. and 99·8° C.

Temperature of Plate = 15·6° C.		Temperature of Plate = 99·8° C.	
Field.	$R \times 10^7$ .	Field.	$R \times 10^7$ .
8,200	-2·61	8,000	-2·00
22,000	-2·76	21,500	-2·06

The coefficient  $R$  is negative and is but slightly affected by the strength of the magnetic field. The numerical value of  $R$  decreases very considerably with increasing temperature.

Temperature coefficient of  $R$  in field of 22,000 units = -0·0038.

*Zinc.* Thickness of plate = 0·15 mm.

*Thermomagnetic Temperature and Potential Effects.*—These were measured in magnetic fields of three different strengths, and at temperatures of 46·4° C., 74·4° C., and 128·3° C.

Temperature of Plate = 46·4° C.			Temperature of Plate = 74·4° C.			Temperature of Plate = 128·3° C.		
Field.	$S \times 10^7$ .	$Q \times 10^4$ .	Field.	$S \times 10^7$ .	$Q \times 10^4$ .	Field.	$S \times 10^7$ .	$Q \times 10^4$ .
7,700	+1·04	+1·24	7,700	+0·95	+1·35	7,700	+0·83	+1·30
13,500	+1·05	+1·26	13,300	+0·98	+1·34	13,500	+0·80	+1·26
21,100	+1·06	+1·32	21,200	+0·97	+1·44	21,400	+0·82	+1·20

The coefficients  $S$  and  $Q$  are both positive, and they vary slightly with variation of magnetic field. The value of  $S$  decreases steadily with increasing temperature. The value of  $Q$  increases up to a certain temperature, beyond which an increase in temperature produces a decrease in  $Q$ .

Temperature coefficient of  $S$  in field of 21,000 units for range—

46·4° C. to 74·4° C. = -0·0030.

74·4° C. to 128·3° C. = -0·0030.

“ “ “ “ Q “ “ “ “ 46·4° C. to 74·4° C. = +0·0032.

“ “ “ “ “ “ “ “ 74·4° C. to 128·3° C. = -0·0031.

*Hall Effect*.—This was measured at temperatures of  $15\cdot9^{\circ}$  C. and  $101\cdot5^{\circ}$  C.

Temperature of Plate = $15\cdot9^{\circ}$ C.		Temperature of Plate = $101\cdot5^{\circ}$ C.	
Field.	$R \times 10^7$ .	Field.	$R \times 10^7$ .
7,700	+1·40	7,800	+0·95
13,200	+1·41	13,770	+0·98
21,300	+1·43	21,300	+0·99

The coefficient R is positive, and increases slightly with increasing magnetic field, but decreases with increasing temperature.

Temperature coefficient of R in field of 21,300 units =  $-0\cdot0037$ .

*Aluminium*. Thickness of plate = 0·25 mm.

*Thermomagnetic Effects*.—These were measured at  $40^{\circ}$  C.

Coefficient S in field of 22,700 units =  $-6\cdot3 \times 10^{-8}$ .

„ Q „ 22,700 „ =  $-3\cdot9 \times 10^{-5}$ .

*Hall Effect*.—This was measured at  $14\cdot5^{\circ}$  C.

Coefficient R in field of 21,600 units =  $-1\cdot15 \times 10^{-7}$ .

#### DISCUSSION OF RESULTS.

It will be seen from the following table that, although the Hall and thermomagnetic temperature effects vary both in magnitude and sign from metal to metal, the ratio  $\frac{R}{S}$  is for all the metals tested of the same positive sign, and does not vary greatly in magnitude. The ratio is not greatly affected by change of temperature, except in the case of nickel.

Metal.	Field.	Value of Ratio $\frac{R}{S}$ .	
		Temperature $44^{\circ}$ C.	Temperature $100^{\circ}$ C.
Nickel . . .	{ 8,500	1·32	1·83
	{ 23,000	1·37	1·90
Iron . . .	{ 8,000	1·34	1·44
	{ 22,000	1·36	1·49
Copper . . .	{ 8,000	1·18	1·15
	{ 21,000	1·09	1·18
Zinc . . .	{ 8,000	1·19	1·07
	{ 21,000	1·20	1·09
Aluminium . . .	22,700	1·83	...

The ratio  $\frac{Q}{R}$  varies from metal to metal. It is positive for all the metals tested except copper. The values are given in the following table:—

Field strength = 21,000.

Metal.	$\frac{Q}{R}$ at 45° C.	$\frac{Q}{R}$ at 100° C.
Nickel . . . .	$+3.25 \times 10^3$	$+6.0 \times 10^3$
Iron . . . .	$+1.39 \times 10^3$	$+1.18 \times 10^3$
Copper . . . .	$-0.68 \times 10^3$	$-0.73 \times 10^3$
Zinc . . . .	$+1.07 \times 10^3$	$+1.40 \times 10^3$
Aluminium . . . .	$+0.34 \times 10^3$	...

It may be of interest to consider these results in their relation to the electron theory. In the development of this theory some difficulty arises in explaining the variation of the signs of the transverse effects. Drude \* in his investigation assumes the existence of positive and negative carriers both taking part in the transmission of the heat and electric currents. Drude finds the following expressions for the Hall and thermomagnetic transverse effects:—

$$R \dagger = c \left( \frac{y_1 x_2 - y_2 x_1}{x_1 + x_2} \right)$$

$$Q = - \frac{p}{\sigma} (\sigma_1 x_1 y_2 + \sigma_2 x_2 y_1)$$

$$S = \frac{1}{c\sigma} (\sigma_2 y_1 - \sigma_1 y_2),$$

where

$$x_1 = \frac{d(\log n_1)}{dt}, \quad x_2 = \frac{d(\log n_2)}{dt},$$

$$\begin{aligned} y_1 &= ev_1, & y_2 &= ev_2, \\ \sigma_1 &= e^2 v_1 n_1, & \sigma_2 &= e^2 v_2 n_2, & \sigma &= \sigma_1 + \sigma_2. \end{aligned}$$

$n_1$  and  $n_2$  are the numbers of positive and negative carriers respectively in each unit volume of the metal.

$e$  = the magnitude of the charge on each carrier.

$v_1$  and  $v_2$  are the average velocities impressed on the positive and negative carriers respectively by unit electric field.

$p$  and  $c$  are constants.

Now if  $x_1$  and  $x_2$  have the same sign,  $R$  and  $S$  are both differential effects

\* *Ann. d. Phys.*, vol. i., 1901.

† The expression for  $R$  given by Drude has been multiplied by  $\sigma$  in order to make it applicable to the coefficient  $R$  as defined in this paper.

and may of course vary in sign from metal to metal, but R and S will both have the same sign in the same piece of metal. This is quite in accordance with experiment.

In the case of the coefficient Q, it will be seen that a diversity of sign can only occur if  $x_1$  and  $x_2$  have opposite signs, which is not a very likely proposition. In any case, there would be no reason to expect any close relation between the sign of Q and the sign of R. Experiment shows, however, that these coefficients have in general, though not always, the same sign.

Again, in order to account for the fact that the ratio  $\frac{R}{S}$  has nearly the same value for all metals, it must be supposed that one group of carriers is of small importance compared with the other.

It would seem, therefore, that the difference in the signs of the transverse effects in different metals must be referred to some cause other than the participation of positive as well as of negative carriers in the transmission of the current.

Sir Joseph Thomson\* has suggested that the reversal of the Hall effect in iron may be due to the fact that the magnetic field close to a molecule is in the opposite direction from the magnetic field in the free space between molecules. The Hall effect would thus be a differential effect, and the reversed sign would be easily accounted for.

A. W. Smith† has put this to the test of experiment by measuring the Hall effect in iron at various temperatures up to 1000° C. No reversal of the sign of the Hall effect was observed, although iron loses its magnetic properties at a temperature considerably below 1000° C. A simple inspection of the values of the Hall effect for different metals is sufficient to show that there is no direct relation between the sign of the Hall effect and the magnetic properties of the metal, for the effect is positive in both iron and zinc, while it is negative in both nickel and copper.

Nevertheless, a differential action such as Thomson has suggested would carry us far towards an explanation of the experimental results, if it could be supposed to occur in all metals whether magnetic or not. Such a differential action would apply with equal force to all the transverse effects, and would thus account for the experimental relation between R and S. The fact that the relation between Q and R is not so simple is not difficult to understand, for in the case of the thermomagnetic effect the temperature gradient in the plate sets up a potential gradient along

\* *Corpuscular Theory of Matter.*

† *Phys. Rev.*, Jan. 1910.

the plate, and this will have an effect on the value of  $Q$ . In consequence of this, it is not to be expected that the ratio  $\frac{Q}{R}$  will be so nearly constant as  $\frac{R}{S}$ .

On examining the experimental results, it will be seen that there is no simple relation between the temperature coefficient of  $Q$  and those of  $R$  and  $S$ . This is easily accounted for by the fact, previously pointed out, that the potential effect is influenced by causes other than those which determine the other two effects. On the other hand, some relation is to be expected between the temperature coefficients of  $R$  and  $S$ .

On the assumption that only negative carriers need be considered, the electron theory leads to the expressions

$$R = A \frac{e\lambda u}{8aT} \quad S = B \frac{e\lambda u}{8aT},$$

where  $e$  = the electronic charge

$\lambda$  = mean free path,

$u$  = mean square velocity of electrons,

$aT$  = K.E. of electron at absolute temperature  $T$  due to its linear motion.

$A$  and  $B$  are factors depending upon the distribution of the magnetic field within the metal.

Now, if  $R$  and  $S$  are influenced in the same way by the distribution of the magnetic field, the value of  $\frac{R}{S}$  is unity, and should be independent both of field strength and temperature. There is a rough approximation to this in the case of copper and zinc. In the case of iron the value of  $\frac{R}{S}$ , although considerably greater than unity, does not vary greatly with variation of either field strength or temperature.

In the case of nickel, however, a very considerable discrepancy is apparent. It is difficult to account for this discrepancy except on the assumption that the values of  $R$  and  $S$  do not depend in quite the same way upon the distribution of the magnetic field. This is not inconceivable, since in the case of the heat current the electrons are moving with velocities which remain constant between two collisions, whereas in the case of the electric current the velocities of the electrons are subject to an acceleration.

The galvanomagnetic temperature effect, which has not been considered in this paper owing to the difficulty experienced in measuring it in such

metals as copper, zinc, and aluminium, is of considerable theoretical interest. In accordance with the theory of the differential field, the sign of this effect should change along with the sign of the Hall effect, whereas in accordance with Drude's statement of the theory it should be of the same sign in all metals. Zahn has measured the effect in several metals, but it so happens that, owing to the metals used, no definite conclusions can be drawn from the results. The author hopes to carry out further experiments on this point.

*(Issued separately August 4, 1914.)*

## XVI.—Some Factorable Continuants. By W. H. Metzler, Ph.D.

(MS. received May 15, 1914. Read June 15, 1914.)

1. IN the *Transactions of the South African Philosophical Society* for January 1905, Dr Thomas Muir gives the most general continuant resolvable into factors by means of a given set of line-multipliers. He starts with the multipliers and determines the continuant resolvable by them. At the end of his paper he gives another continuant and its factors, but not the line-multipliers, which he says "is equally interesting in itself and equally full of promise as a base for investigation." Throughout his paper Muir is dealing with one of the two determinant factors of order  $n$  into which every centro-symmetric continuant of order  $2n$  can be broken up. Starting with the larger continuant of which Muir's is a factor, one of the objects of this paper is to determine a set of row and column multipliers that will cause the continuant to break up into quadratic factors and thence into linear factors. Other and more general types of continuants are given which these same multipliers reduce to quadratic factors. Another and more convenient way to determine the factors of these determinants is obtained in the form of reduction formulas. It is also shown how, for the two parts of order  $n$  into which the larger continuant of order  $2n$  breaks up, the linear factors come out by reduction.

2. The determinant in question is

$$\begin{aligned}
 & a \frac{(2n-1)\beta}{2n-1} \\
 & \frac{1.(\beta+2n-2)}{3-2n} a \frac{(2n-2)(\beta+1)}{2n-3} \\
 & \quad \frac{2.(\beta+2n-3)}{5-2n} a \frac{(2n-3)(\beta+2)}{2n-5} \\
 T_a \equiv & \quad \dots \\
 & \frac{(2n-2)(\beta+1)}{2n-3} a \frac{1.(\beta+2n-2)}{3-2n} \\
 & \quad \frac{(2n-1)\beta}{2n-1} a \\
 & = \{a^2 - \beta^2\} \{a^2 - (\beta+2)^2\} \dots \{a^2 - (\beta+2n-2)^2\},
 \end{aligned}$$

and the multipliers are

$$\begin{aligned}
 C_{2k-1} &+ \frac{k(2n-4k-3)}{(2n-2k-1)} C_{2k+1} + \dots \\
 &+ \frac{|k+h-1|}{|k-1 \cdot |h|} \cdot \frac{(2n-4k+3)(2n-4k+1) \dots (2n-4k+3-2 \cdot \overline{h-1})}{(2n-2k+1)(2n-2k-1) \dots (2n-2k+1-2 \cdot \overline{h-1})} C_{2k+2h-1} + \dots, \\
 C_{2k} &+ \frac{k(2n-4k+1)}{(2n-2k-1)} C_{2k+2} + \dots \\
 &+ \frac{|k+h-1|}{|k-1 \cdot |h|} \cdot \frac{(2n-4k+1)(2n-4k-1) \dots (2n-4k+1-2 \cdot \overline{h-1})}{(2n-2k-1)(2n-2k-3) \dots (2n-2k-1-2 \cdot \overline{h-1})} C_{2k+2h} + \dots, \\
 R_{2k} &- \frac{(k-1)(2n-4k+5)}{(2n-2k+1)} R_{2k-2} + \dots \\
 &+ (-)^k \frac{|k-1|}{|k-h-1 \cdot |h|} \cdot \frac{(2n-4k+5+4 \cdot \overline{h-1})(2n-4k+3)(2n-4k+5) \dots (2n-4k+3+2 \cdot \overline{h-2})}{(2n-2k+1)(2n-2k+3) \dots (2n-2k+1+2 \cdot \overline{h-1})} R_{2k-2h} + \dots \\
 R_{2k+1} &- \frac{k(2n-4k+3)}{(2n-2k+1)} R_{2k-1} + \dots \\
 &+ (-)^k \frac{|k|}{|k-h \cdot |h|} \cdot \frac{(2n-4k+3+4 \cdot \overline{h-1})(2n-4k+1)(2n-4k+3) \dots (2n-4k+1+2 \cdot \overline{h-2})}{(2n-2k+1)(2n-2k+3) \dots (2n-2k+1+2 \cdot \overline{h-1})} R_{2k-2h+1} + \dots,
 \end{aligned}$$

where  $C_i$  and  $R_j$  represent the  $i$ th column and  $j$ th row respectively.

3. The work of finding these line-multipliers will not be given here, though a few words as to the method used may be of interest. A series of consecutive non-zero elements along and parallel to the principal diagonal, beginning with that in the  $2r$ th row and  $2r$ th column, were written down, and the various multipliers for these general rows and columns determined under the conditions that when all the operations were completed the resulting determinant was such that all the elements, say below the principal diagonal, were zero, except those in the odd places of the line immediately below the principal diagonal, in which case the determinant obviously breaks up into quadratic factors, each of which is the difference of two squares.

4. As an illustration take the determinant of order eight

$$\left| \begin{array}{ccccccccc}
 a & & \beta & & & & & & \\
 \beta+6 & a & & 6(\beta+1) & & & & & \\
 -5 & & & 5 & & & & & \\
 & & & & & & & & \\
 2(\beta+5) & a & & 5(\beta+2) & & & & & \\
 -3 & & & 3 & & & & & \\
 & & & & & & & & \\
 3(\beta+4) & a & & 4(\beta+3) & & & & & \\
 -1 & & & 1 & & & & & \\
 & & & & & & & & \\
 4(\beta+3) & a & & 3(\beta+4) & & & & & \\
 1 & & & -1 & & & & & \\
 & & & & & & & & \\
 5(\beta+2) & a & & 2(\beta+5) & & & & & \\
 3 & & & -3 & & & & & \\
 & & & & & & & & \\
 6(\beta+1) & a & & \beta+6 & & & & & \\
 5 & & & -5 & & & & & \\
 & & & & & & & & \\
 \beta & & a & & & & & & \\
 \end{array} \right|$$

and perform the operations

$$\begin{aligned} C_1 + C_3 + C_5 + C_7, & \quad C_3 + \frac{6}{5}C_5 + \frac{3}{5}C_7, \quad C_5 - C_7, \\ C_2 + C_4 + C_6 + C_8, & \quad C_4 + \frac{2}{3}C_6 - C_8, \quad C_6 - 9C_8, \\ R_8 + 9R_6 - 5R_4 - 5R_2, & \quad R_6 - \frac{2}{3}R_4 - \frac{1}{3}R_2, \quad R_4 - R_2, \\ R_7 + R_5 - \frac{9}{5}R_3 - \frac{1}{5}R_1, & \quad R_5 - \frac{6}{5}R_3 + \frac{1}{5}R_1, \quad R_3 - R_1, \end{aligned}$$

and we have

$$\left| \begin{array}{cc|c} a & \beta & \\ \beta & a & \frac{6}{5}(\beta+1) \\ 0 & a & \frac{5}{3}(\beta+2) \\ \frac{3}{5}(\beta+2) & a & 4(\beta+3) \\ 0 & a & \frac{3(\beta+4)}{-1} \\ -\frac{1}{3}(\beta+4) & a & \frac{2(\beta+5)}{-3} \\ 0 & a & \frac{\beta+6}{-5} \\ -5(\beta+6) & a & \end{array} \right| = (a^2 - \beta^2)(a^2 - \overline{\beta+2}^2)(a^2 - \overline{\beta+4}^2)(a^2 - \overline{\beta+6}^2) = (a+\beta)(a+\beta+2)(a+\beta+4)(a+\beta+6) = (a-\beta)(a-\beta-2)(a-\beta-4)(a-\beta-6).$$

5. Other and more general types of continuants than  $T_a$  given in art. 2, whose quadratic factors are brought out by the same set of line-multipliers, are the following :

$$T_b \equiv \left| \begin{array}{cc|c} a & \frac{(2n-1)\beta}{2n-1} & \\ \frac{a+(2n-2)\gamma}{3-2n} & b & \frac{(2n-2)(a+\gamma)}{2n-3} \\ \frac{2\{\beta+(2n-3)\delta\}}{5-2n} & a & \frac{(2n-3)(\beta+2\delta)}{2n-5} \\ \dots & & \dots \\ \frac{(2n-2)(\beta+\delta)}{2n-3} & a & \frac{\beta+(2n-2)\delta}{3-2n} \\ \frac{(2n-1)a}{2n-1} & b & 2n \end{array} \right|$$

$$= \{ab - a\beta\}\{ab - (a+2\gamma)(\beta+2\delta)\} \dots \{ab - (a+\overline{2n-2} \cdot \gamma)(\beta+\overline{2n-2} \cdot \delta)\},$$

and

$$\begin{aligned}
 T_a &= \frac{(2n-1)\alpha(\beta+2n-2)}{2n-1} \\
 &\quad \gamma \frac{(\delta+2n-2)}{3-2n} \quad b \quad \frac{(2n-2)\{\delta(\gamma+2n-3)+\gamma\}}{2n-3} \\
 &\quad \frac{2\{\beta(a+2n-3)+a\}}{5-2n} \quad a \quad \frac{(2n-3)\{a(\beta+2n-4)+2\beta\}}{2n-5} \\
 T_b &\equiv \dots \\
 &\quad \frac{(2n-2)\{a(\beta+2n-3)+\beta\}}{2n-3} \quad a \quad \frac{\beta(a+2n-2)}{3-2n} \\
 &\quad \frac{(2n-1)\delta(\gamma+2n-2)}{2n-1} \quad b \quad 2n \\
 &= \{ab - a(\beta+2n-2)\delta(\gamma+2n-2)\} \{ab - (a + \overline{\beta+2n-4+2\beta})(\delta + \overline{\gamma+2n-4+2\gamma})\} \dots \\
 &\quad \{ab - \beta(a+2n-2)\gamma(\delta+2n-2)\}.
 \end{aligned}$$

If in  $T_b$  we change the sign of  $\beta$  and  $\delta$ , which is equivalent to changing the signs of the elements below the principal diagonal, the signs between the terms of the binomial factors would be plus instead of minus.

6. If in  $T_b$  we put :

(1)  $b=a$ ,  $a=\beta$ , and  $\gamma=\delta=1$ , it reduces to  $T_a$ ;

(2)  $\gamma=\delta=0$ , all the factors are alike and we have

$$T_b = (ab - a\beta)^n,$$

or if, in addition,  $b=a$  and  $a=\beta$ ,

$$T_b = (a^2 - \beta^2)^n;$$

(3)  $\gamma=-a$  and  $\delta=-\beta$ , two factors become alike and

$$T_b = (ab - a\beta)^2(ab - 9a\beta) \dots (ab - \overline{2n-3}^2 a\beta);$$

(4)  $\alpha=\gamma=\frac{1}{\beta}=\frac{1}{\delta}$  and  $b=a$ , then

$$T_b = (a^2 - 1^2)(a^2 - 3^2) \dots (a^2 - \overline{2n-1}^2).$$

This, as far as the factors are concerned, is equivalent to putting  $\alpha=\beta=\gamma=\delta=1$ ,

or  $\alpha=\gamma=\frac{1}{k}$  and  $\beta=\delta=k$ , where  $k$  is any number,

or  $\beta=\alpha=2n-1$ ,  $\gamma=\delta=-2$ ,

or  $\beta=\alpha=2n-1$ ,  $\gamma=\delta=-1$ ;

(5)  $\beta = -\alpha = 2n - 1$ ,  $\gamma = -\delta = 2$ , then

$$T_b = (a^2 + 1^2)(a^2 + 3^2) \dots (a^2 + 2n-1^2),$$

which is Elliott's form.\*

(6)  $a = \beta = 0$ ,  $\gamma = \delta = 1$ , and  $b = a$ , then

$$T_b = a^2(a^2 - 2^2)(a^2 - 4^2) \dots (a^2 - 2n-2^2);$$

(7)  $a = \beta = 1$ ,  $\gamma = \delta = \frac{1}{2}$ , and  $b = a$ , then

$$T_b = (a^2 - 1^2)(a^2 - 2^2) \dots (a^2 - n^2);$$

(8)  $a = -\beta = 1$ ,  $\gamma = -\delta = \frac{1}{2}$ , and  $b = a$ , then

$$T_b = (a^2 + 1^2)(a^2 + 2^2) \dots (a^2 + n^2).$$

7. If in  $T_c$  we put :

(1)  $\delta = \alpha$ ,  $\gamma = \beta$ , and  $b = a$ , then

$$T = \{a^2 - a^2(\beta + 2n - 2)^2\} \{a^2 - (a \cdot \beta + 2n - 4 + 2\beta)^2\} \dots \{a^2 - \beta^2(a + 2n - 2)^2\};$$

(2)  $\delta = \gamma = \beta = a$  and  $b = a$ , then

$$T_c = \{a^2 - a^2(a + 2n - 2)^2\}^n;$$

(3)  $\delta = \gamma = \beta = a = b = a$ , then

$$T_c = a^{2n}(2n + a - 1)^n(3 - a - 2n)^n,$$

or if  $a + 2n - 2 = x$ , then

$$T_c = (1^2 - x^2)^n(x - 2n - 2)^n.$$

If  $a = 1$ , then

$$T_c = 2^{2n}\{n(1 - n)\}^n;$$

(4)  $\delta = \gamma = \beta = a = 1$ ,  $b = a = x(2n - 1)$ , then

$$T_c = (2n - 1)^{2n}(x^2 - 1^2)^n.$$

If  $x = 1$ ,  $T_c = 0$ .

8. It will be observed that in (2) of articles 6 and 7 we have a determinant of the  $2n$ th order expressed as the  $n$ th power of a determinant of the second order.

From (4) of article 7 it is seen that the determinant

$$\begin{vmatrix} x & 1 \\ 1 & x & 2n-2 \\ 3-2n & x & 2n-3 \\ & \frac{2}{5-2n} & x & 2n-3 \\ & & \ddots & \\ & & & x & 1 \\ & & & 1 & x & 2n \end{vmatrix} = (x^2 - 1^2)^n.$$

\* *Proceedings of London Mathematical Society*, vol. xxxiii. p. 229.

9. There is another and simpler way of getting the factors of these continuants. For instance, if in  $T_a$  we add to every odd column the sum of all the odd columns which follow it, and add to every even column the sum of all the even columns which follow it, then subtract from every row the second row above it, the determinant breaks up into  $(a^2 - \beta^2)$  and a determinant of order  $(2n - 2)$ , which on interchanging the denominators of conjugate elements is a determinant of exactly the same form with  $n$  one less and  $\beta$  two more. Thus, if  $T_a$  be represented by  $f_{2n}(a, \beta)$ , then

$$f_{2n}(a, \beta) = (a^2 - \beta^2)f_{2n-2}(a, \beta + 2).$$

In precisely a similar manner, if  $T_b$  is represented by  $f_{2n}(ab, a\beta)$ , then

$$f_{2n}(ab, a\beta) = (ab - a\beta)f_{2n-2}(ab, \overline{a+2\gamma} \cdot \overline{\beta+2\delta}),$$

and if  $T_c$  is represented by  $f_{2n}(ab, a\beta, \gamma\delta)$ , then

$$f_{2n}(ab, a\beta, \gamma\delta) = \{ab - a(\beta + 2n - 2)\delta(\gamma + 2n - 2)\}f_{2n-2}(ab, a\beta + 2\beta, \gamma\delta + 2\gamma).$$

10. It is easily seen that if  $T_a$  is represented by  $f_n(a) \cdot F_n(a)$ , then

$$f_n(a) = (a + \overline{2n-1}) \cdot f_{n-1}(-a) = (a - \overline{2n-1})(a - \overline{2n-3}) \cdot f_{n-2}(a)$$

and

$$F_n(a) = (a - \overline{2n-1}) \cdot F_n(-a) = (a - \overline{2n-1})(a + \overline{2n-3}) \cdot F_{n-2}(a),$$

which shows that the signs between the terms of the factors are alternately positive and negative for  $f_n(a)$ , and negative and positive for  $F_n(a)$ .

The operations which show this are the following:—

- (1) Add all the other columns to the first.
- (2) Add to every column after the first the second column following it.
- (3) Subtract from each row the second row above it.
- (4) Subtract the first row from the second.
- (5) Interchange the denominators of conjugate elements in the reduced determinant.

11. If in  $T_b$  we put  $b=a$ ,  $a=\beta$ , and  $\delta=\gamma$ , then it breaks up into factors which we may represent by  $f_n(a, a, \gamma)$  and  $F_n(a, a, \gamma)$ , where  $f_n(a, a, \gamma)$  is the sum of two terms, and  $F_n(a, a, \gamma)$  the difference of the same two terms.

Using the same set of operations, it is seen that

$$f_n(a, a, \gamma) = (a + a)f_{n-1}(a, a + 2\gamma, \gamma)$$

and

$$F_n(a, a, \gamma) = (a - a)F_{n-1}(a, a + 2\gamma, \gamma).$$

That is, the linear factors of  $T_b$  with the positive sign between the terms all belong to  $f_n(a, a, \gamma)$ , and those with the negative sign all belong to  $F_n(a, a, \gamma)$ .

Similarly, if in  $T_c$  we put  $b=a$ ,  $\delta=a$ , and  $\gamma=b$ , and if

$$T_c = f_n(a, a\beta) \cdot F_n(a, a\beta),$$

then

$$f_n(a, a\beta) = \{a + a(\beta + 2n - 2)f_{n-1}(a, a\beta + 2\beta)\}$$

and

$$F_n(a, a\beta) = \{a - a(\beta + 2n - 2)F_{n-1}(a, a\beta + 2\beta)\}.$$

SYRACUSE UNIVERSITY,

*28th April 1914.*

(Issued separately September 3, 1914.)

## XVII.—The Analytical Study of the Mechanism of Writing.

By James Drever, M.A., B.Sc. *Communicated by Dr ALEXANDER MORGAN.*

(MS. received March 16, 1914. Read June 1, 1914.)

IN the new and rapidly developing experimental science known as "Experimentelle Pädagogik" in Germany, "Pédagogie expérimentale" in France, and "Experimental Pedagogy" or "Experimental Education" in this country and in America, two well-marked and not entirely consistent tendencies have been hitherto manifest. On the one hand, there has been a tendency, more particularly in Germany, to develop the work in the new field on the lines of experimental psychology, and to employ almost exclusively the apparatus and methods of that science. On the other hand, there has been a tendency, to a very marked extent in this country and in America, to endeavour to carry on experimental work entirely without the aid of exact and elaborate apparatus, eschewing, even regarding as "tabu," the methods of the psychological laboratory. Both tendencies are perhaps more or less inevitable, and both to a certain extent may be said to have been justified by results. Nevertheless, there are certain obvious dangers and defects inherent in both, and the whole situation is itself dangerous for the new science.

If we commit ourselves too exclusively to the employment of psychological apparatus and the method of the psychological laboratory, there is danger of our experimental education becoming merely a branch of experimental psychology, which might involve in the first place the neglect of certain fields of study, in which such methods and apparatus are quite inapplicable, and, in the second place, a dangerous warping of our attitude, aim, and evaluation, consequent upon our psychological viewpoint and our restricted field. If, again, we endeavour to carry on our experimental work as far as possible without the use of exact and elaborate apparatus, no objection can be made to the thing in itself, but the temptation is strong to avoid such detailed and fine analytical work as demands the use of precise measuring apparatus, and more or less elaborate recording apparatus, which in the long run is almost bound to lead to our science becoming exceedingly unscientific, by our contenting ourselves with experimental investigations of the kind that any teacher can carry out in any schoolroom, and then deluding ourselves with the idea that elaborate and complex statistical treatment of our results will give them scientific

validity. Worst of all is the antagonism between the two groups of workers in the same field, which is all the more dangerous because the one group is mainly composed of psychologists, who know little of the practical work of education, and rather look down upon the practical teacher, and the other group of practical teachers, who have merely a superficial acquaintance with laboratory psychology, and distrust the psychologist.

This condition of unstable equilibrium, if it can be so described, has characterised the early stages in the development of other experimental sciences in the past, notably of experimental psychology itself. The condition will pass, but only when the new science comes to its own in a developed laboratory equipment and a developed technique, which are peculiar to itself and not merely borrowed from another science. It is obvious that experimental pedagogy must always owe a considerable debt to experimental psychology, and also that a great deal of good work may be done with the simplest apparatus. But there are certain fundamental problems of the school, and of life from the school point of view, all analytic problems demanding accurate analytical methods, which must be entirely ignored or only superficially noticed, if we confine ourselves to either or both of these lines of approach. It would seem, therefore, that some of the most interesting, and, if not the most important and practically valuable, at any rate most significant work in the new field is that which undertakes the analytical study, under laboratory conditions and by means of laboratory apparatus, of complex processes characteristic of the work of the school, from the teacher's rather than the psychologist's point of view.

Such complex processes as reading, writing, and reckoning, either as acquired "dexterities" or in the acquiring, may be cited as illustrating the field for analytical investigation offered by the school. To the extent that such processes are fundamental in school work, their investigation should logically occupy a central position in the new experimental science. Considerable progress has already been made, chiefly in Germany and America, in the analytical study of the reading process. The main purpose of the present paper is to indicate how a similar study may be made of the writing process. This purpose will be best achieved by describing some pieces of apparatus which have been devised with a view to the analysis of various elements in the mechanism of writing; for the analysis of the various factors involved in writing is obviously the first step towards its scientific study. The three pieces of apparatus described are all intended to isolate elements in the manual mechanism, and they all yield graphic records which may be examined at our leisure and compared with the actual writing itself.

## I. HAND MOVEMENT APPARATUS.

The chief movements made in writing are those of the forearm, of the hand, and of the fingers. Of these the only movements presenting any difficulty for analysis are those of the fingers, and the finger movements are at the same time the most interesting. The isolation of the finger movements can be obtained by a process of elimination. In the actual writing we have the resultant of all the movements. The hand movement is the resultant of all the movements except those of the fingers. Hence, if we can trace the hand movement, the difference between this and the writing will give us the part played by the finger movement.

Professor Charles H. Judd has devised and described an apparatus for tracing the hand movement during writing (*Genetic Psychology for Teachers*, New York, 1907). Our apparatus is an improved form of this. In Judd's apparatus a broad strip of metal, bent so as to grip the fifth metacarpal bone of the right hand, is bent back a second time on its upper surface, so as to hold a wooden pin, to which a tracing arrangement is attached by a short metal bar with hinges at each end, allowing free movement in the plane of the wooden pin and the writing or tracing style. The tracing style is cylindrical in shape and brought to a rounded point with slits so as to hold ink like a pen point, while it moves freely in a longitudinal direction through a range of about  $1\frac{1}{2}$  inches within a light frame. The point is kept resting on the paper by gravity alone, and the longitudinal play is intended to allow for different inclinations of the back of the hand to the plane of the paper in different individuals and at different points in the writing.

Judd's apparatus is defective in several respects. In the first place, the position of the tracing arrangement is itself very awkward, since its plane is almost parallel to the back of the hand, and in writing it seems to drag along the surface of the paper, sometimes interfering considerably with the movement of the hand, and always distracting the attention of the writer. In the second place, the joints are not sufficiently rigid and the free movement at the joints intensifies the dragging and distracting behaviour of the tracing style, while it also allows the hand to move without the tracing point moving. In the third place—and this is the chief defect—gravity cannot be relied upon to keep the point constantly on the surface of the paper, especially where there are sudden and rapid changes in the inclination to the paper of the back of the hand.

In order to remedy these defects and get an apparatus on which we can rely for a true record of the hand movement, it is necessary to attach the

tracing arrangement differently to the metal strip, to keep the writing point against the surface of the paper by means of a spring, and to prevent such movement of the whole tracing arrangement as will tend to cause it either to fail to respond to any movement of the hand, or to interfere with the attention of the writer or his free hand movement. In the apparatus shown (fig. 1) these objects are secured by attaching a metal pin tangentially to the metal strip where it curves over on to the back of the hand, and fitting the tracing arrangement to a tube which passes over this pin and is movable along the pin, being fastened by a screw in any position that may be necessary for adjustment. All the joints are arranged for



FIG. 1.

adjustment and not for free movement. Finally, by means of a spiral spring the tracing point after adjustment is kept in contact with the paper. The trace itself is given by a capillary glass tracing tube or by a lead pencil, the holder for which occupies the place of the tracing style in Judd's apparatus.

As an indication of the kind of work that may be done with this apparatus, some tracings are shown (fig. 2), but the results hitherto obtained may also be briefly summarised.

1. Normally, in careful adult writing, and more especially in pen writing, the finer movements in the formation of the letters are due to the fingers. As the writing is increased in speed, the hand may take over a larger and larger share of the movement, until with very rapid writing the movements are sometimes nearly all hand movements.

2. The main movement of the hand in writing is alternately a rotation about an axis in the wrist and about an axis in the elbow with careful

writing, but as the writing increases in speed the rotation about the wrist axis tends to disappear.

3. In the writing of children the part played by finger movement is very variable. In general, hand movement predominates even in

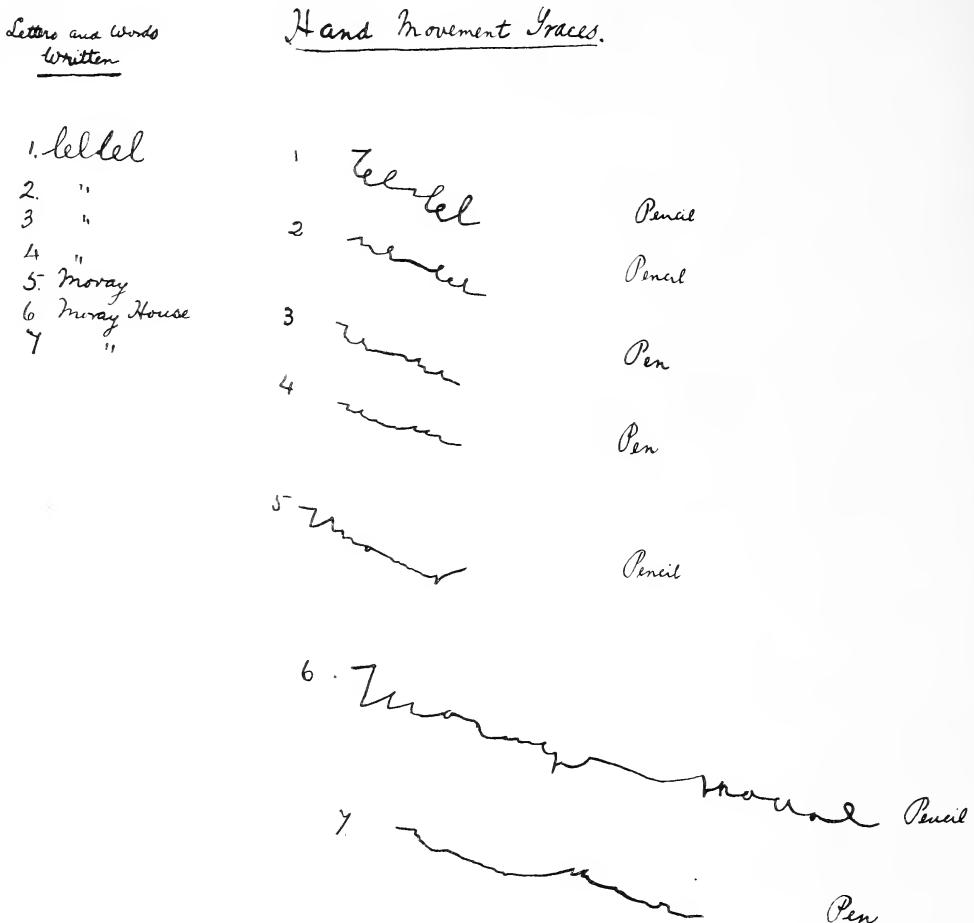


FIG. 2.

the formation of the letters, but this must not be regarded as a universal principle.

## II. GRIP PRESSURE APPARATUS.

So far as the writer knows, no one has hitherto attempted to obtain a record of the pressure of grip in writing. The problem undoubtedly presents considerable difficulties, but is a very interesting one. The apparatus shown (fig. 3), which would therefore appear to be the first attempt to get such a record, has several more or less obvious defects, but

may be regarded as indicating the general lines upon which any such apparatus must be constructed. The essential part of the apparatus is the arrangement for receiving the grip in such a way as to enable us to record its pressure. This is constructed of rubber and is double walled. In its construction two teats are used, a large and a small. These are placed one inside the other, the space between their walls being filled with mercury and sealed. Finally, a narrow glass tube is passed into the inner space, and that too is filled with mercury until the mercury stands about two inches up the tube.

To begin with, a single teat was used, but it was found that, immediately under the fingers, with a moderately firm pressure, all the mercury was expelled, and the rubber sides pressed together. Consequently it was impossible to record the full pressure with this arrangement. This defect is remedied by the double teat arrangement with the sealed space between



FIG. 3.

the walls. The record of grip pressure is obtained in the usual way by connecting the upper end of the glass tube, which projects above the metal holding tube, by means of rubber tubing to a recording tambour.

The most serious defect of this apparatus is its weight, and this is largely due to the use of mercury. It might be possible to replace the mercury with a lighter liquid, if one could be obtained which neither affected the rubber nor evaporated to any great extent from the inner space. It is impossible to use merely an air space between the teats, since this makes the holding part of the apparatus much too soft and introduces thereby a very disturbing factor.

### III. POINT PRESSURE APPARATUS.

The pressure on the writing point itself has already received a considerable amount of attention, and has been made the basis for several interesting discussions, bearing not only on the psychology of writing, but also on the study of defective and feeble-minded children, and of the effects of drugs like alcohol on the motor co-ordinations in writing.

Hitherto the apparatus employed to record what is called *par excellence* writing pressure has in every case recorded the pressure on the writing surface rather than on the writing point. Kraepelin employed what he called a "Schriftwage," which consisted of a plate supported by springs

and mechanically connected to a lever for recording on a smoked surface. The paper was placed on the pressure plate and the pressure in writing on the paper was recorded by the recording lever. Meumann similarly employed a pressure plate, but supported it on an air cushion pneumatically connected with a recording tambour. The chief defect of any such arrangement, apart from the complications introduced into the writing process itself, is that variations in pressure on the writing plate may be due to variations which have no necessary connection with the writing itself, but are the result of more or less accidental changes in the position of the hand or wrist relatively to the plate.

The original form of the apparatus shown (fig. 4), which has now been modified in some minor respects, was first described by the writer in the *Journal of Experimental Pedagogy*, March 5, 1913. The essential feature of the apparatus is that it records the pressure upon the writing point

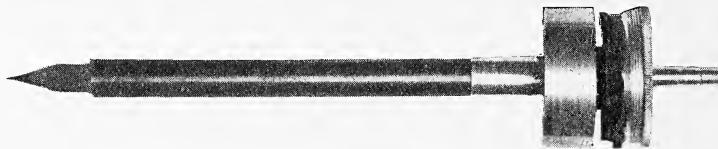


FIG. 4.

itself by receiving the pressure of the top end of the writing instrument on a receiving tambour. To this a holding tube is attached into which either pencil or pen is slipped. By means of a guiding tube, which serves as a holder, the pressure is kept normal to the surface of the tambour. In order to lessen friction, as well as to prevent side movements of the pen or pencil, a ring—or sometimes two—is placed inside the guiding tube, and this just allows the pen or pencil to move freely up and down. By connecting the receiving tambour with a recording tambour we get the record of point pressure. It is not yet certain whether a light spiral spring inside the receiving tambour, in such position that the writing instrument presses against it, is an advantage or not.

For the two pressure recording instruments the names "Grip Pressure Cheirograph" and "Point Pressure Cheirograph" might be suggested. Both of them might be found serviceable, not merely in the study of writing pressure for the purposes of the science of experimental pedagogy, but in the science and practice of medicine for the diagnosis of defects in motor co-ordination; as we have indicated, writing pressure, that is point pressure, has already been studied to some extent from this point of view. The study of grip pressure might also be expected to throw some light on writers' cramp.

## IV. RESULTS OF STUDY OF WRITING PRESSURE.

Traces obtained with the two pieces of pressure apparatus are shown (fig. 5). At the same time it might be well to indicate some of the more important results obtained in the investigation of writing pressure.

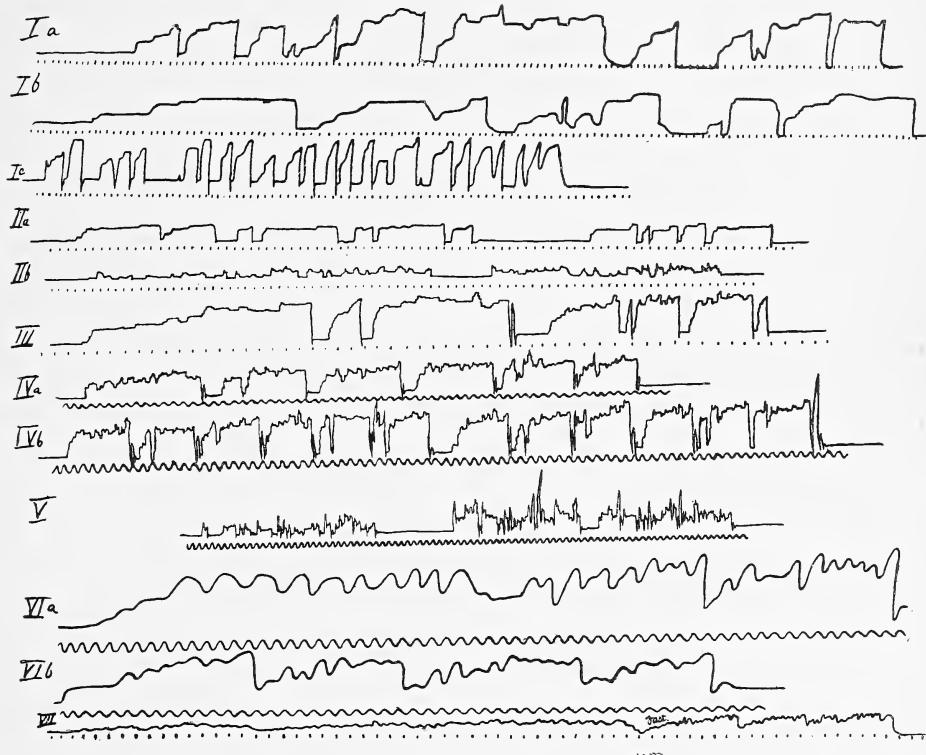


FIG. 5.

- I. to III. Point pressure tracings from children. Time record in seconds by Jacquet Chronograph.
- Ia. Child of six. Words "The cow gives us milk."
- Ib. Child of six (first attempt at script). Words "A man can."
- Ic. Child of six (printing). Words "A man can run."
- IIa. Pencil writing, and IIb. Pen writing of child of eight. Words "Moray House School," written twice in each case.
- III. Child of eleven. Pencil writing. Words "Moray House School," written twice.
- IV. and V. Point pressure tracings from adults. Time record in  $\frac{1}{2}$  secs. by vibrating spring.
- IVa. Pencil writing, ordinary rate. Words "Moray House School," written twice.
- IVb. Pencil writing by same subject, maximum rate. Words "Moray House School," written four times.
- V. Pen writing, slow and fast. Words "Moray House School," written once slow and twice fast.
- VI. and VII. Grip pressure tracings from adults and child of eleven. Time records for adults in  $\frac{1}{2}$  secs. and for child in secs.
- VIA. Adult pencil writing. Words "Moray House School."
- VIB. Adult pen writing. Words "Moray House School Moray."
- VII. Child's pencil writing. Words "Moray House School" twice, slow and fast.

The most interesting results are probably those indicative of the differences between adult writing and child writing. The grip pressure of the adult nearly always shows a rhythmical rise and fall of pressure, which is almost as regular in the tracing as the vibrations in the tracing of a

tuning fork, although with increased speed of writing the amplitude of the pressure changes sensibly diminishes. The rhythm is also shown in children's writing from about the age of ten, but the irregularities are very marked. It is difficult to get any reliable results with our grip pressure apparatus at an earlier age. Analogous phenomena appear in the case of point pressure. The point pressure trace of adult writing shows a characteristic "rippled" top on each wave of pressure, indicating more or less rhythmical increase and diminution of pressure. In the child's writing this characteristic is entirely absent before the age of about eleven, and we have for our pressure trace either a more or less continuous line, or a line that is simply "crooked," without any regularity in its crookedness. These phenomena are probably in the main phenomena of co-ordination, but they also have a psychological interest, as we shall see presently.

A second characteristic difference between adult and child writing may be regarded as due partly also to co-ordination phenomena and partly to psychical phenomena. It is well known that the practised reader does not recognise the several letters of a word individually, nor does he speak them individually, in reading a word, but reads the word, as it were, with a single total impulse, either of recognition or of speech. Similarly, the adult writer writes a word, not with a separate impulse for each letter, but with a single impulse for the whole word. In learning to write, however, the child learns first to draw the shapes of the letters, and there is a separate impulse for each stroke or letter or group of letters, according to the drawing unit with which the child is dealing, and this gradually changes from the stroke to the letter, from the letter to the group of letters, from the group of letters to the whole word, as the child progresses. These differences are well indicated in the point pressure traces. The adult trace shows at once that each word is written as a whole. The child, learning to write, shows equally unmistakably the units for which there is a single drawing impulse.

Even when the child is taught from the beginning to write continuously, the traces can still be easily distinguished by the lack of rhythmical pressure variations in the child's trace. This difference, although in the main a phenomenon of co-ordination, is at the same time indicative of the nature of the psychical impulse that guides the writing. It is due in part to the fact that writing is a form of language, while drawing is not. The rhythm is present because it is the word that is before consciousness, not the shape or figure to be drawn.

Moreover, the language unit is the sentence, not the word, and this is generally clearly shown both in speech and in reading. Are there any

traces of this in writing? There are to some extent. In adult writing we can often mark off the phrases, at any rate, by the subordination of the individual pressures for each word to a single maximum pressure on some part, usually the end, of the phrase. While the child is drawing and not writing we naturally look in vain for any such characteristic. It must be noted, however, that the writing of clerks, whom we may consider professional writers, tends to lose this last language mark and tends to show an approximately uniform pressure.

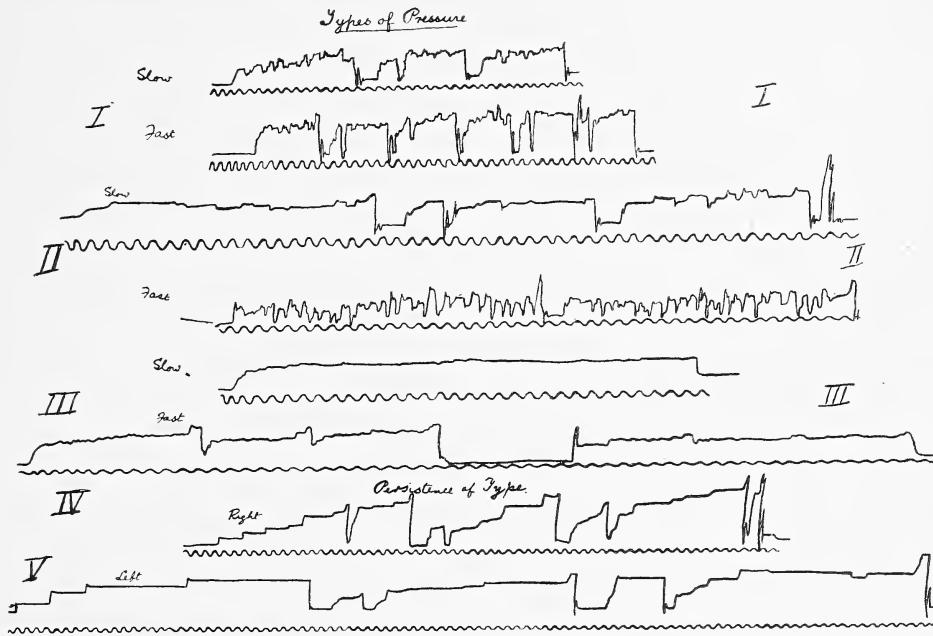


FIG. 6.

I. "Masculine" type.

II. "Feminine" type.

III. "Mechanical" or "Clerical" type.

IV. Right-hand writing, and

V. Left-hand writing without practice by  
same subject.

Previous workers have sought to distinguish, on the basis of point pressure traces, different types of writing and writers (fig. 6). As an introduction to any such attempt, it must be stated emphatically that the point pressure trace is as characteristic of an individual as his hand of writing or his signature, and even in left-handed writing, without practice with the left hand, individual characteristics reveal themselves in the pressure trace. Nevertheless, it does seem as if there were distinct types of writer. Two adult types have been generally distinguished by previous investigators. The one tends to show a single maximum of pressure for each word or phrase written as a whole, and tends to increase writing pressure with increased speed of writing; the other tends to show several

maxima of pressure in the word or phrase, or to write with an approximately uniform pressure, as in the case of clerk's writing already cited, and also tends to show decrease of pressure with increased speed of writing. The first has been called the "masculine" type, the second the "feminine." It seems desirable, however, to distinguish the two varieties appearing under the second type, and to recognise three types. The "clerk" type is quite as marked as either of the others, and is quite as distinct from the "feminine" as the "feminine" from the "masculine." Another characteristic mark of this third or "clerk" type is that the writing speed is normally very near the maximum. When such writers are asked to increase their speed, they may do so to a slight extent, but often all that happens is a breaking down of the uniformity of pressure ordinarily shown without any significant increase of speed, and sometimes the speed actually decreases as an accompaniment of this breakdown of pressure uniformity, while the subject thinks he is writing faster than before. A better name for this "clerk" type, and more descriptive of its chief characteristics, might be "mechanical" type.

There seems little reason to doubt that a considerable development of our knowledge of the writing process will take place along the lines of investigation indicated in this paper. We might even look forward to the founding of a real science of graphology. At all events, many points of interest to the teacher, and some of interest to the nerve and brain specialist, the alienist, or the general physician must be revealed.

(Issued separately September 3, 1914.)

XVIII.—Abnormal Echinoids in the Collection of the Royal Scottish Museum. By James Ritchie, M.A., D.Sc., Royal Scottish Museum; and James A. Todd, M.A., B.Sc. Communicated by WILLIAM EAGLE CLARKE. (With a Plate.)

(MS. received May 16, 1914. Read June 1, 1914.)

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I.

“ECHINI are a particularly good group in which to study questions of variation, because here variations can usually be expressed in very definite terms of numerical or other equally positive character.”\* On this account, and because, in spite of much description, the variants liable to occur in sea urchins have not yet been exhausted, the three specimens described below are recorded. Each of these exhibits a pronounced abnormality in the major symmetries. Two of them resemble another abnormal echinoid in the same collections, already discussed in *Proc. Zool. Soc.*,† in lacking part of a definite ambulacrum; but the means by which the tests have accommodated themselves to changed conditions of growth differ markedly in each of the three cases. The third specimen exhibits, in place of the normal five-radiate arrangement, almost perfect hexamery—a type of abnormality very different from that of the first two specimens. For in these the distortion is due to incomplete development caused by interference with the processes of growth, while there the hexamery is a fundamental change in symmetry, is congenital in origin, and probably represents the type of variation known as duplication of parts.

\* R. T. Jackson, “Phylogeny of the Echini, with a Revision of Palaeozoic Species,” in *Mem. Boston Soc. Nat. Hist.*, vol. vii., 1912, p. 51.

† Ritchie and M’Intosh, *Proc. Zool. Soc.*, 1908, p. 646.

Before proceeding to the description of these specimens, we would draw attention to a few facts of more general interest.

I. *Regulation*.—It is clear that where so large an area as an ambulacrum or inter-ambulacrum ceases to grow, after temporary development, much rearrangement in plates is necessary in order that the potential gulf in the test should be spanned. Two distinct modes of regulation occur. In the first place, the remaining plates in the neighbourhood of the abnormality may themselves wholly compensate for the deficiency by abnormal development in length and breadth. Such is the case in the *Echinus esculentus* already referred to, where, an ambulacrum only having dropped out, the two inter-ambulacral series of areas 4 and 5 have, by orderly increase, shared in filling in the space;\* or in Philippi's case of *Echinus melo*,† where the place of an ambulacrum with its associated inter-ambulacral series (*i.e.* a ray) is taken by a single inter-ambulacral series from each of areas 1 and 5.

Or, in the second place, orderly growth of normal series of plates may be replaced or supplemented by the addition of abnormal plates varying much in shape and size. These are sometimes arranged with an approach to bilateral symmetry, as in inter-ambulacral areas 2 and 3 of the *Ambylopneustes* described by Hawkins,‡ and as in the *Ambylopneustes* described below (text-fig. 1), or they may form an irregular medley, as in the specimen of *Echinus esculentus* here described (Plate, fig. 1).

It seems to be a general rule, however, that neither of these modes of regulation altogether compensates for the primary disturbance, for in every case growth seems to have been retarded, and the abnormal area is indicated by a depression in the test and occasionally by a marked distortion of the apical area from its normal position.

II. *Duplication of Parts*.—Various stages of the phenomenon of duplication have been described in Echinoids (see p. 250). These have represented duplicity only in partial degree, but it is possible that the hexamerous *Echinus* described below exhibits almost perfect duplication of both ambulacral and inter-ambulacral areas, and so completes the series of duplication stages.

III. *Ocular Plates and Coronal Growth*.—It is generally held that the growth of new plates in ambulacral and inter-ambulacral series proceeds from the oculars. The case is stated strongly by Lambert.§

\* *Proc. Zool. Soc.*, 1908, p. 646.

† *Archiv f. Naturg.*, iii. p. 241, pl.

‡ *Proc. Zool. Soc.*, 1909, text-fig. 227.

§ Lambert, "Note sur un cas de monstrosité de l'apex chez l'*Echinocorys vulgaris*," *Bull. Soc. Yonne*, 1890, xliv.

"Les centres vitaux de l'apex sont dans les ocellaires et non dans les génitales, et c'est seulement à l'abri et au contact des ocellaires que se forment les nouvelles anules ambulacrariaires ou que naissent les anules interradiales." Again Jackson, in his monumental and masterly monograph,\* says: "The ocular plates seem to exert a controlling influence in the building up of the corona, as below and in immediate contact with the oculars originate the coronal plates both ambulacral and inter-ambulacral" (p. 35). And again: "If this is true, then the loss of an ocular would cause a failure to develop of the plates that normally went with it; also an abnormal position of an ocular should cause an abnormal distribution of the associated coronal plates" (p. 36).

The present abnormal specimens offer two comments on these statements. In the hexamerous *Echinus esculentus*, the ocular of the posterior ambulacrum (say VI.) is wholly subtended by genital 5, which extends some distance on both sides of it (Plate, fig. 2, and text-fig. 3). Yet this derangement of the ocular as regards its relations with the genital plates has not affected the growth of the coronal plates, which spring in normal manner from the sides of the ocular. On the other hand, in each of the deficient *Echinus esculentus* and *Amblypneustes* one ocular plate is awanting, and nevertheless coronal plates have still continued to be formed all along the exposed margins of the genitals (text-figs. 1 and 2). These plates are very irregular in shape and do not belong to the normal coronal series, but are sufficient to show that the growth areas are not associated in any essential way with the ocular plates.

**IV. Preservation of Specimens.**—It may be worth drawing attention to the fact that the specimens, though dried, had not been "cleaned." In two cases, therefore, the sex was able to be distinguished from the shrivelled reproductive organs, and in one case examination of the cruder internal structures was made.

## II. EXAMPLES OF INCOMPLETE DEVELOPMENT.

### (i) *Amblypneustes ovum* (Lamarck).

The specimen is a male example of *Amblypneustes ovum* collected at the National Park, Wilson's Promontory, Victoria, N.S.W., in October 1910, and presented to the Museum among a number of others by Lord Carmichael of Skirling.

The test, when examined, was dry and denuded of spines. Its maximum

\* Jackson, R. T., "The Phylogeny of the Echini, with a Revision of the Palaeozoic Species," *Mem. Boston Soc. Nat. Hist.*, vol. vii., 1912.

horizontal diameter was 3·7 cm., its height 3·2 cm. It possesses five ambulacral and five inter-ambulacral areas, but ambulacrum IV. (Lovén's notation) has its apical extremity separated from the apical system by about 1 cm., and possesses no corresponding ocular plate. The space intervening between the apex of the incomplete ambulacrum and the apical system is filled in by an aggregation of modified inter-ambulacral plates, more or less irregularly arranged, but with each individual plate approximately symmetrical about its longitudinal diameter. Retardation in the growth

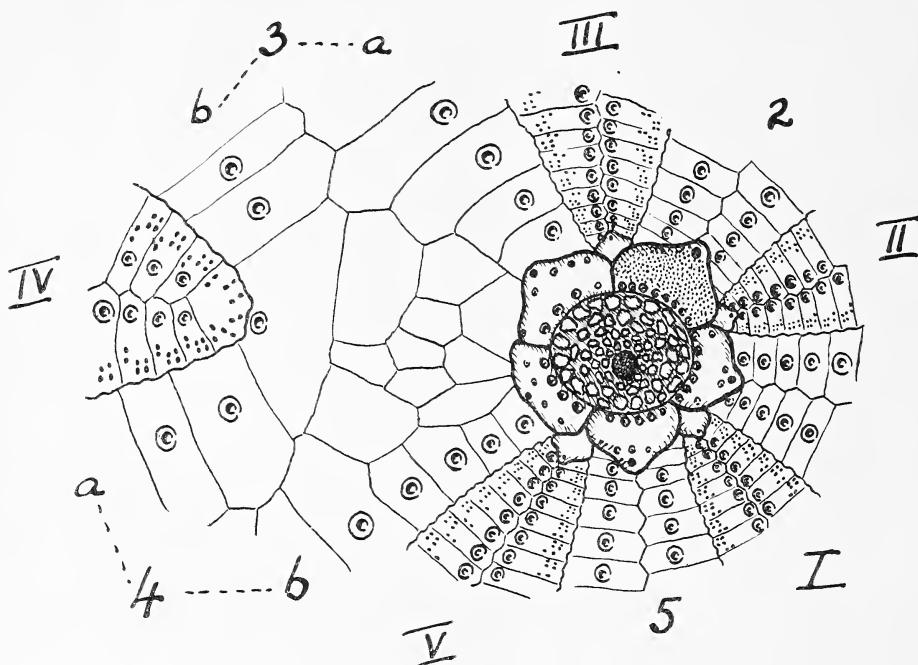


FIG. 1.—Incomplete Development in *Amblypneustes ovum*. Apical area and surrounding coronal plates  $\times 4\frac{1}{2}$ .

of this area seems to have had the effect of preventing the apical system from attaining its customary polar position, so that, though still centrally placed (*i.e.* immediately above the mouth), it is overtopped in height by the upper parts of radial areas I. and II. The apical system, but for the absence of the ocular plate corresponding to the incomplete ambulacrum, presents the normal pentagonal symmetry. A minor irregularity appears in inter-ambulacral area V., where, about 1 cm. from the edge of the peristome, there is a small papillary excrescence about 4 mm. in diameter. This is due to modification in three of the inter-ambulacral plates, two of which are occluded, and the intercalation of three additional small demi-plates which give rise to the protuberance.

A general type of regulation occurs in the *Echinus* described by Ritchie and M'Intosh (*loc. cit.*), where a simple increase in the length of the interambulacral plates compensated for and bridged the potential gulf left by the disappearance of the ambulacrum. But here the mode of regulation is essentially different. It would seem that before the ambulacrum ceased to be formed some disturbance occurred in the growth area, for a couple of exceedingly large interambulacral plates in proper series have been formed. Subsequently to the assumed damage the ocular plate was cast off or absorbed, and then a large growth area at the external margins of the two genital plates became continuous and gave rise to an enormous median, roughly triangular plate which succeeds the detached end of the ambulacrum and terminates two half-rows of interambulacral plates. The remaining space between this and the apical system is filled in, not by regular interambulacral plates, but by a group of irregular casual plates, the group being roughly symmetrical about a median longitudinal axis.

This type of regulation is a stage between the complete unharmed interambulacral areas (exhibited in the *Echinus esculentus* described by Ritchie and M'Intosh, or in areas 4 and 5 of the *Amblypneustes* recorded by Hawkins \*), and the complete disappearance of a total ray, as occurs in the specimens recorded by Bell † and Philippi.‡

*Soft Parts.*—So far as could be distinguished, the badly preserved genitalia presented the normal five-partite arrangement and contained male elements.

(ii) *Echinus esculentus*, Linn.

The specimen was obtained by Mr F. G. Pearcey in the Cromarty Firth at a depth between  $8\frac{1}{2}$  to  $16\frac{1}{2}$  fathoms. It contained shrivelled female reproductive organs. Even in the dry condition in which it was preserved, when still covered with spines, it showed marked irregularity of outline. This in plan was trapezium-shaped. There was a distinct flattening of the test in the part which lay between the vertex and the long side of the trapezium, and the apical system was so distorted that it lay on this flattened surface, only one edge reaching up to the summit of the test. The maximum horizontal diameter of the test was 5 cm., its height 3 cm. There were the usual five teeth in Aristotle's lantern.

The spines having been removed, there was revealed the type of abnormality shown by the specimen of *Amblypneustes* described above, but in a more extreme degree; for here ambulacrum V. had almost dis-

\* Hawkins, *Proc. Zool. Soc.*, 1909, part ii. p. 714, figs. 226-230.

† Bell, *Jour. Linn. Soc.*, vol. xv., 1881, p. 126, pl. v.

‡ Philippi, see Bateson, *Materials for the Study of Variation*, London, 1894, p. 443, fig. 137.

appeared (Plate, fig. 1). The peristome and mouth parts were normal, but in the apical system abnormality was strikingly apparent. The genital plates of areas 1, 2, and 3 (the last split into two segments) were present approximately in their normal positions, and associated with them, in the customary arrangement, were the oculars I., II., III., and IV. It is certainly remarkable that in spite of the spreading out of the plates surrounding the periproct not one of the oculars was "insert." The remainder of the periphery of the periproct was bounded by a single rank

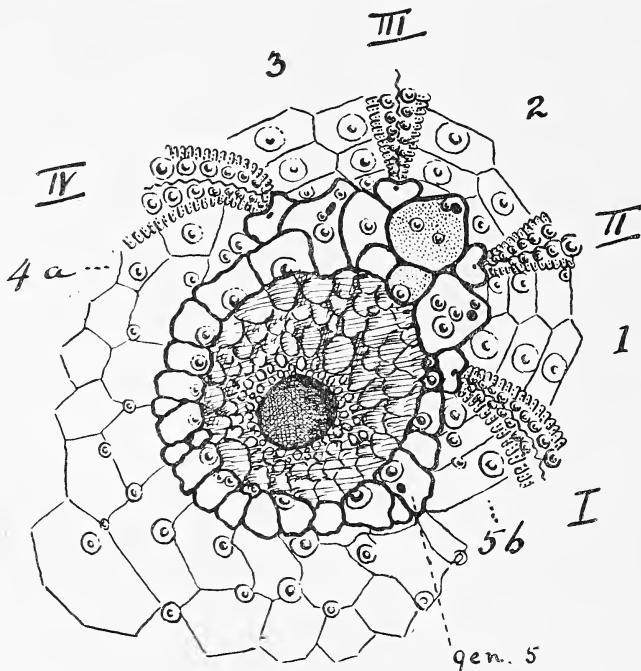


FIG. 2.—Incomplete Development in *Echinus esculentus*. Apical area and surrounding coronal plates  $\times 3$ ; gen. 5, supposed 5th genital plate.

of small plates, each more or less rectangular in outline, with several smaller triangular plates thrust in to complete the almost circular outline of the series. This abnormal series contained in all nineteen plates and demi-plates, with two additional small circular plates between ocular I. and the margin of the periproct. The plates were similar in size and shape, and the majority bore primary tubercles, but one, the third from ocular I., was perforated by a distinct pore and possibly represents genital 5, although in the dried state no genital products were associated with it. The madreporite was divided into two lobes, both perforate.

Outside the ambulacral area the derangement and consequent regulation was exceedingly extensive. Apart from the one disturbed area, the

"morphological units" have remained intact, so that the left row of inter-ambulacrum 4 and the right row of inter-ambulacrum 5 were practically normal and were associated normally with the ambulacral areas in their respective "arms." On the other hand, the inter-ambulacral rows of arm V. bent away from their ambulacral area, the intervening space thus created being filled in by irregular small plates. But they ceased to exist at about the level of the truncated ambulacrum. Still a large area, measuring 12·5 mm. in height from the termination of the truncated ambulacrum to the apical chain of plates, and 28 mm. from side to side, remained to be accounted for. This was filled in by a series of irregular and generally small plates arranged in lines roughly parallel to the circular periphery of the periproct. Considerable flattening has taken place in this growth area, which has thus been covered with the minimum of material.

Of ambulacrum V. only about 1 cm. remained. The two pore rows, instead of approaching each other and meeting aborally to form a closed area, were parallel throughout, the open aboral end being filled in by small plates. The individual rows of ambulacral plates in the disturbed area have suffered in different ways. The row to the left was perfect so far as it went as regards pore-pairs, but the right-hand row was shorter and for the latter half of its length was destitute of pores, except for three scattered and imperfect pore-pairs.

Aristotle's lantern was removed, and showed the normal skeletal arrangements.

*Soft Parts.*—The dried remains of the soft parts were unsatisfactory. Three genital glands or portions of them were present, corresponding to the genital pores in areas 1, 2, and 3, but there was no trace of gland in connection with the abnormally situated pore supposed to be genital 5. The intestine was much broken, but it was noted that an upward loop in the middle of the abnormal area actually crossed the lower portion of the apical area, and that the intestine doubled on itself towards the anus in area 4, instead of, as usually occurs, in area 3.

### III. TOTAL VARIATION FROM FIVE TO SIX-RAYED FORM.

#### *Echinus esculentus*, Linn.

Only one case of this type has been observed in the collections—an example of *Echinus esculentus*, Linn., obtained in the Cromarty Firth by Mr F. G. Pearcey, at a depth between 8½ and 16½ fathoms. Examination of the shrivelled reproductive organs proves it to have been female.

The specimen was preserved in a dry condition and, even clothed with

spines, it presented an unusually depressed appearance, the maximum diameter being 10 cm., the height 5 cm. Yet the hexradiate symmetry was so little apparent that it had escaped notice.

On the test denuded of spines the presence of six ambulacra and six inter-ambulacra was very marked, depression in the inter-ambulacral areas giving the ambitus an approach to regular hexagonal shape. A peculiar feature is common to all the inter-ambulacral areas. These, instead of narrowing gradually towards their subtending genital plates, widen out about a centimetre from the summit, so that the recently formed plates are as large as, or even larger than, some of their predecessors. On

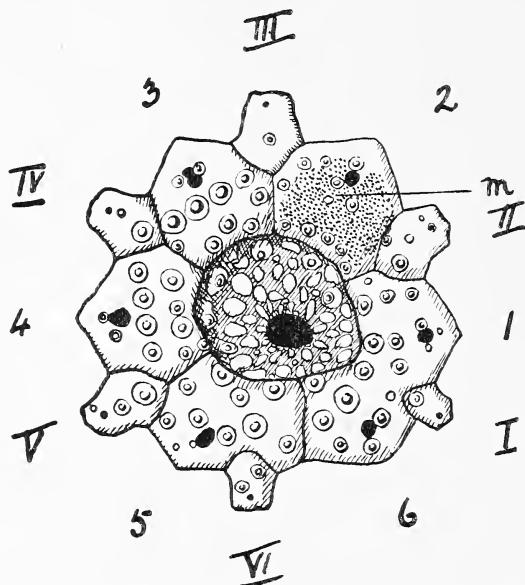


FIG. 3.—Hexamery in *Echinus esculentus*. Apical area  $\times \frac{2}{3}$ ;  $m$ , madreporite.

the aboral surface hexamery is apparent in the peristome. Six teeth are present, the lantern itself is six-partite, the elements of the parts being normal, while the buccal tube feet are arranged in six pairs.

The apical system, while presenting a hexradiate appearance, is less regular. The genital plates are five in number. Those in areas 2, 3, and 4 (according to Lovén's system, and assuming that the madreporite retains its normal position in area 2) are normal in shape and position. Genital plate 5 has a greater lateral diameter than usual; opposite area 5 it is normal in shape, but it is continued for a short distance opposite area 6. The remainder of the apical system is filled in by a double plate which subtends both of the areas 1 and 6. This is practically equivalent to two normal plates laterally adnate. The genital pores are normal in position, there

being two individuals on the double plate. The pore on genital plate 3 is doubled, the apertures being incompletely separated.

There are six oculars. Those opposite ambulacral areas II., III., IV., and V. are normal in shape and position. Ocular VI. lies in a notch in genital 5, by which plate alone it is subtended; and ocular I. lies in a similar notch opposite the middle of the double genital plate. Each of these is about half the usual size.

The anus lies nearest to the right-hand corner of genital 5.

The removal of Aristotle's lantern disclosed the fact that, in the soft parts also, hexamery prevailed, for the reproductive organs were developed in six equal inter-radial rays. The alimentary canal, although longer than usual, followed the normal course even in detail, for the intestine doubled upon itself in inter-ambulacral area 2, and its last point of attachment was in inter-ambulacral area 3, as in normal specimens.

Clear cases of spontaneous variation in the major symmetries of sea urchins are very rare. Of the most likely of such cases—those in which complete rays are added—few have been recorded, and still fewer have been satisfactorily described. As Bateson in his *Materials for the Study of Variation*, 1894, mentions only two cases of "total variation to a six-rayed form," and as Jackson's list (*op. cit.*, 1912, p. 46) omits several recorded cases, we give the following short summary of all the examples of "total" hexamery we have been able to discover, with the view of simplifying future researches:—

#### RECORDED CASES OF TOTAL HEXAMERY.

##### *Regularia.*

(1) *Amblypneustes* sp.: hexamerous specimen (no further description). Haacke, *Zool. Anz.*, 1885, p. 506.

(2) *Echinus esculentus*: 6 teeth, pairs of buccal tube feet, ambulacral and inter-ambulacral zones, ocular and genital pores, ocular and genital plates, two of the latter adherent—described in present paper.

(3) *Tripneustes esculentus*: 6 teeth, ambulacra and inter-ambulacra, oculars and genitals, two of the latter adherent. Jackson, *Mem. Boston Soc. Nat. Hist.*, vol. vii., 1912, p. 46.

(4) *Toxopneustes lividus*: 6 teeth, pairs buccal tube feet, ambulacra, genitals, and oculars. Ribaucourt, *Comptes rendus Ac. Sci.*, vol. cxlvii., 1908, p. 92.

(5) *Paracentrotus (Strongylocentrotus) lividus* (artificially reared): 6 teeth, terminal tentacles, and ambulacra. Delage, *Comptes rendus Ac. Sci.*, vol. cxlv., 1907, p. 546.

(6) *Strongylocentrotus dröbachiensis*: 6 teeth, ambulacra, inter-ambulacra, oculars, and genitals, two of the latter adherent. Jackson, *op. cit.*, p. 47.

(7) *Strongylocentrotus dröbachiensis* as No. 6.

(8) Species unnamed—hexamerous specimen (no further description), seen by Ribaucourt and mentioned by him, *loc. cit.*

#### *Irregularia.*

(9) *Galerites albogalerus*: 6 ambulacra and inter-ambulacra. Meyer, *Nov. Acta Ac. L. C. Nat. Cur.*, vol. xviii., 1836, p. 294, pl. xiii., figs. 6 and 7.

(10) *Galerites* sp., six-rayed. Laur., *S. B. Ges. Isis*, 1894, p. 6.

(11) *Pyrina ovulum*: a sixth ray added at posterior, displacing periproct, 6 oculars, 5 genitals. Seguin, *Feuill. Nat.*, sér. 4, No. 376, 1902, p. 81, figs. 1 and 2.

The descriptions of many of these specimens are insufficient to indicate whether or not the hexamerous arrangement prevailed in all the organs. As to the *Pyrina ovulum* recorded by Seguin, that author suggests that the addition of a fifth genital plate and corresponding ocular indicates a reversion to an earlier and more typical symmetry than that of *Pyrina*. Of those regular sea urchins which have been described with any attempt at detail, perfect hexamery appears to have prevailed in cases 4 and 5 of the above list, all the organs having been, so far as one can judge, perfectly formed. The origin of such cases cannot be further particularised than as due to spontaneous meristic variation.

In the case of the *Echinus esculentus* here described the apical area gives a clue to the situation of the abnormality. Oculars II., III., IV., and V. are similar in size, whereas the remaining two oculars resemble each other in being each about half the size of a normal individual. This fact, and the irregularity of the neighbouring genital plates, indicate that the additional areas were added in the right posterior segment. Further, the union between genitals 1 and 6 suggests that the latter is an imperfect double of the former, the imperfection of the duplication making necessary the abnormal compensatory extension of genital 5. We suggest, then, but with reserve, that this *Echinus* may represent a case of almost perfect reduplication of radii, inter-ambulacral area 1 and its flanking series of ambulacra being repeated in area 6 with its associated ambulacral series. If such be so, the present specimen completes the series of already known cases of partial reduplication of radii (see Bateson, *Materials for Study of Variation*,

p. 446). Stewart has described an example of *Amblypneustes griseus*, in which an ambulacrum only was imperfectly duplicated; Cotteau a specimen of *Hemaster batnensis*, in which an ambulacrum only was completely duplicated; Gautier an *Hemaster latigrunda*, in which an ambulacrum was completely duplicated and an inter-ambulacrum imperfectly; and in the present *Echinus esculentus* both ambulacral and inter-ambulacral series show perfect duplication.

Along with our *Echinus* must be reckoned the specimens of *Tripneustes esculentus* and *Strongylocentrotus dröbachiensis* (two cases) described by Jackson; for in all of these, two of the six genital plates are adherent, apparently indicating again the residue of the almost perfect duplication of a ray. Regarding this curious phenomenon of two fused genitals with an ocular between them which he found in the only cases (three in number) of complete hexamery discovered amongst 50,000 sea urchins, Jackson says: "It is certainly most extraordinary that this parallel structure should exist in three specimens, and indicates what I have elsewhere pointed out, how very definite extremely rare variation may be." It adds to the wonder, and to the evidence of definiteness of particular variations, that the specimen above described, belonging to still a different genus from Jackson's, should repeat for a fourth time in hexamerous Echini this curious abnormality.

The evidence as to means of growth-compensation controverts the findings of Jackson, who found that "in the six-rayed specimen [*Tripneustes* but also in his other specimens] evidently the space gained to add the extra ambulacrum and inter-ambulacrum is attained by building ambulacra of practically the usual width, but narrowing all the inter-ambulacra equally to much less than the usual width. This emphasises the conclusion gathered from normal Echini that the inter-ambulacrum is essentially a space-filler and adapts itself to fill what space is available between the ambulacra which are the most essential structures." In view of these statements the six-rayed *Echinus* was compared, as to relative proportions of ambulacra and inter-ambulacra, with a normal specimen of, as nearly as possible, the same size, with these results:—

	Circumference at Ambitus.	Average Width of Inter-ambulacral Areas.	Average Width of Ambulacral Areas.
Normal <i>Echinus</i> . . .	318 mm.	41 mm.	23 mm.
Six-rayed <i>Echinus</i> . . .	314 mm.	33 mm.	19 mm.

Clearly the ambulacral areas have suffered reduction here as well as the inter-ambulacral, and the extent of reduction in the two types of area is roughly proportional, the diameter relationship between the ambulacral and inter-ambulacral areas of the normal specimen being as 1 to 1.783, and of the six-rayed specimen as 1 to 1.737. The proportionally greater loss in diameter of the inter-ambulaera is obviously too small to support Jackson's statement.

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#### EXPLANATION OF PLATE.

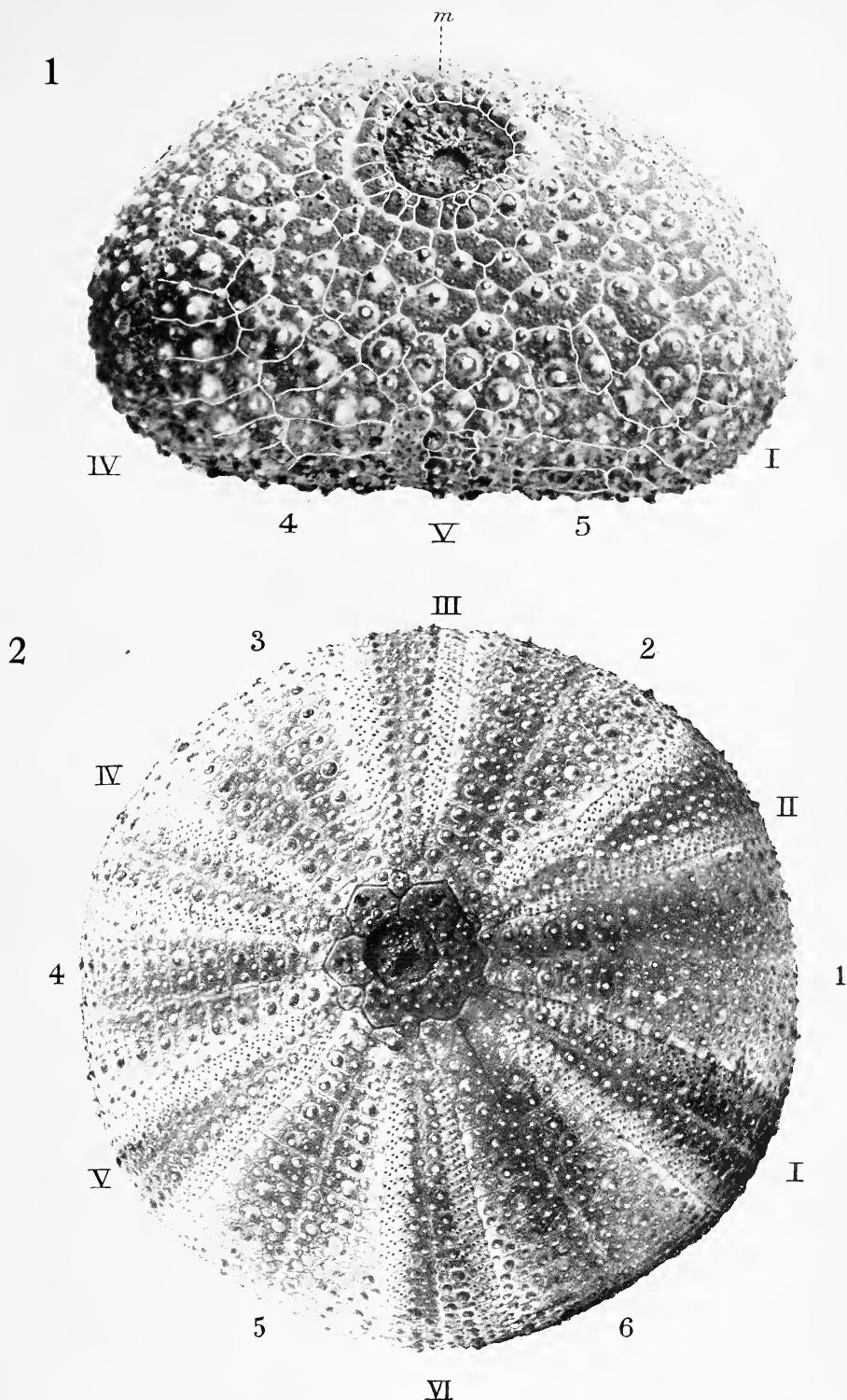
Roman and Arabic numbers mark the ambulacral and inter-ambulacral areas according to Lovén's notation.

Fig. 1. Incomplete development in *Echinus esculentus*. Moray Firth specimen viewed from flattened, abnormal aspect, showing the termination of imperfect ambulacrum V., the very numerous and irregular regulation plates or "space-filers" in the corona, and the asymmetrical position and abnormal plates of the apical area.  $\times 2$ .  $m$  = madreporite.

Fig. 2. Hexamery in *Echinus esculentus*. Apical aspect of Moray Firth specimen. Natural size.

(Issued separately September 4, 1914.)

RITCHIE AND TODD—ABNORMAL ECHINOIDS.





XIX.—Description of a Projection-Model of the 600-Cell in Space  
of Four Dimensions. By D. M. Y. Sommerville, M.A., D.Sc.,  
Lecturer in Mathematics, University of St Andrews. (With a Plate.)

(Read May 4, 1914. MS. received June 1, 1914.)

§ 1. In 1880 Stringham<sup>1</sup> proved that in space of four dimensions there exist six and no more regular rectilinear figures, whose boundaries are regular polyhedra. The same result was arrived at independently by Hoppe<sup>2</sup> in the following year. In 1883 Schlegel<sup>3</sup> gave an extensive investigation of the same problem, and constructed projection-models of the six regular figures, which were exhibited at the Magdeburg meeting of the Society of German Naturalists in 1884. This series of models was published by the firm L. Brill of Darmstadt and is obtainable from their successors, Martin Schilling in Leipzig.

The models are constructed of brass wire and silk threads, and represent projections of the figures in ordinary space in such a way that there is no overlapping of boundaries. In each case the external boundary of the projection represents one of the solid boundaries of the figure. Thus the 600-cell, which is the figure bounded by 600 congruent regular tetrahedra, is represented by a tetrahedron divided into 599 other tetrahedra; 20 tetrahedra meet at every vertex and 5 at every edge; 12 edges meet at each point; the total number of vertices is 120. At the centre of the model there is a tetrahedron, and surrounding this are successive zones of tetrahedra. The boundaries of these zones are more or less complicated polyhedral forms, cardboard models of which, constructed after Schlegel's drawings, are also to be obtained from the same firm.

§ 2. The model which was constructed and exhibited by the present writer represents an exact stereographic projection of the 600-cell, *i.e.* the centre of projection is taken on the circumscribed hypersphere, and in fact is one of the vertices of the figure. The projection of this vertex would therefore be at infinity, and the 12 edges which meet there would be represented by lines proceeding to infinity from the vertices of the regular icosahedron, which is the outermost accessible boundary of the projection.

<sup>1</sup> "Regular Figures in  $n$ -dimensional Space," *Amer. J. Math.*, **3**, 1–14.

<sup>2</sup> "Regelmässige linear begrenzte Figuren von vier Dimensionen," *Arch. Math.*, Leipzig, **67**, 29–44.

<sup>3</sup> "Theorie der homogenen zusammengesetzten Raumgebilde," Halle, *Nova Acta Acad. Leop.*, **44**, 343–459.

In the model these infinite edges have been omitted, so that the model is to that extent incomplete. The projection of the vertex which is opposite the centre of projection forms the centre of the model, and the successive zones of vertices are very simple and regular. Starting from the outside—

Zone A is the vertex at infinity (1 vertex).

Zone B is a regular icosahedron (12 vertices).

Zone C is a regular dodecahedron (20 vertices).

Zone D is a regular icosahedron, whose vertices are not joined to one another. In the model these vertices are joined to the vertices of zone C by wires painted black, forming pyramids on the faces of the dodecahedron (12 vertices).

Zone E, the mesial zone, is the semi-regular polyhedron called the icosidodecahedron, which is bounded by 20 triangles and 12 pentagons (30 vertices).

Zone — D is similar to zone D, and its vertices are joined by black wires to the vertices of zone — C, forming pyramids on the faces of a dodecahedron (12 vertices).

Zone — C is similar to C, *i.e.* a regular dodecahedron (20 vertices).

Zone — B is similar to B, *i.e.* a regular icosahedron (12 vertices).

Zone — A is the centre (1 vertex). (Total number of vertices 120.)

The edges which join up the vertices of each zone are of brass wire, and, with the exception of the edges joining zones C, D, and — C, — D, the edges joining different zones are of differently coloured silk threads. All the threads which join the vertices of the same two zones, and those of the corresponding zones on the other side of the mesial zone, are of one colour.

§ 3. The model has been constructed so that the radius of the circumscribed hypersphere is 8 cm.

In making the calculations for the lengths of the edges great use has been made of Schoute's valuable paper<sup>1</sup> which gives the co-ordinates of the 120 vertices in the most symmetrical form, and tabulates the connecting edges. In Schoute's system of numbering, the 120 vertices are numbered from 1 up to 60, and from — 1 to — 60 for the opposite vertices. With reference to the special arrangement of the vertices in the stereographic projection, whereby one vertex is singled out as centre of projection, another system of numbering allows of a more compact table of connecting edges. The vertices of each zone are numbered separately, and so also are the rings or zones of vertices of each zone. A pair of opposite vertices of

<sup>1</sup> "Regelmässige Schnitte und Projectionen des Hundertzwanzigzelles und Sechshundertzelles im vierdimensionalen Raume," Amsterdam, *Verh. K. Akad. Wet.* (1e. Sect.), II. No. 7 (1894).

any zone are numbered  $\pm n$ . Thus, *e.g.*, B 3 and  $-B - 3$  denote opposite vertices of the 600-cell. For the mesial zone  $-E$  is the same as  $E$ , and  $-E 3$  is the same as  $E 3$ .

In zones B and D, the icosahedra, the vertices are numbered 0; 1, 2, 3, 4, 5;  $-1, -2, -3, -4, -5$ ; 0.

In zone C, the dodecahedron, they are numbered 1, 2, 3, 4, 5; **1, 2, 3, 4, 5**;  **$-1, -2, -3, -4, -5$** ;  $-1, -2, -3, -4, -5$ .

In zone E, the icosidodecahedron, they are numbered 1, 2, 3, 4, 5; **1, 2, 3, 4, 5**; 13, 24, 35, 41, 52,  $-13, -24, -35, -41, -52$  (or 31, 42, 53, 14, 25); etc.; *i.e.* the 10 vertices of the equator of this zone are represented by the pairs of numbers which represent the vertices of the adjacent rings to which they are connected, and 31 is the same as  $-13$ .

§ 4. In order that reference may be made to Schoute's tables, Table I. gives the numbers in the present system, which correspond to Schoute's numbers. No. 4 of his system, which is on the axis of  $w$ , is taken as centre of projection. Then  $-4$  is  $-A$ , the centre of the model. The numbers corresponding to the negative numbers of Schoute's system are obtained by changing B, C, D into  $-B, -C, -D$ , and changing the sign of the number.

Table II. gives the edges of the 600-cell.

Table III. gives the co-ordinates of the vertices, according to Schoute, but the plane of  $x, y, z$  has been moved so as to pass through A, *i.e.*  $w$  has been changed into  $2(e+1)-w$ . The symbol  $e = \sqrt{5}$ .

Table IV. gives the lengths of the edges of the projection.

§ 5. In the projection a number of groups of points become coplanar. These are the projections of points which lie in the same hyperplane passing through the centre of projection. The groups of points in the original figure which are so projected form zones of the same form as the zones B, C, D, E, and their centres lie in the zones B, C, D, E respectively.

Thus, taking the point B0 as centre, we have the icosahedron—

$$\boxed{\text{A}; \text{ B } 1, 2, 3, 4, 5; \text{ C } 1, 2, 3, 4, 5; \text{ D } 0,}$$

which is projected into a plane figure.

With centre C1 we have first the icosahedron—

$$\text{B } 0, 3, 4; \text{ C } 1, 2, 5; \text{ D } 0, 3, 4; \text{ E } 1, 3, 4,$$

and next the dodecahedron—

$$\boxed{\text{A}; \text{ B } -1, 2, 5; \text{ C } 3, 4, 2, -3, -4, 5; \\ -\text{C } 1; -\text{D } 0, 3, 4; \text{ E, } 2, 5, 2, 5, 13, 14,}$$

and this last is projected into a plane figure.

With centre D0 we have the icosahedron—

$$B\ 0; \ C\ 1, 2, 3, 4, 5; \ E\ 1, 2, 3, 4, 5; \ -D\ 0;$$

the dodecahedron—

$$B\ 1, 2, 3, 4, 5; \ D\ 1, 2, 3, 4, 5; \ E\ 1, 2, 3, 4, 5; \ -C\ 1, 2, 3, 4, 5;$$

and the icosahedron—

A ; C <b>1, 2, 3, 4, 5</b> ; -D 1, 2, 3, 4, 5; -B 0,
--

and the last figure becomes a plane figure in the projection.

With centre E1 we have the icosahedron—

$$C\ 3, 4; \ D\ 0, 1; \ E\ 2, 5, 3, 4; \ -D\ 0, 1; \ -C\ 3, 4,$$

the dodecahedron—

$$\begin{aligned} & B\ 0, 1; \quad C\ 2, 5, 3, 4; \quad D\ 2, 5; \quad E\ 3, 4, 31, 41; \\ & -B\ 0, 1; \quad -C\ 2, 5, 3, 4; \quad -D\ 2, 5; \end{aligned}$$

the icosahedron—

$$B\ 2, 5; \ C\ 1, -1; \ E\ 2, 5, 35, 42; \ -C\ 1, -1; \ -B\ 2, 5;$$

and the icosidodecahedron—

A ; B 3, 4, -3, -4; C <b>2, 5, -2, -5</b> ; D 3, 4, -3, -4; E 1, -1, 25, 52; -A ; -B 3, 4, -3, -4; -C 2, 5, -2, -5; -D 3, 4, -3, -4,
---

the latter being projected into a plane figure.

§ 6. The equations of transformation for the stereographic projection with centre at the point A, *i.e.* the origin, and plane of projection  $w=2(e+1)$ , are :

$$\frac{x}{x'} = \frac{y}{y'} = \frac{z}{z'} = \frac{2(e+1)}{w'},$$

where  $x', y', z'$ ,  $w'$  are the co-ordinates of a vertex of the 600-cell and  $x, y, z$  those of its projection.

TABLE I.—NOTATION FOR THE 120 VERTICES OF THE 600-CELL, (a) SCHOUTE'S NOTATION, (b) NOTATION USED IN THE PRESENT PAPER.

a.	b.								
1	E 52	13	B 0	25	B 2	37	B 3	49	E 2
2	E 1	14	B 1	26	B 5	38	B 4	50	E 5
3	E 1	15	B -1	27	B -5	39	B -4	51	E 4
4	A	16	-B 0	28	-B 2	40	-B 3	52	E -3
5	C 5	17	C 1	29	D 2	41	C 5	53	E 5
6	C 2	18	C -1	30	D 5	42	C 2	54	E 2
7	C 4	19	C 1	31	D -5	43	C -2	55	E 42
8	C -3	20	-C 1	32	-D 2	44	-C 5	56	E 53
9	-C 5	21	D 0	33	C 4	45	D 3	57	E 3
10	C 3	22	D 1	34	C 3	46	D 4	58	E 4
11	C -4	23	D -1	35	C -3	47	D -4	59	E 41
12	-C 2	24	-D 0	36	-C 4	48	-D 3	60	E 13

TABLE II.—THE EDGES OF THE 600-CELL.

(To complete the table (1) perform a cyclic permutation of the numbers 1, 2, 3, 4, 5 in each row, keeping 0 unaltered ; (2) change the sign of every number in a row ; (3) change A, B, C, D into  $-A$ ,  $-B$ ,  $-C$ ,  $-D$  and *vice versa*.)

A	joined to each B.
B	0 joined to A ; B 1, 2, 3, 4, 5 ; C 1, 2, 3, 4, 5 ; D 0.
1	" " A ; B 0, 2, -3, -4, 5 ; C -1, 3, 4, 3, 4 ; D 1.
C	1 " " B 0, 3, 4 ; C 1, 2, 5 ; D 0, 3, 4 ; E 1, 3, 4.
1	" " B -1, 3, 4 ; C 1, -3, -4 ; D -1, 3, 4 ; E 1, 13, 14.
D	0 " " B 0 ; C 1, 2, 3, 4, 5 ; E 1, 2, 3, 4, 5 ; -D 0.
1	" " B 1 ; C -1, 3, 4, 3, 4 ; E 1, 31, 41, 3, 4 ; -D 1.
E	1 " " C 3, 4 ; D 0, 1 ; E 2, 5, 3, 4 ; -D 0, 1 ; -C 3, 4.
1	" " C 1, 1 ; D 3, 4 ; E 3, 4, 13, 14 ; -D 3, 4 ; -C 1, 1.
13	" " C 1, -3 ; D -1, 3 ; E 1, -3, 53, 14 ; -D -1, 3 ; -C 1, -3.

TABLE III.—CO-ORDINATES OF THE VERTICES OF THE 600-CELL. EDGE=4.

(The order of the combinations of sign, corresponding to the vertices going from left to right in a row, is  $++$ ,  $+ -$ ,  $- +$ ,  $- -$ . To complete the table change all the signs in each row.)

	x	y	z		x	y	z
A : $w=0$	0	0	0	D : $w=2e$ ; -D : $w=2e+4$			
-A : $w=4(e+1)$	{ 0	0	0				
B : $w=e-1$ ; -B : $w=3e+5$							
0, 1 2, -5 3, 4	0 e+1 $\pm 2$	$\pm 2$ 0 e+1	e+1 $\pm 2$ 0	0, 1 2, -5 3, 4	0 e+3 $\pm(e+1)$	$\pm(e+1)$ 0 e+3	e+3 $\pm(e+1)$ 0
C : $w=e+1$ ; -C : $w=3(e+1)$				E : $w=2(e+1)$			
1, 1 4, 3 5, -2 5, 2 -3, -4	0 $\pm 2$ e+3 $\pm(e+1)$ $\pm(e+1)$	e+3 0 $\pm 2$ e+1 e+1	$\pm 2$ e+3 0 e+1 -(e+1)	52 1 1 2 3, 13, 41, -4 2, -3, 5, -4 5, 42, 2, 35	2(e+1) 0 0 2 $\pm(e+3)$ $\pm(e+1)$ $\pm(e+3)$	0 2(e+1) 0 2 $\pm(e+1)$ $\pm(e+3)$ $\pm(e+1)$	0 0 2(e+1) 0 $\pm(e+1)$ $\pm(e+3)$ 2

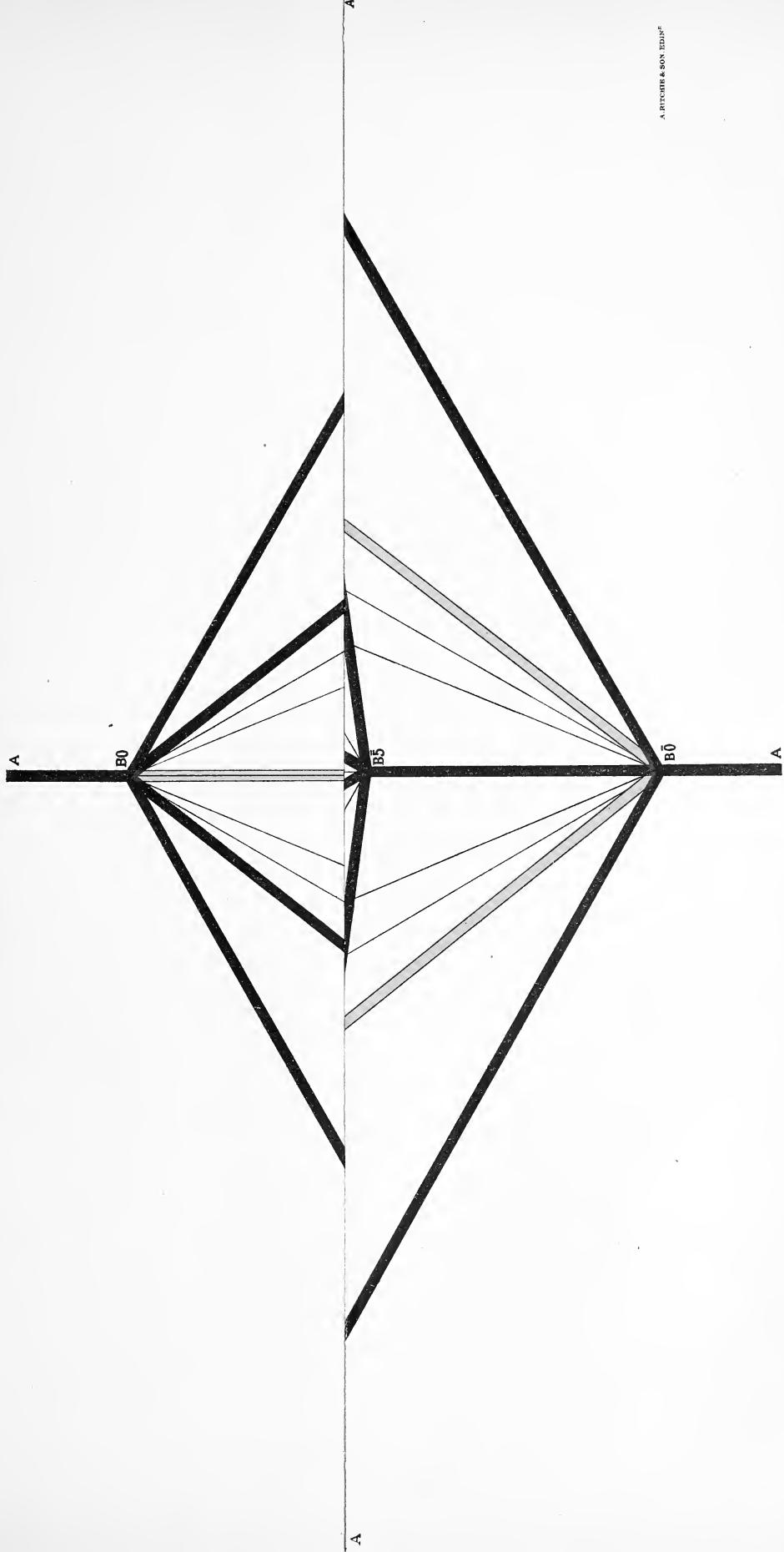
TABLE IV.—LENGTHS OF EDGES OF THE PROJECTION OF THE 600-CELL.

Joining Zones.	No. of Edges.	Length of Edge, when—		
		Edge of 600-cell = 30.	Radius of Circumscribed Hypersphere	
			= 30.	= 1.
- B to - B	30	$6(5-e)$	$6(3e-5)$	0·3416
- C, - C	30	20	$10(e-1)$	0·4120
- C, - D	60	$10(e-1)\sqrt{3}$	$10(3-e)\sqrt{3}$	0·4411
E, E	60	30	$15(e-1)$	0·6180
D, C	60	$6e\sqrt{10+2e}$	$6e\sqrt{10-2e}$	1·0515
C, C	30	60	$30(e-1)$	1·2361
B, B	30	$30(3+e)$	$30(e+1)$	3·2361
B, A	12	$\infty$	$\infty$	$\infty$
- A, - B	12	$3\sqrt{10(5-e)}$	$6e\sqrt{5-2e}$	0·3249
- B, - C	60	$2e\sqrt{6(5-e)}$	$4\sqrt{15(5-2e)}$	0·3752
- B, - D	12	$12e\sqrt{5-2e}$	$6e\sqrt{50-22e}$	0·4016
- C, E	60	$10\sqrt{6}$	$5(e-1)\sqrt{6}$	0·5046
- D, E	60	$15(e-1)\sqrt{2}$	$15(3-e)\sqrt{2}$	0·5402
- D, D	12	$6e\sqrt{10-2e}$	$12e\sqrt{5-2e}$	0·6498
E, D	60	$6e\sqrt{5+e}$	$6e\sqrt{5-e}$	0·7435
E, C	60	$30\sqrt{2}$	$15(e-1)\sqrt{2}$	0·8740
D, B	12	$12e\sqrt{5+2e}$	$6e\sqrt{10+2e}$	1·7013
C, B	60	$30(1+e)$	60	2·0000

The accompanying plate represents a symmetrical orthogonal plane projection of the three-dimensional projection on exactly  $\frac{1}{2}$  the scale of the model. Zones B and - B are in black, C and - C are in red, and E is in blue. The vertices are denoted as in the text, but for compactness the “minus” is put as a mark over the letter or number.

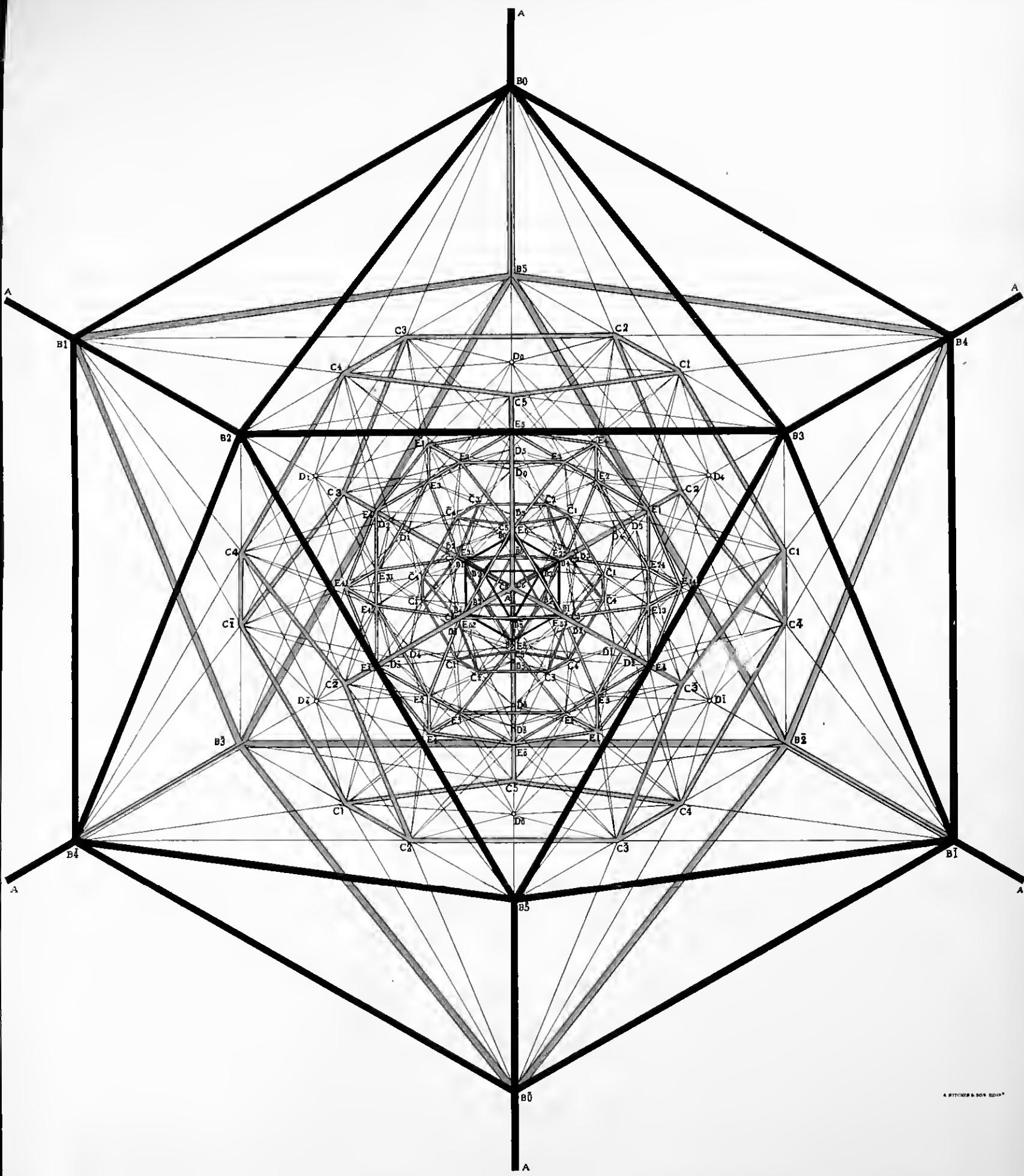
(Issued separately September 29, 1914.)

SOMMERVILLE: FOUR DIMENSIONAL FIGURE.





## SOMMERSVILLE: FOUR DIMENSIONAL FIGURE.





**XX.—Changes of Electrical Resistance accompanying Longitudinal and Transverse Magnetizations in Iron and Steel.** By Professor C. G. Knott, D.Sc.

(Read May 4, 1914. MS. received October 1, 1914.)

In January 1913 I communicated a paper on the changes of resistance of nickel when subjected to a combination of longitudinal and transverse magnetic fields (1). The following paper contains an account of exactly similar experiments with iron and steel.

Each steel or iron strip formed the core of an anchor-ring coil which was double-wound, with two exactly equal coils of copper wires. When the current was passed through the two contiguous coils in series in the same direction the metal cores were magnetized longitudinally. When the current was passed in opposite directions through the two coils there was no magnetization produced in the cores, but the heating effect was the same as in the first case. At the beginning of each experiment the current was applied in the latter or unmagnetizing arrangement, and was sustained for a sufficient time to permit the temperature to become practically constant. With reversal of the current in the one half of the enveloping coil a longitudinally magnetizing force was established within the region occupied by the iron or steel core. By means of a succession of reversals and re-reversals the core could be subjected to a cyclical variation of magnetizing force, while the temperature remained practically constant.

Six layers of the magnetizing coil were wound round each core, the number of windings in each layer being in accordance with the following table.

Layer.	Number of Windings in Magnetizing Coil.	
	Steel Core.	Iron Core.
I.	156	184
II.	130	176
III.	120	184
IV.	128	180
V.	112	186
VI.	160	180
Total Windings	806	1090

The steel core formed a circle of 6 cm. diameter, and the iron core one of 7.3 cm. diameter. The larger size of the iron core accounts for the greater number of windings in each layer.

Applying the usual approximate formula, we find that a current of one ampere passing through the magnetizing coils will produce fields of 53.7 and 59.6 in the steel-core and the iron-core anchor-ring respectively.

The transverse field was applied by means of a specially designed electromagnet with cylindrical pole pieces, the air gap between which could be altered with ease. The anchor-ring coil under investigation was placed symmetrically in the air gap, so that the axis of the anchor-ring passed through the centres of the pole pieces. The magnetic fields established in the air gap for various lengths of air gap and strengths of current passed through the coils of the electromagnetic were measured by means of a Grassot Fluxmeter. The lines of force established in the air gap ran across the coiled strip of iron or steel, that is, transverse to the direction in which the resistance was being measured.

The method of experimenting was identical with that described in detail in the former paper (1).

The iron or steel strip formed the greater part of one arm of a Wheatstone Bridge, an approximate balancing being secured by adjustment of the point of contact on a stretched wire. The combined system of conductors forming the Wheatstone Bridge was made part of a circuit through which a small steady current was passed from a secondary cell. When this current was flowing steadily through the circuit, one of the known resistances in the Bridge was altered slightly in a definite manner by introducing a large resistance shunt in parallel with this resistance. The deflection obtained on the galvanometer, being due to a measurable disturbance in the balance, was essentially a standardizing of the deflection. This calibrating shunt being thrown out of connection, the iron or steel strip which formed the opposing branch in the Bridge was then magnetized. The disturbance due to this cause at once declared itself by a corresponding deflection on the galvanometer scale. This deflection, taken in conjunction with the deflection formerly produced in the standardizing experiment, gave the means of calculating the change of resistance accompanying a given magnetization.

The galvanometer used in these experiments was a D'Arsonval galvanometer of the Ayrton-Mather design, and was found eminently satisfactory on account of its steadiness and sensitiveness.

As in the previous experiments with nickel, the deflections were obtained by reversing the steady current through the Wheatstone Bridge, the

reading produced when the current was in the one direction being subtracted from the mean of the readings immediately preceding and succeeding with the current in the other direction. Five successive sets of such triplets of readings were taken as quickly as possible: (1) with no magnetizing force applied, (2) with the magnetizing force applied in, say, the positive direction, (3) with no magnetizing force applied, (4) with the magnetizing force applied in the negative direction, (5) with no magnetizing force applied. Each triplet gave a first difference of deflections; and from the five first differences two second differences were obtained by subtracting the second from the average of the first and third, and the fourth from the average of the third and fifth. The average of these two second differences was the final value of the deflection due to the application of the magnetizing force. By means of the standardizing experiment this final value was reduced to absolute measure in the form  $dN/N$ , where  $N$  is the resistance of the iron or steel strip.

The calibration experiment involved the observation of at least nine distinct readings; and the final value of the deflection in the experiment just described involved fifteen distinct readings. Hence the value of any one of the ratios,  $dN/N$ , is deduced from twenty-four distinct galvanometer readings.

A complete set of observations for any given pair of fields, the one longitudinal and the other transverse, required four groups of the fifteen readings just described. The first group was obtained with no transverse field, the longitudinal field being put on and removed twice with change of direction between the first and second applications. In the second group the transverse field was applied and kept steadily in action, the longitudinal field being put on and off with reversal of direction as before. In the third group the longitudinal field was kept steadily applied in its turn, and the transverse field was put on and off exactly as the longitudinal field was manipulated during the first and second groups. Finally, in the fourth group the longitudinal field was thrown off altogether and the transverse field applied and removed by itself in a cyclic manner, as was done with the longitudinal field in the first group.

The field which was put on and off with reversal of direction is distinguished as the "cyclic field"; and the other, which for the time is being maintained, is called the "steady field."

For other details of the method, and for the investigation of the complete theory, reference may be made to the earlier paper.

In the Appendix, which contains all the measured values of the changes of resistance, and in what follows here, the horizontal field will be re-

presented by  $h$  and the transverse field by  $t$ . The corresponding changes of resistance will be represented by capital letters H and T in accordance with the following convention.

The four changes of resistance which form one set will be H, H(T), T, T(H), with the meanings

H = effect of cyclic  $h$ , no transverse field existing;

H(T) = " " "  $h$  superposed on steady transverse field;

T = " " "  $t$ , no longitudinal field existing;

T(H) = " " "  $t$  superposed on steady longitudinal field.

RESULTS FOR STEEL:  $dN/N \times 10^4$ .

Longitudinal Field.		Transverse Fields in ( ).			
		(864)	(1282)	(2141)	(3781)
(53·7)	H	+ 2·84	+ 2·72	+ 2·77	+ 2·89
	H(T)	+ 0·3	+ 0·1	+ 0·19	+ 0·01
	T	- 4·59	- 5·53	- 6·29	- 7·05
	T(H)	- 7·55	- 9·12	- 9·86	- 10·37
(104)		(843)	(1268)	(2111)	(3706)
	H	+ 5·15	+ 5·19	+ 5·29	+ 4·75
	H(T)	+ 0·86	+ 0·31	+ 0·29	+ 0·22
	T	- 4·44	- 5·64	- 6·31	- 6·73
(157)	T(H)	- 10·34	- 11·39	- 12·25	- 12·4
		(606)	(1268)	...	(3706)
	H	+ 6·47	+ 6·69	...	+ 6·45
	H(T)	+ 3·12	+ 1·08	...	+ 0·5
	T	- 3·99	- 5·91	...	- 6·77
	T(H)	- 7·75	- 12·53	...	- 14·3

RESULTS FOR IRON:  $dN/N \times 10^4$ .

Longitudinal Field.		Transverse Fields in ( ).			
		(898)	(1282)	(2156)	(3796)
(59·6)	H	+ 3·83	+ 4·58	+ 3·76	+ 3·79
	H(T)	+ 1·17	+ 0·49	+ 0·36	+ 0·04
	T	- 2·7	- 5·48	- 7·84	- 8·98
	T(H)	- 4·92	- 10·11	- 11·51	- 12·46
(120·4)		...	(1282)	(2156)	(3781)
	H	...	+ 6·25	+ 6·83	+ 6·24
	H(T)	...	+ 2·43	+ 0·52	+ 0·09
	T	...	- 5·73	- 7·97	- 9·41
	T(H)	...	- 10·83	- 13·0	- 15·8

The various values are tabulated in the foregoing tables. The bracketed numbers in the first column on the left give the values of the longitudinal fields; and the bracketed numbers in the horizontal rows give the values of the transverse fields. The remaining numbers are the changes of resistance produced by the field or combination of fields indicated by the symbol in the second column.

In these experiments there is no evidence of what others have observed, namely, an *increase* of resistance in low and moderate transverse fields. For example, Grunmach (2), in three out of the four recorded experiments with iron, obtained increase of resistance up to fields of 7000 or 8000 Gauss, after which the change became a decrease rapidly increasing in value as the field was taken stronger. In like manner, he obtained with nickel an increase of resistance up to field 700, and thereafter decrease as the transverse field was made stronger.

I have always been very doubtful of the reality of this initial increase of resistance; and a recent paper by Messrs W. Morris Jones and J. E. Malam (3) seems to me to establish the fact that when nickel is accurately placed in the transverse field the change of resistance is always a decrease. In my own earlier experiments with nickel spirals in transverse fields I was never satisfied that I had the spiral absolutely perpendicular to the field until I had got rid of this apparent initial increase in low fields. When very thin wires are used, the difficulty of eliminating all chance of a resolved longitudinal effect becomes greatly increased. For the change of resistance depends undoubtedly upon the magnetization within the metal. In very thin wires the transverse magnetization cannot be very much greater than the transverse magnetizing force, whereas in the early stages the longitudinal magnetization is much greater than the longitudinal magnetizing force. A little consideration will show that a comparatively small resolved component of the magnetizing force along some part of the wire may easily be accompanied by a longitudinal magnetization large enough to produce a resistance change of positive sign able to overbalance the very small decrease due to the transverse magnetization.

All this danger of having present an uneliminated longitudinal component is obviated in the experiments now described by the use of ribbons instead of wires of iron and nickel. For in the first place it is a comparatively simple matter to set the coiled strip or ribbon with its width accurately along the lines of force; and in the second place, even if the adjustment were not quite accurate, the magnetization along the width of the metal would be considerable, so that any possible resolved longitudinal

effect would not be large enough to mask the effect of the transverse field. It seems to me, therefore, that attempts to explain the supposed increase of resistance in low transverse fields are quite uncalled for. What requires theoretical explanation is the *decrease* of resistance of both iron and nickel in transverse fields, and the *increase* of resistance in longitudinal fields.

This conclusion receives further support that in the case of cobalt Grunmach (2) obtained only a decrease of resistance. The cobalt was not in the form of a thin wire, but was a strip 0·2 mm. thick and 0·5 mm. broad coiled in a double flat spiral. With such a form there was less chance of error of adjustment. Consequently no increase of resistance was obtained in the lower fields.

With the doubtful exception of tin in the lowest field, all the other metals experimented with by Grunmach showed increase of resistance in transverse fields (2). These metals were silver, cadmium, tantalum, platinum, tin, gold, palladium, zinc, copper, and lead.

Through the kindness and by the help of Principal A. Crichton Mitchell, late of Travancore, I am able to add to these mercury. Professor Mitchell prepared a thin mercury column in a spiral glass tube of a convenient size to be inserted in the air-gap of the electromagnet which I used for establishing the transverse fields in the present experiments. Substituting the mercury spiral for the iron or steel ribbon in the arrangement described above, I measured the change of resistance in four different fields. The results are given in the following short table, in which the first row gives the values of the transverse field in Gauss, and the second the corresponding changes of resistance per 10,000.

CHANGE OF RESISTANCE OF MERCURY IN TRANVERSE MAGNETIC FIELDS.

Transverse Field.	2064	3801	5263	6473
$dR/R \cdot 10^4$	+0·11	+0·31	+0·43	+0·64

The relation between these sets of numbers is not linear, nor does a parabolic law satisfy them very satisfactorily. Nevertheless, assuming the formula  $dR/R = At^2$ , where  $t$  is the transverse field, we find

$$A = 1\cdot7 \times 10^{-12},$$

a result of the same order as for other non-magnetic metals.

[*Note added November 19, 1914.*—My attention has been drawn to a paper published in 1910 in the *Nuovo Cimento* (5), in which Dr G. Rossi

gives certain results for the change of resistance of mercury in a transverse magnetic field. Treating his numbers in the same way, I find that A has the value  $6 \times 10^{-13}$  or  $5 \times 10^{-13}$  for mercury filaments of diameter 0·7 mm. or 0·5 mm. respectively. The diameter of the mercury filament used in the experiment just described was almost exactly 1 mm. The discrepancies are considerable; and it is difficult to believe that the effect in mercury should depend on the diameter of the filament within the limits indicated.]

Now in field 3750 the corresponding changes of resistance per 10,000 in iron and steel are respectively -6·9 and -9·2, that is, twenty or thirty times the numerical value for mercury.

In the earlier experiments with nickel the highest transverse field reached was only 815; but it was obvious that in much higher fields the change of resistance would not exceed the value  $-95 \times 10^{-4}$ , that is, about ten times the value for steel. Changes numerically equal to those given above for iron and steel were obtained for nickel in fields of only twenty and thirty Gauss respectively.

It is well to bear in mind that, as proved in the earlier paper, the numerical value of the change due to a given transverse field is a function of the width of the strip of the magnetic metal, for the simple reason that on that width also depends the value of the magnetization.

I now pass on to the consideration of the main object of the research, namely the influence of a steadily maintained magnetic field upon the changes of resistance due to a cyclically applied field at right angles to the former.

With regard to the numbers given in the Table three pages back, it should be noted that the last figure in the measured changes of resistance is of no value, being well within the limits of experimental error.

The smaller number of data for the iron strip was due to the overheating of the magnetizing coil round the strip and the consequent breaking down of the insulation between the contiguous turns of the coil. But the nature of the results is obviously the same in both metals, and may be expressed qualitatively in the following words:—

1. Under the influence of longitudinal magnetization the electric resistance of iron and steel is increased; but this increase is notably diminished when the longitudinal magnetizing force is superposed cyclically upon a steadily sustained transverse magnetization. In the highest transverse fields used the change of resistance due to the superposed longitudinal field was in most cases very small, being a small fraction of the value when the longitudinal field acted alone.

2. Under the influence of a transverse magnetization the electric resistance of iron and steel is diminished; and this diminution becomes markedly greater when the transverse field is superposed cyclically upon a steadily maintained longitudinal field. In certain cases the change of resistance due to the transverse magnetizing force was more than doubled when this field was superposed upon the steadily maintained longitudinal field.

It will be seen on referring to my earlier paper on the behaviour of nickel under crossed magnetic fields (1) that as regards the effect of the steady longitudinal field upon the change of resistance accompanying the application of a transverse field, exactly the same kind of phenomena are obtained with the iron and steel.

On the other hand, as regards the effect of the steady transverse field upon the change due to the superposed longitudinal field, there was a peculiarity in the behaviour of nickel which is not found in the case of iron or steel. This peculiarity was that when the steady transverse field was above a certain value the change of resistance due to the superposed longitudinal field altered in sign, that is, the resistance was diminished, not increased.

In the earlier experiments with nickel the arrangements did not permit the application of such large fields as were possible in the later experiments with iron and steel. Yet much greater values of the resistance change were obtained with the nickel than with the iron or steel, although these were subjected to much higher magnetizing forces. This will appear from the comparisons made in the short table below, in which the changes of resistance in practically the same strengths of magnetizing fields are set side by side. The approximate values of the fields are given below each group of measurements.

CHANGES OF RESISTANCE PER 10,000.

	Longitudinal Field Cyclic.			Transverse Field Cyclic.			
	Nickel.	Steel.	Iron.	Nickel.	Steel.	Iron.	
H	+66	+2·8	+3·7	T	- 91	- 4·3	- 2·7
H(T)	-17	+0·3	+1·2	T(H)	-192	-7·6	-4·9
<i>h</i> =	60	54	60	<i>t</i> =	800	700	900
<i>t</i> =	800	800	900	<i>h</i> =	60	54	60

The main features of the phenomena here described are contained in this short table. The similarity of the effects produced in iron and nickel suggests that we are dealing with a fundamental property of ferromagnetic

substances; but of this we cannot be certain until the same experiments have been carried out with cobalt. I hope also to be able to make a similar study of the properties of bismuth. Meanwhile, I leave over any further theoretical discussion. I cannot, in fact, add anything to what was said in the earlier paper; and I am not aware that anyone has been able to make even a plausible suggestion as to the molecular mechanism on which these phenomena in crossed magnetic fields depend.

My thanks are due to Miss J. G. Dunlop and Miss M. Jazewska, who determined for me with great care the change with temperature of the resistance of the iron ribbon.

#### APPENDIX.

##### RESULTS AS REDUCED IN LABORATORY NOTE-BOOK, ARRANGED APPROXIMATELY ACCORDING TO DATE IN THE YEAR 1913.

The numbers in the columns headed Resistance Change give the changes of resistance, estimated per 10,000, of the metal strip.

*h* and *t* represent respectively the longitudinal and transverse fields.

The temperatures are calculated from the resistances of the metal ribbon.

#### IRON.

Date, Fields, Temp.	Cyclic Field.	Steady Field.	Resistance Change.	Date, Fields, Temp.	Cyclic Field.	Steady Field.	Resistance Change.
July 22	<i>h</i>	None	+ 3·69				
<i>h</i> =59·6	<i>h</i>	+ <i>t</i>	+ 1·12				
<i>t</i> =898	<i>h</i>	- <i>t</i>	+ 1·22				
37° C.	<i>h</i>	None	+ 3·96				
	<i>h</i>	+ <i>t</i>	+ 1·17				
	<i>t</i>	None	- 2·7				
	<i>t</i>	+ <i>h</i>	- 4·72				
	<i>t</i>	- <i>h</i>	- 5·12				
<i>h</i> =59·6	<i>h</i>	None	+ 4·58	July 23	<i>h</i>	None	+ 6·25
<i>t</i> =1282	<i>h</i>	<i>t</i>	+ 0·49	<i>h</i> =120·4	<i>h</i>	<i>t</i>	+ 2·43
37° C.	<i>t</i>	None	- 5·88	<i>t</i> =1282	<i>h</i>	None	- 5·73
	<i>t</i>	<i>h</i>	- 10·11	160° C.	<i>t</i>	<i>h</i>	- 10·83
	<i>t</i>	None	- 5·09				
<i>h</i> =59·6	<i>h</i>	None	+ 3·76	<i>h</i> =120·4	<i>h</i>	None	+ 6·83
<i>t</i> =2156	<i>h</i>	<i>t</i>	+ 0·36	<i>t</i> =2095	<i>h</i>	<i>t</i>	+ 0·52
37° C.	<i>t</i>	None	- 7·84	160° C.	<i>t</i>	None	- 7·97
	<i>t</i>	<i>h</i>	- 11·51		<i>t</i>	<i>h</i>	- 13·9
<i>h</i> =59·6	<i>h</i>	None	+ 3·79	<i>h</i> =120	<i>h</i>	None	+ 6·24
<i>t</i> =3781	<i>h</i>	<i>t</i>	+ 0·04	<i>t</i> =3796	<i>h</i>	<i>t</i>	+ 0·09
37° C.	<i>t</i>	None	- 8·98	160° C.	<i>t</i>	None	- 9·41
	<i>t</i>	<i>h</i>	- 12·46		<i>t</i>	<i>h</i>	- 15·8

## STEEL.

Date, Fields, Temp.	Cyclic Field.	Steady Field.	Resistance Change.	Date, Fields, Temp.	Cyclic Field.	Steady Field.	Resistance Change.
July 26 $h=53\cdot7$ $t=864$ $34^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 2.84 + 0.30 - 4.59 - 7.55	July 25 $h=104$ $t=843$ $70^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 5.15 + 0.86 - 4.44 - 10.34
$h=53\cdot7$ $t=1282$ $34^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 2.72 + 0.10 - 5.53 - 9.12	$h=104$ $t=1268$ $70^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 5.19 + 0.31 - 5.64 - 11.39
$h=53\cdot7$ $t=2141$ $34^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 2.77 + 0.19 - 6.29 - 9.86	$h=104$ $t=2111$ $70^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 5.29 + 0.29 - 6.31 - 12.25
$h=53\cdot7$ $t=3781$ $34^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 2.89 + 0.01 - 7.05 - 10.73	$h=104$ $t=3706$ $70^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 4.75 + 0.22 - 6.73 - 12.4
July 29-30 $h=158$ $t=606$ $160^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 6.47 + 3.12 - 3.99 - 7.75				
$h=156$ $t=1268$ $160^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 6.69 + 1.08 - 5.91 - 12.53				
$h=156$ $t=3706$ $160^\circ C.$	$h$ $h$ $t$ $t$	None $t$ None $h$	+ 6.45 + 0.50 - 6.77 - 14.3				

## REFERENCES.

- (1) C. G. KNOTT, "Changes of Electrical Resistance accompanying Longitudinal and Transverse Magnetizations in Nickel," *Proc. R.S.E.*, xxxiii, p. 200 (1913).
- (2) L. GRUNMACH, "Über den Einfluss transversaler Magnetisierung auf die electrische Leitungsfähigkeit der Metalle," *Ann. d. Physik*, vol. xxii, p. 141, 1906.
- (3) W. MORRIS JONES, B.Sc., and J. E. MALAM, "The Electrical Resistance of Nickel in Magnetic Fields," *Phil. Mag.*, April 1914.
- (4) C. G. KNOTT, "Magnetization and Resistance of Nickel at High Temperatures," Part 2, 1906, *Trans. R.S.E.*, xlvi, p. 547.
- (5) G. ROSSI, "Variazioni di resistenza del mercurio e delle amalgame di bismuto nel campo magnetico." *Il Nuovo Cimento*, 1911, serie vi, tomo ii.

(Issued separately December 14, 1914)

## OBITUARY NOTICES.

Dr A. C. L. G. Günther, M.A., Ph.D., M.D., LL.D., F.R.S., etc.  
By William C. M'Intosh.

(Read June 15, 1914.)

THE death of Dr Albert Charles Lewis Gotthilf Günther, who was elected to the Honorary Fellowship of this Society in 1895, has deprived science of the most distinguished ichthyologist of his day, and one whose labours in other departments of zoology were no less noteworthy. He was born in Esslingen in South Germany on the 3rd October 1830, his father being "Siftungs-Commissar" in Esslingen and "Estates-Bursar" in Möhringen, a descendant of a family which had been known in the locality for hundreds of years—indeed the Swabian branch of the Günther family was settled in and about Möhringen on the Filder Plateau at the beginning of the fifteenth century. His mother was Eleonora Nagel, whose family originally came from Bremen. Albert was the eldest son, and was sent for his early education to the Gymnasium at Stuttgart (1837-47); and subsequently he studied at the Universities of Tübingen (1847-52, 1856-57), Berlin (1853), and Bonn (1854-55), thus gaining a wide experience of University life and a breadth of culture which had an important influence on his future career. Descended from a line of clergymen, family tradition destined him for the ministry of the Lutheran Church, for which, indeed, he was trained at the Theological College of Tübingen, and for which he passed the qualifying examination. His natural bent, however, was wholly in another direction, and, after taking the degree of Ph.D. in 1852, he decided to study science and medicine, taking the degree of M.D. at the same University in 1862. Before this, however, he had chosen zoology as the field of his labours, and had published his first paper on a Distome as well as a treatise on *Fische des Neckars*, with the coloured figure of a form new to the river (1853), and a *Handbuch der medicinischen Zoologie* (1858). Visiting England in 1855, he met Sir Richard Owen and Dr John Edward Gray, who had been interested in the former work, and a friendship sprang up between them—resulting in the selection of Dr Günther, in October 1857, to arrange and describe the Fishes, Amphibians, and Reptiles in the British Museum; as well as to prepare

catalogues of the greater part of the collections. Thus settled with definite work before him, and amidst congenial surroundings, Dr Günther laboured incessantly at his great task ; and though the apartments, which were cellar-like, in the old Museum in Bloomsbury were far less cheerful than in the New Natural History Museum at South Kensington, yet his interest and energy never flagged. From the first the Fishes, Batrachians, and Reptiles were prominent in his studies, though Birds and Mammals also received due attention, as shown in various papers to the Zoological Society. Thus his work in the latter group ranged from monkeys, carnivores, rodents, and ungulates to marsupials, and from diverse parts of the globe. Besides accounts of recent birds, he, along with Mr Newton, investigated the extinct birds of Rodriguez. Only a lifelong experience, rigid accuracy, and great natural ability could have enabled him to grasp the salient points of forms pertaining to such diverse types, and this not in single species, but often in hundreds, and whose close resemblances or intricacies of structure were in themselves sources of perplexity.

The extraordinary activity with which he laboured is demonstrated by the long list of his works, memoirs, and papers on all the groups mentioned. Amongst the more important are such as *The Geographical Distribution of Reptiles* (1858), in which he had forestalled many interesting features subsequently described by others ; the memoir on *Ceratodus*, the lung-fish of the Burnett and Mary rivers of Queensland ; that on the structure of *Hatteria* (*Sphenodon*) from New Zealand ; “On the Giant Tortoises” ; and the vast array of papers on the Fishes, Amphibians, Reptiles, and occasionally Birds and Mammals, of every important British expedition, as well as collections from every quarter of the globe—from Pole to Pole, and from river, lake, sea, and land. The mere perusal of the titles of his papers is no light task, whilst every one is the record of a painstaking, laborious research. Mr E. A. Smith, one of his colleagues, estimates that, besides the works and larger memoirs, there were about 300 papers published in the Journals of the London Societies, and that the whole of his writings occupy about ten thousand pages, illustrated by a very large number of fine plates and text-figures. It is a record remarkable alike for its unswerving devotion and notable results, and affords a splendid example to younger men. He accomplished much of this work when burdened with the cares of administration, preparing official reports “in connection with individual members of the staff, monthly and annual reports of progress and work accomplished, the supervision and editing of catalogues and guides issued by his department, besides the consideration of all proposed acquisitions”\* and the con-

\* E. A. Smith, *Zoologist*, March 1914, p. 115.

tinual correspondence. Moreover, to his fellow-workers, such as Charles Darwin and A. Russel Wallace, he was of much service in the chapters on the distribution and classification of Fishes, Amphibians, and Reptiles.

The memoir on *Ceratodus* in the *Philosophical Transactions* is one of special interest, as it details the structure and relationship of a Dipnoan fish, the ancestors of which were separated by the long gap between the present and the Devonian and Carboniferous periods. Yet the persistence of type, as pointed out by Dr Günther, is most remarkable. Further, those early representatives were not the beginners of a series, "but the last of many preceding developmental stages."

His labours in the British Museum resulted in the issue of eight volumes of the Catalogue of the Fishes, a work of immense research, patient investigation, and accurate description. In this work (4000 pages) he pays a tribute to Johannes Müller's ordinal arrangement, though he was not satisfied that the coalesced pharyngeal bones are of sufficient importance to unite the Acanthopterygii and Malacopterygii into one order. An idea of the vast labour spent on this task may be obtained by glancing at the number of species dealt with, no less than 6843 being well established, whilst 1682 others are doubtful. The carrying out of this gigantic task in the cellars of the old British Museum in Bloomsbury shows the indomitable energy of the investigator as well as his thorough grasp of the subject. It is indeed doubtful if such a task will ever again be attempted on the same lines, at least without the physical collapse of the investigator. Two volumes of a Catalogue of *Batrachia salientia* and Colubrine snakes complete the series of ten volumes. Moreover, the Ray Society published his fine work, with numerous illustrations by Ford, on the Reptiles of British India. His daily work in the British Museum ranged over snakes from West Africa and South America to those from Siam and Australia; fishes from the most recent British dredging expeditions, those from fresh waters in every quarter of the globe, and from the neighbouring seas; amphibians from widely distant regions; birds and mammals from diverse localities, and often of great interest. Amongst his other works are the *Challenger* volumes on the shore and deep-water fishes collected in the great expedition. The subject of the deep-sea fishes had long been of special interest to Dr Günther, and we may imagine the delight he felt in the study of no less than 266 species belonging to this category—many of weird form, with remarkable sensory appendages and phosphorescent organs. As he himself has stated, the *Challenger* series laid a broad and sure foundation to our knowledge of the abyssal fish-fauna, and he incorporated all the most recent work of the Norwegian, American French, and British investigators.

of the deep sea. In the introduction to this fine treatise his experienced remarks on phosphorescence and on the nature and distribution of deep-sea fishes are of great value and interest. This volume is illustrated by no less than 72 plates, many of them double, and admirably drawn by Mintern Bros., the successors of G. H. Ford.

His report on the shore fishes collected by the *Challenger* was published before the preceding treatise, and comprised an account of 1400 species, of which 94 were new to science. Only a skilled ichthyologist could thus have worked out the collection with such rapidity, for it was issued in 1886, when Sir Wyville Thomson was still at the head of affairs. Rare forms from the tropical Atlantic, Bermuda, the temperate zone of the South Atlantic, of the Antarctic Ocean, the temperate zone of the South Pacific, of Japan, and the neighbouring regions were accurately described and figured. This and the foregoing volume would alone have made a reputation. Moreover, it gave Dr Günther an opportunity of widening our views with regard to the mutual relations of the fishes of deep and shallow water, and of demonstrating the wide range of many forms both in depth and locality.

One of his greatest services to the science of zoology as a whole, and one in which his work has directly proved a boon to all his fellow-workers, is the *Record of Zoological Literature*, which he founded in 1867 and edited for several years. Investigators have thus a ready means of making themselves acquainted with contemporary work in every country. This step alone would have earned the thanks of every zoologist, and its continuance to-day by the Zoological Society shows its permanent importance. The work must have given Dr Günther much thoughtful labour and care, and could only have been undertaken by one in a central position, and with the co-operation of a wide circle of zoological friends at home and abroad.

His *Introduction to the Study of Fishes* (1880) is another treatise which has had a widespread popularity—from the masterly way in which the author handled a subject to which he had devoted the best part of his life. No student of the group can find a more comprehensive yet concise treatise in any language, and none having an equal amount of reliable information. His chapters on the distribution of fishes—geological and geographical—are especially full of experienced remarks.

Though Dr Günther in his early days made a few of his own drawings, he soon became so occupied that it was necessary to employ others, and he was fortunate in securing for many years the services of G. H. Ford—who was *facile princeps* in lithography during his day, and who in the delineation of the lower vertebrata has never been surpassed—and he

acquired a special skill in illustrating the memoirs of Dr Günther, whose appreciation of a fine drawing was ever forthcoming.

Entering the British Museum in 1857, he by and by was appointed on the staff, and he rose step by step till in 1875 he became Keeper of the Zoological Department in succession to his friend Dr J. E. Gray, and he held this post for twenty years. His record in this institution is remarkable—as beneficial to the Museum as creditable to himself. His catalogues have already been alluded to, and the vast array of original contributions to the Royal, Zoological, and Linnean Societies formed an unbroken succession from first to last. The latter alone would have made a great reputation, yet they were but fragments of his daily work in perfecting the numerous collections committed to his care, in carrying out the endless duties of administration, and in devising improvements. Moreover, the construction of the New Natural History Museum at South Kensington, the scheme of Sir Richard Owen, likewise gave him increased responsibilities in connection with the arrangement of the galleries and cases, and still more with the transfer of the vast and valuable collections to their new premises. This task, perhaps, brought out the administrative talents and practical skill of the Keeper of the Department more prominently than anything else, and well merited the special minute of the Trustees on its successful completion. Amidst the array of vans, lorries, cabs, and conveyance by hand, no specimen of value was lost or broken. Nor was the rearrangement in the new Museum less successfully carried out, though not a few serious obstacles were encountered. Thus when the cases for the mammals on the ground floor were being arranged, it was found that the architect's ornamental projections on the walls were inimical to satisfactory adjustment, and thus this Class had to be placed on the first floor. He also insisted on the advantages of a separate building for specimens preserved in spirit, both for the greater safety of the extensive collections in jars, and for the security of the other portions of the magnificent building.

Some idea of the extent of the National Collection may be gained when it is mentioned that in 1880 there were 1,300,000 zoological specimens, and that when Dr Günther retired in 1895 there were 2,245,000. Known all over the world for his labours in zoology, and having an extensive acquaintance with naturalists and travellers, much of this progress was due to his tact and personal influence—and, it may be added, to his personal example, for from his earliest days he was a field-naturalist as well as a scientific author, and he never missed an opportunity of adding to the collections in the British Museum, whether as the result of his own

dredging and collecting expeditions, or by securing from friends such rare forms, for example, as *Leptocephali*.

In connection with the fittings of the National Collection at South Kensington, it is also interesting to remember that he favoured the construction of metal cases instead of wood, though the Government did not adopt this plan—probably on the score of expense. He was indeed one of the earliest in this country to show the advantages of such cases now fitted up in the most advanced museums. Further, from an early period of his career in the Museum he saw the importance of having a reference library in addition to a general library in connection with the Zoological Department, and he persistently exerted himself to carry out this aim. The severance of the collections from the proximity of the great library in Bloomsbury made this the more necessary, and now the New National History Museum has an important and invaluable general as well as a special zoological library—an inestimable boon to visiting naturalists as well as to the staff.

Yet another side of Dr Günther's services in the British Museum merits attention, viz. the development of the systematic work in the Museum. Thus he succeeded in increasing the scientific staff gradually from 4 to 13, and by a skilful modification of the duties of the attendants he managed to relieve the trained men from menial duties and enlist their services in highly skilled museum-work. Thus the scientific staff had at their disposal a body of experienced and reliable practical aids, so that their progress was rendered both rapid and satisfactory.

His services as a Vice-President of the Royal and Zoological Societies, and President of the Linnean Society, must have entailed a large absorption of his time and energies—especially as many of his memoirs and papers were communicated to one or other.

It might be supposed that one so constantly and so actively engaged in the pursuit of science had little time for attending to the interests of visitors to the collection. Yet, if he had done nothing more than inaugurated the fascinating and instructive cases containing the nesting of birds as now exhibited in the Museum, such would have been memorable. No feature in the great collection is more popular than these life-like illustrations of the British nesting birds of both sexes, their eggs, newly hatched young, and their environment. As he himself has stated, it was essential that the actual birds which made the nest, with their eggs or young, should be secured, and the surroundings taken from the spot, the only artificial elements being flowers, leaves, or structures which could not be preserved satisfactorily. In the case of such birds as the bustard and the ruff, the remarkable plumage and attitudes of the males

form an additional attraction in these charming scenes. None but a skilful field-naturalist in whose mind the actual scenes had imprinted themselves could have designed these wonderful cases; and Dr Günther has often said that he gained as much real knowledge from Nature as from the splendid libraries at his command.

His work in the other departments, viz. Mammals and Birds, was no less noteworthy. Every important and unimportant expedition consigned to him the fishes, amphibians, and reptiles, and occasionally the birds and mammals, and his conscientious treatment of them was uniformly the same, whilst his personal influence with the collectors was a constant source of rich additions to the National Museum.

By Dr Günther's recommendation many valuable collections were added to the British Museum, such as the Gould Collection of Birds, the Oates Collection of the Birds of Pegu, Goodwin-Austin's Indian Birds, the Sclater Collection of Birds, Capt. Shelley's African Birds, the Saville-Kent Corals, the Baly Collection of Phytophaga, the Bates Collection of Heteromera, the Zeller Lepidoptera, the Keyserling Arachnida, the Moore Indian Lepidoptera, the Pascoe Coleoptera, the Morelet Land and Freshwater Shells, the Atkinson Coleoptera and Rhynchota, the Grote North American Lepidoptera, and the Parke Foraminifera.

His great knowledge of zoology and ichthyology in particular, as well as his familiarity with the habits of animals, caused his services to be much sought after by Government Commissions and municipal bodies in regard to their fresh waters. Thus he reported on the pollution of the Thames and on that of several trout and salmon rivers. His evidence on the pollution of the Lower Thames was of great importance as well as conclusive, for his careful experiments proved the effects of such on fishes, and he indicated the length of time they would survive in various kinds of polluted water, *e.g.* sewage, effluents from gas-works, ink-works, etc. He went, for instance, minutely into the question, surveying the Lower Thames in a steam-vessel placed at his disposal by the Metropolitan Board of Works, and thus was enabled to give reliable advice to that body. His evidence in connection with the "yellow fins" of the Allan Water was another example of his acuteness and caution in dealing with a contested point.

Moreover, Dr Günther was ever ready to encourage local collections of objects of natural history, and his gifts to provincial museums, of tame birds for private parks and aviaries, are gratefully remembered. One of his last donations was that to the University Museum of St Andrews, to which he presented about fifty exquisitely coloured birds, ranging from Reeve's pheasant and the capercaillie to humming-birds, the group of the

Pittas being especially noteworthy for their striking coloration. The majority came from the collection of A. Russel Wallace, though some, such as the young kestrels, were reared by himself.

Since he came to England in 1856 he took an interest in the marine fauna—indeed in that year a local publication included his contributions to the marine fauna of Brighton. His holidays were generally devoted to the increase of the Museum's marine or freshwater fishes and other forms. At St Andrews he collected in a day or two various fishes and ten species of marine annelids. An excellent sailor, he sometimes was the only effective naturalist on board a boat or yacht, as, for example, when the distinguished Professor Kölliker of Wurzburg requested his aid off the south coast of England. His tanks for the preservation of the large fishes always accompanied him in these excursions. None enjoyed the freedom of forest, moor, or hill, or the quietude of a river bank more than he, and thus he gained an intimate knowledge of Nature—both animate and inanimate—so important for the head of the Zoological Department of the National Museum. This knowledge, gained by close observation on the Continent of Europe, in Britain, and in the adjoining seas, made him a delightful companion, and there were few who were more welcome than he at the country-seats both of England and Scotland. Moreover, he was an excellent shot—a reminiscence, perhaps, of his military experiences in South Germany—and an expert angler. At one time he took an active interest in the introduction of the Sheat-fish (*Silurus glanis*) to English waters, and with success; but the voracious habits of these large fishes proved disastrous to the salmonoids, and the attempt was not repeated.

Quite lately he prepared for the Trustees a brief account of the changes in the British Museum (Natural History) from 1858 to 1895—that is, during the period of his official connection with the institution. The continuous stream of important additions, many of which were due to the influence of the Keeper himself, the increase of the assistants, the inauguration of systematic publications by the staff, the transference of the greatest collection of the kind in the world from the old to the new quarters, and the introduction of every modern improvement in arrangement, are told with the characteristic modesty and restraint of the veteran investigator.

Dr Günther was the recipient of many honours both at home and abroad. He was a Vice-President of the Royal and Zoological Societies, President of the Linnean Society, President of the Biological Section of the British Association, and a Fellow of most of the learned societies at home and abroad. He was awarded a Royal Medal by the Royal Society, and the Gold Medal of the Linnean Society.

Dr Günther had a tall, somewhat lightly-built, wiry physique which for nigh sixty years stood without a break the stress and strain of official life, the unhealthy atmosphere in the old cellar in the basement at Bloomsbury, and the incessant demands of scientific work. His hair was fair, eyes blue, and his complexion fresh. Throughout his long period of public service, he was never known to have sick-leave. Of strong personality, and resolute when he had once formed a conclusion, yet he was not only an agreeable colleague, but a warm friend to a large circle of acquaintances. In his home he was one of the kindest parents, ever ready to sacrifice himself for the happiness of his family, who had an equally warm attachment to him. Of active habit, and delighting in his garden and his pets, he was ever busy and cheerful. His first home at Hampton Wick, and those subsequently at Surbiton and at Kew Gardens, all reflected the tastes of an enthusiastic naturalist whose pleasure lay in everything with life. His myrtles and other shrubs and trees at Surbiton, his maiden-hair tree and collection of rare shrubs and plants at Kew Gardens, his aviaries, house-pets, and his observations on the birds in Kew Gardens, were a never-failing source of interest and information to himself and others. His health suffered some years ago from a severe attack of pneumonia, but lately was satisfactory until an abdominal affection necessitated an operation from which he did not rally. He was buried in the quiet cemetery at Richmond, mourned by a large circle of scientific friends.

Dr Günther was twice married. His first wife, Roberta M'Intosh, of St Andrews, made the exquisite coloured figures of marine animals, many of which have been published by the Ray Society; their son, Robert, is a Fellow and Tutor of Magdalen College, Oxford, and the author of various able works and memoirs. Dr Günther's second wife, who, with a son, survives him, was Theodora Dowrish Drake, from Cornwall, a lineal descendant of a brother of Sir Francis Drake.

Dr Günther will ever be remembered as a great systematic zoologist who had early and independently worked out many of the problems of the distribution of animals which subsequently were more prominently associated with other names, as an original investigator and *facile princeps* in Fishes, Amphibians, and Reptiles, and as a man of untiring energy, remarkable power of penetration, and of great administrative capacity. Moreover, the interests of the public and of scientific workers at home and abroad were ever safe in his hands. Nowhere will the results of his life-long labours be more keenly appreciated than in the British Museum, the distinguished staff of which paid the last tribute to the veteran zoologist in the peaceful cemetery at Richmond.

John Sturgeon Mackay, M.A., LL.D. By George Philip, M.A., D.Sc.

(Read July 6, 1914.)

THE task which the Society has entrusted to me of putting on record some suitable memorial of the life and work of Dr John S. Mackay is one which I feel honoured in undertaking. At the same time I am conscious of my own limitations in attempting to give to the scientific world a biographical notice of one who was rightly looked on as one of the most learned men of the day, and who possessed all the graces which a well-stored mind can bestow, along with those subtle and ingratiating qualities of the heart which cast such a magnetic influence on all who were privileged to know him. Dr Mackay was, in very truth, the *beau ideal* of a scholar and a gentleman, and death has removed from the circle of his friends one who will long be missed. His death took place at his residence, 69 Northumberland Street, Edinburgh, on March 25 of this year.

John Sturgeon Mackay was born at the village of Auchencairn, Kirkcudbrightshire, on October 22, 1843, so that at the time of his death he was in his seventy-first year. While he was yet an infant, his parents removed to Perth, and there he spent his boyhood and received his early education. At Perth Academy he showed that aptitude for learning which later brought him great distinction, and it is well to note here that his preliminary education laid the foundation of both his linguistic and mathematical studies. The biographer of the late Professor Chrystal in the Society's *Proceedings* makes a similar remark; so that we have these two conspicuous instances at least of men who combined mathematical with classical or linguistic talent. One would fain recall here the advice given by Lagrange to Cauchy's father when consulted by him as to the proper education for his boy: "Do not allow your son to open a mathematical book nor to touch a single diagram until he has finished his classical studies." To the end Dr Mackay was a strenuous supporter of the old-fashioned classical education, and never ceased to deplore the modern trend of early specialisation, holding that preliminary education ought to be devoted to the cultivation of all the faculties, and not to the development of any one at the expense of the others.

After a school career that gave great promise of later distinction, Dr Mackay proceeded to St Andrews University, where he followed the

usual course at that time imposed on aspirants to a degree. The highest honours in mathematics and classics were won by him, and one of his fellow-students, himself a man of eminence, has told me that he was looked upon as the ablest man of his year. His original intention on leaving the University was to enter the ministry of the United Presbyterian Church, and with that in view he attended the Theological Hall in connection with that body in Edinburgh. Theology, as it was presented to him fifty years ago, was not to his taste, and he decided to renounce his intention of qualifying for admission to the Church, and to take up teaching as a profession. His first situation was on the mathematical staff of his old school, Perth Academy, so that, as he was fond of relating, he had as a predecessor William Wallace, afterwards the eminent occupant of the Chair of Mathematics in Edinburgh University. His stay in Perth was short—two years, I think,—and in 1866 he received an appointment as mathematical master in Edinburgh Academy, an institution which retained his services until he retired in 1904. His long connection with this well-known school had far-reaching effects both on the school and on Dr Mackay himself. To the very last he took unabated interest in all that pertained to the life of the school, and showed the most unswerving loyalty to everything connected with it. Indeed, at the beginning of the present year, when his eyesight failed him, he was engaged in compiling a register of pupils who attended the Academy since its establishment in 1824. Many of his pupils have risen to great eminence in various walks of life, both at home and abroad, and few of them revisited Edinburgh without spending some hours with their old master, whom they were proud to reckon among their friends. His affection for his pupils was real and genuine, and he followed their careers with a truly paternal interest.

Dr Mackay was singularly well suited for a teacher. His ready sympathy and kindly disposition immediately secured for him the goodwill of his pupils, while his great learning and nobility of character were so evident that they must have exercised a very powerful influence for good on the whole school. His well-stored mind was ever ready to give of its contents; and, while some men in such circumstances look on their learning as wasted, Dr Mackay, quite otherwise, thought nothing too good for his boys. A pupil of his own, a distinguished man of letters of this city, has put on record the following appreciation, and I cannot do better than quote his words: “In reviewing the list of those with whom he happens to have been brought into contact, the present writer can think of few more richly endowed than he with the qualities which really matter. He was eminently straight, he was eminently loyal, and he was eminently magnanimous. It

is of less consequence, yet not to be recalled without a pang, that he had a delightful sense of humour, which, coupled with the control he possessed over his vast stores of learning, rendered him the most charming of companions. A school may reckon itself fortunate which has inscribed on the roll of its masters the name of so learned, so accomplished, and so good a man as was John Sturgeon Mackay.”\*

His retirement from active duty dates from 1904, and as he was comparatively vigorous he looked forward to a period of great usefulness. He still spent two months or so of the year on the Continent, and also continued his mathematical researches. But latterly his intimate friends noticed a diminishing vitality, and although he came back every year refreshed and invigorated by the change, it was evident that the heavy self-imposed strain of many years was now beginning to tell on him. In January of this year, failing eyesight was the first indication that things were not right; and as this condition grew steadily worse, it became evident that it was symptomatic of very serious weakness, and after lingering for a few weeks he passed peacefully away on Wednesday, March 25. Having his time so fully taken up with more congenial pursuits, Dr Mackay took little or no interest in those affairs that bring men prominently into the public eye. To his friends he showed a warm and affectionate disposition ; stimulating in his criticism but never censorious, he had the happy faculty of saying the right thing and doing the right thing at the right time. Anything in the nature of sham, morally or intellectually, was specially abhorrent to him, and he very readily detected it. But those who showed even in a small degree an inclination to do something more than merely “put in the day ” found in him a staunch friend, willing to do his utmost in assisting them in their work, and by his kindly and well-directed counsel enabling them to bring their labours to a happy issue. His reputation for accurate scholarship extended beyond the confines of his own country, and he was frequently appealed to for information by savants all over the world. Included among his intimate friends were such well-known men in the domain of mathematical science as Neuberg, d’Ocagne, Laisant, and Aubert in Belgium and France, Moritz Cantor in Germany, and Robert Tucker in our own country.

The Royal Society did him the honour of electing him to a Fellowship in the year 1882 ; and although he admitted the prior claims of the Edinburgh Mathematical Society for his support in the matter of original papers, he did useful work as a member of the Council and also as a member of its Library Committee. By making him an Honorary Fellow

\* See *Edinburgh Academy Chronicle* for May of this year.

ten years ago, the Society showed its appreciation of the great service Dr Mackay rendered to scientific learning. His extensive knowledge of books was recognised by his appointment as a member of the Permanent International Bibliographical Association. His *alma mater*, the University of St Andrews, readily granted him the highest distinction she could offer and in 1884 conferred on him the degree of LL.D. He served two periods as Examiner in Mathematics in St Andrews, and for many years he occupied a similar position on the Examining Board of the Chartered Accountants' Society of Scotland. He was elected by the Edinburgh Mathematical Society as its first President, and it is not the least of his claims to our remembrance that he gave such whole-hearted support to its affairs that it was a constant pleasure to him to see it grow from a small beginning, with a membership of two score, to its present position of influence, with a membership of two hundred and fifty scattered over the four quarters of the globe. His zeal for the welfare of the Society never diminished, and until within the last few years, when his health began to decline, he was seldom absent from its meetings. As was to be expected from such an accomplished French scholar as he was, he took a very prominent part in the work of the Franco-Scottish Society, and attended several of its excursions through France.

In giving an account of the scientific work of the late Dr Mackay, it will be simplest to deal with it in the historical order of its development. At the outset, it is no exaggeration to say that the whole domain of pure geometry, in so far as it deals with plane figures, came under his notice, and a list of his published papers will show that he enriched almost every part of the subject by discoveries of more or less importance. A very prominent place must be assigned to his knowledge of Greek geometry. His great command over Latin and Greek made him singularly well qualified to deal with this fascinating subject, and only a mere chapter of accidents prevented him from obtaining the full honour to which his labours entitled him. The seventeenth- and eighteenth-century geometers like Commandinus, Edmund Halley, and Robert Simson had studied and edited, as far as they could, the works of Archimedes, Apollonius, Euclid, and Diophantos, and fairly complete collections of the works of these mathematicians were available; but very little attention had been paid to the writings of Pappus, one of the latest of the Alexandrian school of mathematicians. Dr Mackay made up his mind to supply the defect, and for many years he spent his vacations working patiently and laboriously at the MSS. of Pappus in the British Museum and in the Continental libraries, collating and translating them. He had practically finished his task, when Hultsch, the celebrated

German commentator, published his three-volume edition of Pappus, and Dr Mackay took no further steps to bring his out. This is all the more regrettable as British scholarship could well have stood a native edition of Pappus; and although Dr Mackay very magnanimously admitted that his Pappus was in no way superior to that of Hultsch, it is not to be doubted but that mathematical literature would have been greatly richer to-day if his book had been published. I understand that Sir T. L. Heath is soon to add Pappus' "Mathematical Collections" to his excellent editions of Archimedes, Apollonius, Diophantos, and Euclid, and so remove the stigma that English mathematicians are no longer interested in Greek mathematics. Dr Mackay was unfortunate, too, in coming so soon after Allman, whose researches in Greek geometry appeared first in *Hermathena* and afterwards in book form. These circumstances to a certain extent robbed him of the full honour due to his original work, but, nevertheless, he was looked upon as one of the foremost living authorities on Greek mathematics. His reviews of Heath's *Diophantos* and of Gow's *History of Greek Mathematics* in the *Academy* give us an insight into his grasp of the subject, and make us regret all the more that we have not a work from his own pen dealing with the early history of geometry. He was *par excellence* the man to have done it.

These studies naturally led on to the work of the Scottish geometers, Robert Simson and Matthew Stewart, who were more Euclid than Euclid himself in their methods of geometrical analysis, and Dr Mackay subjected their works to a most exhaustive examination. To mention only one of the results that followed from this, I might note that he finally settled the question as to who was the original discoverer of the so-called Simson Line, and he showed that Robert Simson has no claim to that honour, but that the theorem in question is due to William Wallace, who published it under a *nom de plume* in the *Mathematical Repository* (old series), ii, 111.\* Popular periodicals of the type of the *Repository*, the *Lady's and Gentleman's Diary*, etc., were forms of mathematical literature that flourished in our country from the middle of the eighteenth to the middle of the nineteenth century, and were supported very greatly by non-academic mathematicians. These journals gave incontestable proof that mathematical science, and particularly geometry, was very widely studied in our country, and was a source of pleasure and amusement to many whose daily avocations required physical rather than intellectual energy. Many of the problems dealt with were of a high order, and afterwards formed a prominent part of geometrical science. The existence of the nine-point

\* See Dr Mackay's paper in *Edin. Math. Soc. Proc.*, vol. ix.

circle, properties of symmedians and symmedian points, etc., were early discussed in the diaries. Dr Mackay made a close study of these journals, and the results of his labours were communicated to the French Association for the Advancement of Science at their Congress at Besançon in 1893, in a paper entitled "Notice sur le journalisme mathématique en Angleterre."

Dr Mackay's original papers were practically all published in the *Proceedings* of the Edinburgh Mathematical Society, and they constitute the most valuable record in our language of the geometry of the triangle. It is quite impossible to give here even the titles of all his papers, but it may be stated that no earnest student of any branch of plane geometry can afford to neglect his writings.\* They deal with the nine-point circle, the six scribed circles of a triangle, isogonals, symmedians, and isogonic centres of a triangle. Perhaps his most valuable contributions are "The Triangle and its Six Scribed Circles," published in vol. i, vol. ii, and vol. xi of the above *Proceedings*, and "The Symmedians of a Triangle and their Concomitant Circles," in vol. xiv. The first of these two occupied several years of his leisure, and to make it as complete as possible he enlisted the services of such well-known geometers as Tucker, Neuberg, Fuhrmann, and d'Ocagne. We may judge of the completeness of the work when we know that it occupied 1600 quarto pages of MS. His paper on the "Symmedians of a Triangle" made known for the first time in an English journal the remarkable properties of the K points and of the Tucker group of circles which have as particular cases the first and second Lemoine circles, the Taylor circle, and the Adams's circle.

Dr Mackay was also the author of the articles "Calendar" and "Geometry" in *Chambers's Encyclopaedia*, and "Euclid" in *Encyclopaedia Britannica*. The interesting and learned article on "Numeration" in the jubilee volume of the Chartered Accountants' Association of Scotland is also from his pen.

Of his books the most important is his *Elements of Euclid* (W. & R. Chambers, Edinburgh, 1884). Like many others, it is based on the well-known edition of Robert Simson, but it shows a vast improvement on any previous text-book. Every page of it shows evidence of ripe scholarship, and it possesses what no other text-book we know possesses, viz. references to original memoirs and authorities and full historical notes. Writers of mathematical text-books in general carefully avoid introducing such personal elements, and thereby in our view make a very great

\* A list of these papers will be found in the index volume of the Edinburgh Mathematical Society.

mistake. The idea that the subject has reached its present condition by the labours of many workers, largely obscure, is very helpful to learners, and gives a humanistic trend to the study of geometry. A Key to the *Elements* was published in 1885.

It is almost needless to say that Dr Mackay did not view with favour the departure from the Euclidean sequence. He held that some logical sequence is necessary, and that Euclid's is superior to any more recent innovations. Signs are not wanting that his views are now being shared by a growing number of mathematicians, who detect in our present system too much looseness and slovenliness. He was requested to write a text-book of geometry in accordance with the recent movement; and although he complied with the request and produced his *Plane Geometry*, books i–iii in 1904, and books iv–v in 1905, they naturally have not the characteristic features of the earlier work. His *Arithmetic Theoretical and Practical* appeared in 1899, and forms one of the soundest and most illuminating books we have on the subject.

This short account of his work will show the great service Dr Mackay rendered to mathematical learning, and the loss the scientific world has sustained by his death.

**Professor John Gibson.** By Principal A. P. Laurie, D.Sc.

(MS. received October 26, 1914. Read December 7, 1914.)

JOHN GIBSON was born in Edinburgh on May 13, 1855, and was educated at Edinburgh Academy. He afterwards studied chemistry at Heidelberg under Bunsen, Kirchhoff, Kopp, and others, working for five consecutive sessions in Bunsen's laboratory, and graduating in 1876 as Doctor of Philosophy.

On returning to Edinburgh, he became assistant under Professor Crum Brown; later on, in 1881, being appointed chief assistant in the laboratory, where he taught for eleven years. In 1892 he was appointed Professor of Chemistry in the Heriot-Watt College, a post which he held up to the day of his death.

Gibson was, above all things, an analyst. He seems to have developed his original interest in chemical analysis under Bunsen, and to the end of his life he remained in the very first rank of analysts, and always regarded that part of the teaching in the department as of the utmost importance.

As an example of his capacity for analytical research, we cannot do better than take his report on "An Analytical Examination of Manganese Nodules, with special reference to the Presence or Absence of the Rarer Elements," which was published in the *Challenger Reports—"Deep Sea Deposits,"* in 1891, and involved an original research in analytical methods. All those who had the good fortune to be students under him have benefited by his enthusiastic appreciation for, and exact knowledge of, analytical methods.

While in Edinburgh University, Gibson carried out a large number of observations for the Fishery Board on the composition of sea waters, more especially in the North Sea, and he also made an investigation into some of the rare earths. Years of investigation were devoted to the study of these rare earths, and the separation of pure salts from them. Unfortunately, all that ever was published on this subject was a short paper on "Glucinum" in the *Transactions of the Chemical Society*, 1893.

Gibson always approached the problem of publication with great unwillingness. When once he completed a research, his interest carried him on to fresh investigations, and it was with great difficulty that he could be persuaded to put pen to paper with a view to publication. As a consequence of this, many valuable researches have been lost to science, and this

is especially the case in connection with glucinum, cerium, lanthanum, and didymium. Large quantities of the minerals were worked up, pure salts prepared, and much work was done, which has no doubt since been confirmed by others, although it may be questioned whether even now all Gibson's results have been re-established.

About the time when the paper on glucinum was published, Gibson started some experiments on the effects of light on such changes as the conversion of chlorine water into hydrochloric acid, the resulting observations being published in a short paper on "Photochemical Action" in the *Proceedings* of the Royal Society of Edinburgh in February 1897. This was followed by a short paper, "A Preliminary Note on a Characteristic of Certain Chemical Reactions" (*Proc. Roy. Soc. Edin.*, Dec. 1897). The origin of these papers was as follows: In studying the action of light on these mixtures, Gibson discovered the fact that the amount of change depended on whether the final result of the reaction was to increase the electrical conductivity of the solution as a whole or to diminish it, there being a tendency for any such solution to move in the direction of increased electrical conductivity. This led him further to investigate the question as to how far other reactions, apart from those caused by light, were influenced by these conditions.

No particular physical value had been, so far, associated with the electrical conductivity of a system as a whole, and the whole direction of research was proceeding towards experiments on very dilute solutions, with a view to the application of the laws laid down by van't Hoff, Arrhenius, Kohlrausch, and Nernst to the problems of electrochemistry. It was probably for this reason that more attention has not been directed to the very interesting results obtained by Gibson in this direction.

In the preliminary paper already referred to he gives examples of the law that many chemical reactions are governed by the tendency of a solution to develop a state of maximum conductivity in the system, these examples being: the dehydration by hydrochloric acid of hydrated cobaltous chloride; the dehydration of sugar by sulphuric acid; the reduction of chromic anhydride by hydrochloric acid; the oxidation of hydrogen iodide by sulphuric acid; and the oxidation of nitric oxide by nitric acid.

In order to carry these investigations further, he decided to redetermine the conductivity curves of some of the best known acids and salts, and devoted a great deal of time and labour to these measurements, with the result that there can be no doubt that the most exact conductivity curves that we have for hydriodic, hydrobromic, and hydrochloric acids, and

ammonium bromide, lithium bromide, and sodium bromide, are those determined by Gibson; whilst for these experiments he devised his electrically controlled thermostat, which is a very perfect instrument of its kind.

The main interest of his work, however, remains as before the study of the relation between maximum conductivity and certain types of chemical change. He showed, for instance, the close relation between this and the precipitation of salts from solution by hydrochloric acid; the behaviour of aqueous solutions of hydrogen chloride towards dissolved oxygen and dissolved chlorine respectively; the oxidation of hydrogen chloride in aqueous solution by chromic acid; the action of hydrochloric acid as an esterifying agent; and the action of hydrogen chloride on acetaldehyde, aldol, and crotonaldehyde, and of hydrochloric acid on cobalt chloride. In addition, he investigated the decomposition of aqueous solutions of hydrogen iodide; the behaviour of nitric acid when exposed to light; and, in more detail, the action of sulphuric acid on sucrose and on formic acid. In all these cases he proved quite definitely that the limit to which the reaction was carried was fixed by the point at which the system as a whole reached its maximum conductivity, and that many reactions were reversed on each side of this maximum conductivity point, proceeding in opposite directions when once the maximum of the curve had been passed.

It is, of course, evident that there are a large number of reactions which are not governed by this condition, and this is one of the reasons why for many years Gibson hesitated to publish his results, as he wished to get some definite law by which he could distinguish between reactions which were governed by the maximum conductivity and those that were not.

It is probably safe to say from his results that all chemical systems which are electrolytes tend towards the point of maximum conductivity, although there may be other forces at work which are sufficiently powerful to conceal this tendency; but whenever we are dealing with balanced reactions in which a very small change of conditions will make the reaction proceed the other way, we find the maximum conductivity of the system is the governing condition. There can be no doubt that we have therefore to look for the widest application of this principle when dealing with plant and animal life, where we have such a delicate balance constantly occurring between two possible directions of chemical change.

Gibson has shown the application of his theory to the change from sugar to starch, and again from starch to sugar, in the leaf of the plant, and he also made a considerable number of experiments—which, un-

fortunately, will now never be published—on the influence on enzymatic reactions of the same condition. His experiments on mustard powder and on crushed bitter almonds have already been published in the paper on “The Significance of Maximum Specific Electrical Conductivity in Chemistry” (*Trans. Roy. Soc. Edinburgh*, xlvi, Part I, No. 6). These will be found well worthy of study by those who are interested in plant chemistry and in enzymatic changes. It is certainly open to question whether one of the controlling conditions of enzymatic reactions will not be found to be the nature of the mineral salts that are present, and the amount of dilution or concentration required to bring the solution of the salt to its maximum conductivity point.

John Gibson was elected a Fellow of the Royal Society of Edinburgh in 1877, and twice served as a member of Council, from 1892 to 1894 and from 1897 to 1900. He was of great service to the Council when papers of a chemical nature were under consideration.

Gibson had just completed the fitting up of the new laboratories at the Heriot-Watt College, and had only entered into possession of them for a couple of months, when his death occurred, on January 1, 1914.

It was a peculiarly hard stroke of fate that he should not have had the opportunity of enjoying for a longer time those laboratories in which he had taken so great an interest, and to the completion of which he had for so long looked forward.

The following is a list of his papers published in the Society's *Proceedings* and *Transactions*:—

#### In the Proceedings, R.S.E.

1. On some Laboratory Arrangements. April 2, 1883. Vol. xii.
2. On Peroxides of Zinc, Cadmium, Magnesium, and Aluminium (with R. M. Morrison). Read July 5, 1880: published 1885 in vol. xiii.
3. On Papers by MM. Haas, Clève, and Lecoy de Boisbaudran, on the Production of Peroxides by means of Peroxide of Hydrogen. April 6, 1885. Vol. xiii.
4. The Action of Sodium Carbonate and Bromine on Solutions of Cobalt and Nickel Salts. February 17, 1890. (*Abstract.*) Vol. xvii.
5. Manganese Deposits in Marine Muds (along with R. Irvine). January 9, 1891. Vol. xviii.
6. On the Chemical Composition of Sea-water. July 3, 1893. Vol. xx.
7. On Photo-chemical Action. February 15, 1897. Vol. xxi.
8. Preliminary Note on a Characteristic of Certain Chemical Reactions. December 6, 1897. Vol. xxii.

9. On a Thermostat electrically heated and regulated (Title only). February 5, 1900. Vol. xxiii.
10. On certain Relations between the Electrical Conductivity and the Chemical Character of Solutions (Title only). May 6, 1901. Vol. xxiv.
11. Preliminary Note on the Conductivity of Concentrated Aqueous Solutions of Electrolytes. November 6, 1905. Vol. xxvi.
12. Eight papers, with others. June 22, 1908. Vol. xxviii.
13. On an Electrically Controlled Thermostat and other Apparatus for the Accurate Determination of the Electrolytic Conductivity of Highly Conducting Solutions (with G. E. Gibson). June 22, 1908. Vol. xxx.
14. On the Precipitation of Soluble Chlorides by Hydrochloric Acid (with R. B. Denison). August 20, 1910. Vol. xxx.

#### In the Transactions, R.S.E.

1. On the Relationship between Concentration and Electrolytic Conductivity in Concentrated Aqueous Solutions. May 11, 1905. Vol. xl.
2. The Significance of Maximum Specific Electrical Conductivity in Chemistry. Read July 13, 1908: published October 20, 1911. Vol. xlvi.



## APPENDIX.

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## LAWS OF THE SOCIETY,

*As revised October 26, 1908.*

[By the Charter of the Society (printed in the *Transactions*, vol. vi. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

### I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Title.  
Honorary Fellows.

### II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.\* Fellows may compound for these contributions on such terms as the Council may from time to time fix.

### III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment.

Payment to  
cease after  
25 years.

### IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

Fees of Non-  
Resident  
Ordinary  
Fellows.

Case of Fellows  
becoming Non-  
Resident.

\* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on January 3, 1831.

At the Meeting of the Society, on January 5, 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

Numbers unlimited.

The number of Ordinary Fellows shall be unlimited.

Fellows entitled to Transactions and Proceedings.

All Ordinary Fellows of the Society who are not in arrear of their Annual Contributions shall be entitled to receive, gratis, copies of the parts of the Transactions of the Society which shall be published subsequent to their admission, upon application, either personally or by an authorised agent, to the Librarian, provided they apply for them within five years of the date of publication of such parts.

Copies of the parts of the Proceedings shall be distributed to all Fellows of the Society, by post or otherwise, as soon as may be convenient after publication.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,\* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall be exhibited publicly in the Society's rooms for one month, after which it shall be considered by the Council. If the Candidate be approved by the Council, notice of the day fixed for the election shall be given in the circulars of at least two Ordinary Meetings of the Society.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

\* "A. B., a gentleman well versed in science (*or Polite Literature, as the case may be*), being to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

## V.

## VI.

## VII.

## VIII.

## IX.

## X.

## XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to Royal Personages, the limitation of numbers specified in Law X.

## XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below\*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one Meeting, and printed in the circulars for Two Ordinary Meetings of the Society, previous to the day of election.

## XIII.

The election of Ordinary Fellows shall take place only at one Afternoon Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

## XIV.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

## XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the Transactions of the Society, and shall superintend the printing of the same.

## XVI.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

\* We hereby recommend \_\_\_\_\_ for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

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## XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be Resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

## XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

## XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the election of Office-Bearers shall be held annually on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

## XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

## XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

## XXII.

Auditor.

At the Extraordinary Meeting in October, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

## XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

## XXIV.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

## XXV.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the hall, for the inspection of the Fellows.

## XXVI.

All articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint.

## XXVII.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

## XXVIII.

If, in the opinion of the Council of the Society, the conduct of any Fellow is unbefitting the position of a Member of a learned Society, or is injurious to the character and interests of this Society, the Council may request such Fellow to resign; and, if he fail to do so within one month of such request being addressed to him, the Council shall call a General Meeting of the Fellows of the Society to consider the matter; and, if a majority of the Fellows present at such Meeting agree to the expulsion of such Member, he shall be then and there expelled by the declaration of the Chairman of the said Meeting to that effect; and he shall thereafter cease to be a Fellow of the Society, and his name shall be erased from the Roll of Fellows, and he shall forfeit all right or claim in or to the property of the Society.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND  
GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1915–1916 for the “best communication on a scientific subject, communicated,\* in the first instance, to the Royal Society during the Sessions 1913–1914 and 1914–1915.” Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science ; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1914–1915, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.
2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1914.
3. The Competition is open to all men of science.
4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.
5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society.

\* For the purposes of this award the word “communicated” shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented \* to the Society during the Sessions 1912-13, 1913-14, whether they may have been given in with a view to the prize or not.

### III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate :

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1915-1916.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented \* to the Society during the two years preceding the fourth Monday in October 1915,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

### IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

\* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

**AWARDS OF THE KEITH, MAKDOUGALL - BRISBANE,  
NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.**

**I. KEITH PRIZE.**

1<sup>ST</sup> BIENNIAL PERIOD, 1827-29.—Dr BREWSTER, for his papers “on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals,” published in the Transactions of the Society.

2<sup>ND</sup> BIENNIAL PERIOD, 1829-31.—Dr BREWSTER, for his paper “on a New Analysis of Solar Light,” published in the Transactions of the Society.

3<sup>RD</sup> BIENNIAL PERIOD, 1831-33.—THOMAS GRAHAM, Esq., for his paper “on the Law of the Diffusion of Gases,” published in the Transactions of the Society.

4<sup>TH</sup> BIENNIAL PERIOD, 1833-35.—Professor J. D. FORBES, for his paper “on the Refraction and Polarization of Heat,” published in the Transactions of the Society.

5<sup>TH</sup> BIENNIAL PERIOD, 1835-37.—JOHN SCOTT RUSSELL, Esq., for his researches “on Hydrodynamics,” published in the Transactions of the Society.

6<sup>TH</sup> BIENNIAL PERIOD, 1837-39.—Mr JOHN SHAW, for his experiments “on the Development and Growth of the Salmon,” published in the Transactions of the Society.

7<sup>TH</sup> BIENNIAL PERIOD, 1839-41.—Not awarded.

8<sup>TH</sup> BIENNIAL PERIOD, 1841-1843.—Professor JAMES DAVID FORBES, for his papers “on Glaciers,” published in the Proceedings of the Society.

9<sup>TH</sup> BIENNIAL PERIOD, 1843-45.—Not awarded.

10<sup>TH</sup> BIENNIAL PERIOD, 1845-47.—General Sir THOMAS BRISBANE, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11<sup>TH</sup> BIENNIAL PERIOD, 1847-49.—Not awarded.

12<sup>TH</sup> BIENNIAL PERIOD, 1849-51.—Professor KELLAND, for his papers “on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations,” published in the Transactions of the Society.

13<sup>TH</sup> BIENNIAL PERIOD, 1851-53.—W. J. MACQUORN RANKINE, Esq., for his series of papers “on the Mechanical Action of Heat,” published in the Transactions of the Society.

14<sup>TH</sup> BIENNIAL PERIOD, 1853-55.—Dr THOMAS ANDERSON, for his papers “on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances,” published in the Transactions of the Society.

15<sup>TH</sup> BIENNIAL PERIOD, 1855-57.—Professor BOOLE, for his Memoir “on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments,” published in the Transactions of the Society.

16<sup>TH</sup> BIENNIAL PERIOD, 1857-59.—Not awarded.

17<sup>TH</sup> BIENNIAL PERIOD, 1859-61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers “on the Horizontal Force of the Earth’s Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally,” published in the Transactions of the Society.

18<sup>TH</sup> BIENNIAL PERIOD, 1861-63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication “on some Kinematical and Dynamical Theorems.”

19<sup>TH</sup> BIENNIAL PERIOD, 1863-65.—Principal FORBES, St Andrews, for his “Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars,” published in the Transactions of the Society.

20<sup>TH</sup> BIENNIAL PERIOD, 1865-67.—Professor C. PIAZZI SMYTH, for his paper “on Recent Measures at the Great Pyramid,” published in the Transactions of the Society.

21<sup>ST</sup> BIENNIAL PERIOD, 1867-69.—Professor P. G. TAIT, for his paper “on the Rotation of a Rigid Body about a Fixed Point,” published in the Transactions of the Society.

22<sup>ND</sup> BIENNIAL PERIOD, 1869-71.—Professor CLERK MAXWELL, for his paper “on Figures, Frames, and Diagrams of Forces,” published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871-73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873-75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875-77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877-79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877-78.

27TH BIENNIAL PERIOD, 1879-81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881-83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883-85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885-87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887-89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889-91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891-93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv., xxxvi., and xxxvii. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893-95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895-97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897-99.—Dr JAMES BURGESS, for his paper "on the Definite Integral  $\frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$ , with extended Tables of Values," printed in vol. xxxix. of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899-1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901-03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., &c., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I., II., likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903-05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905-07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907-09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909-11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911-13.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

## II. MAKDOUGALL-BRISBANE PRIZE.

1<sup>ST</sup> BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2<sup>ND</sup> BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his “Memoir of the Life and Writings of Dr Robert Whytt,” published in the Transactions of the Society.

3<sup>RD</sup> BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. “Icarus,” for his paper “on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology,” published in the Transactions of the Society.

4<sup>TH</sup> BIENNIAL PERIOD, 1864–66.—Not awarded.

5<sup>TH</sup> BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper “on the Connection between Chemical Constitution and Physiological Action,” published in the Transactions of the Society.

6<sup>TH</sup> BIENNIAL PERIOD, 1868–70.—Not awarded.

7<sup>TH</sup> BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper “on the Homological Relations of the Cœlenterata,” published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8<sup>TH</sup> BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper “on the Germ Theory of Putrefaction and the Fermentive Changes,” communicated to the Society, 7th April 1873.

9<sup>TH</sup> BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper “on the Diurnal Oscillation of the Barometer,” published in the Transactions of the Society.

10<sup>TH</sup> BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper “on the Old Red Sandstone of Western Europe,” published in the Transactions of the Society.

11<sup>TH</sup> BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper “on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination,” published in the Transactions of the Society.

12<sup>TH</sup> BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his “Contributions to the Geology of the North-West of Europe,” including his paper “on the Geology of the Faroes,” published in the Transactions of the Society.

13<sup>TH</sup> BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper “on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor,” and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

14<sup>TH</sup> BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers “On the Drainage Areas of Continents, and Ocean Deposits,” “The Rainfall of the Globe, and Discharge of Rivers,” “The Height of the Land and Depth of the Ocean,” and “The Distribution of Temperature in the Scottish Lochs as affected by the Wind.”

15<sup>TH</sup> BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled “History of Volcanic Action during the Tertiary Period in the British Isles,” published in the Transactions of the Society.

16<sup>TH</sup> BIENNIAL PERIOD, 1889–90.—Dr LUDWIG BECKER, for his paper on “The Solar Spectrum at Medium and Low Altitudes,” printed in vol. xxxvi. Part I. of the Society’s Transactions.

17<sup>TH</sup> BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on “The Physical Conditions of the Clyde Sea Area,” Part I. being already published in vol. xxxvi. of the Society’s Transactions.

18<sup>TH</sup> BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx. pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19<sup>TH</sup> BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M’KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society’s publications.

20<sup>TH</sup> BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21<sup>ST</sup> BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled “Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland,” printed in vol. xxxix. of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900-02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl. of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902-04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli. of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904-06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliosimeter," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906-08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908-10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27TH BIENNIAL PERIOD, 1910-12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

### III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856-59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliate Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859-61.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862-65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865-68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868-71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Foetal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871-74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874-77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877-80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880-83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883-86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palaeontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886-89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889-92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892-95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895-98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898-1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix. of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901-04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904-07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907-09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909-11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award : see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911-13.—Dr W. S. BRUCE, in recognition of the scientific results of his Arctic and Antarctic explorations.

#### IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884-87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887-90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890-93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893-96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896-1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900-04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures : his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904-08.—Professor GEORGE CHRYSSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4TH QUADRENNIAL PERIOD, 1908-12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

**PROCEEDINGS OF THE STATUTORY GENERAL MEETING**  
**Beginning the 131st Session, 1913-1914.**

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 27, 1913, at 4.30 p.m.

Principal SIR WM. TURNER, K.C.B., President, in the Chair.

Before the ordinary business of the Meeting commenced, Professor CRUM BROWN, in the name of LADY KELVIN, presented to the Society a Marble Bust of the late LORD KELVIN, and the President, SIR WM. TURNER, received the Bust in the name of the Society. (For account of the Ceremony of the Presentation and Reception, see *Proceedings*, vol. xxxiv, pp. 1-3.)

The Minutes of the last Statutory Meeting, October 28, 1912, were read, approved, and signed.

On the motion of Dr HORNE, seconded by Mr JAS. CURRIE, Mr JOHN ALISON and Mr J. B. CLARK were appointed Scrutineers, and the ballot for the New Council commenced.

The TREASURER's Accounts for the past year, 1912-1913, were submitted; these, with the Auditors' Report, were read, and approved.

The Scrutineers reported that the following Council had been duly elected:—

Professor JAMES GEIKIE, LL.D., D.C.L., F.R.S., F.G.S., President.	} Vice-Presidents.
JAMES BURGESS, C.I.E., LL.D., M.R.A.S.,	
Professor T. HUDSON BEARE, M.Inst.C.E.,	
Professor F. O. BOWER, M.A., D.Sc., F.R.S.,	
Professor SIR THOMAS R. FRASER, M.D., LL.D., F.R.C.P.E., F.R.S.,	
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S.,	} Secretaries to Ordinary Meetings.
Professor SIR E. A. SCHÄFER, M.R.C.S., LL.D., F.R.S.,	
CARGILL G. KNOTT, D.Sc., General Secretary.	
ROBERT KIDSTON, LL.D., F.R.S., F.G.S.,	
Professor ARTHUR ROBINSON, M.D., M.R.C.S.,	} Meetings.
JAMES CURRIE, M.A., Treasurer.	
JOHN S. BLACK, M.A., LL.D., Curator of Library and Museum.	

**ORDINARY MEMBERS OF COUNCIL.**

Professor T. H. BRYCE, M.A., M.D.	Professor E. T. WHITTAKER, Sc.D., F.R.S.
WILLIAM ALLAN CARTER, M.Inst.C.E.	Principal A. P. LAURIE, M.A., D.Sc.
ANDREW WATT, M.A.	Professor J. GRAHAM KERR, M.A., F.R.S.
JAMES H. ASHWORTH, D.Sc.	LEONARD DOBBIN, Ph.D.
JAMES GORDON GRAY, D.Sc.	ERNEST MACLAGAN WEDDERBURN, M.A., LL.B.
Professor R. A. SAMPSION, M.A., D.Sc., F.R.S.	
Professor D'ARCY W. THOMPSON, C.B., B.A., F.L.S.	

Society's Representative on } WILLIAM ALLAN CARTER, M.Inst.C.E.  
 George Heriot's Trust,

On the motion of Professor F. O. BOWER, thanks were voted to the Scrutineers.  
 On the motion of Mr HEWAT, thanks were voted to the Auditors, Messrs LINDSAY, JAMIESON & HALDANE, and they were reappointed.  
 On the motion of Dr KNOTT, thanks were voted to the Treasurer, Mr JAMES CURRIE.

PROCEEDINGS OF THE ORDINARY MEETINGS,  
Session 1913-1914.

FIRST ORDINARY MEETING.

*Monday, November 3, 1913.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The President opened the Session with a short Address.

The following Communications were read:—

1. Atmospheric Electric Potential Results at Edinburgh during 1912. By G. A. CARSE, M.A., D.Sc., and G. SHEARER, M.A., B.Sc. (*With Lantern Illustrations.*)
2. Some Factorable Minors of a Compound Determinant. By Professor W. H. METZLER, A.B., Ph.D.
3. An Analytical Theory of the Equilibrium of an Isotropic Elastic Rod of Circular Section. By Dr JOHN DOUGALL. Communicated by Professor G. A. GIBSON, LL.D.

The following, nominated for Honorary Fellowship, were balloted for, and duly declared elected:—

AS BRITISH HONORARY FELLOWS:—

1. HORACE LAMB, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.
2. Sir WILLIAM TURNER THISELTON-DYER, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew.

AS FOREIGN HONORARY FELLOWS:—

1. GEORGE ELLERY HALE, F.M.R.S., Director of the Mount Wilson Solar Observatory (Carnegie Institute of Washington).
2. EMIL CLEMENT JUNGFLEISCH, Mem. Inst. Fr., Professor of Organic Chemistry in the College of France, Paris.
3. SANTIAGO RAMÓN Y CAJAL, F.M.R.S., Professor of Histology and Pathological Anatomy in the University of Madrid.
4. VITO VOLTERRA, F.M.R.S., Sc.D., Ph.D., Professor of Mathematics and Physics in the University of Rome.
5. CHARLES RENÉ ZEILLER, Mem. Inst. Fr., Professor of Plant Palaeontology in the National Superior School of Mines, Paris.

SECOND ORDINARY MEETING.

*Monday, November 17, 1913.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. On the Fossil Flora of the Staffordshire Coal Fields. Part III.—The Fossil Flora of the Westphalian Series of the South Staffordshire Coalfield. By Dr R. KIDSTON, F.R.S. (*With Lantern Illustrations.*)
2. *Sphaerostoma ovale* (*Conostoma ovale et intermedium*, Williamson), a Lower Carboniferous Oval from Pettycur, Fifeshire, Scotland. By Professor MARGARET J. BENSON, D.Sc. Communicated by Dr R. KIDSTON, F.R.S.
3. Studies on the Pharmacological Action of Tetra-alkyl-ammonium Compounds. I.—The Action of Tetra-methyl-ammonium Chloride. By Professor C. R. MARSHALL, M.D. (*With Lantern Illustrations.*)
4. The Theory of Bigradients from 1859-1880. By Dr THOMAS MUIR, F.R.S.

The following Candidate for Fellowship was balloted for, and duly declared elected:—  
EDWARD PHILIP HARRISON, Ph.D.

## THIRD ORDINARY MEETING.

*Monday, December 1, 1913.*

Professor J. Hudson Beare, B.Sc., M.Inst.C.E., Vice-President, in the Chair.

At the request of the Council the following Address was delivered :—

*Principia Atmospherica: A Study of the Circulation of the Atmosphere.* By W. N. SHAW, LL.D., Sc.D., F.R.S., Director of the Meteorological Office, London. (*With Lantern Illustrations.*)

The following Communications were also read :—

1. Observations on the Auditory Organ in the Cetacea. By Principal Sir WILLIAM TURNER, K.C.B.
2. Note on a Siliceous Sponge of the Order *Hexactinellida* from South Shetland. By Principal Sir WILLIAM TURNER, K.C.B.

The following Candidates for Fellowship were balloted for, and declared duly elected :—JOHN WILLIAM PARE, M.B., C.M. (Edin.), M.D., L.D.S. (Eng.), WILLIAM FRASER, WILLIAM BARRON COUTTS, M.A., B.S., ALFRED OSWALD, and JOHN EDWARD GEMMELL, M.B., C.M. (Edin.).

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## FOURTH ORDINARY MEETING.

*Monday, December 15, 1913.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. Obituary Notice of Dr R. M. Ferguson. By Dr A. E. SCOGAL, M.A.
2. Studies on the Pharmacological Action of Tetra-alkyl-ammonium Compounds. II.—The Action of Tetra-ethyl-ammonium Chloride. III.—The Action of Methyl-ethyl-ammonium Chlorides. By Professor C. R. MARSHALL, M.D. (*With Lantern Illustrations.*)
3. Enzymatic Peptolysis in Germinating Seeds.—Parts I. and II. By Miss DOROTHY COURT, B.Sc. Communicated by Professor E. WESTERGAARD, Ph.D.
4. The Kinetic Energy of Viscous Flow through a Circular Tube. By Professor A. H. GIBSON, D.Sc.
5. Polychæta of the Family Nereidæ collected by the Scottish National Antarctic Expedition. By L. N. G. RAMSAY, M.A., B.Sc. Communicated by Dr J. H. ASHWORTH.

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## FIFTH ORDINARY MEETING.

*Monday, January 19, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. The Place in Nature of the Tasmanian Aboriginal as deduced from a Study of his Calvaria. Part II.—His Relation to the Australian Aboriginal. By Professor R. J. A. BERRY and Dr A. W. D. ROBERTSON.
2. A Study of the Curvatures of the Tasmanian Aboriginal Cranium. By Mr L. W. G. BUCHNER. Communicated by Professor R. J. A. BERRY. (In the absence of Professor BERRY a brief account of the above two papers was given by Dr GERALD LEIGHTON.)
3. The Path of a Ray of Light in a Rotating Homogeneous and Isotropic Solid. By E. M. ANDERSON, M.A., B.Sc. Communicated by the GENERAL SECRETARY.
4. The Anatomy of a New Species of Bathydoris and the Affinities of the Genus: Scottish National Antarctic Expedition. By T. J. EVANS, M.A. Communicated by Dr J. H. ASHWORTH. (*With Lantern Illustrations.*)
5. On the Genus *Porponia* and related Genera: Scottish National Antarctic Expedition. By Professor OSKAR CARLGREN. Communicated by Dr W. S. BRUCE. (*With Lantern Illustrations.*)

The following Candidates for Fellowship were balloted for, and declared duly elected :—JOSEPH PEARSON, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon; SPENCER MORT, M.B., Ch.B., Medical Super-

intendent, Edmonton Infirmary, London, N.; and CHARLES GLOVER BARKLA, D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, Littledean, 34 Priestfield Road, Edinburgh.

Mr W. B. COUTTS, D.Sc., M.A., signed the Roll, and was duly admitted a Fellow of the Society.

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#### SIXTH ORDINARY MEETING.

*Monday, February 2, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

At the request of the Council the following Address was delivered :—

Notes on the Evolution of Antarctica. By T. W. EDGEWORTH DAVID, C.M.G., Hon. D.Sc. Oxon., F.R.S., Professor of Geology in the University of Sydney, N.S.W.; Scientific Officer with the Shackleton Expedition, 1907-09; Leader of Party which reached South Magnetic Pole. (*With Lantern Illustrations.*)

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#### SEVENTH ORDINARY MEETING.

*Monday, February 16, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. The Axial Inclination of Curves of Thermoelectric Force: A Case from the Thermolectrics of Strained Wires. By JOHN M'WHAN, M.A., Ph.D. Communicated by Professor ANDREW GRAY, LL.D., F.R.S.
2. Rupture Strains in Beams and Crane Hooks. By ANGUS R. FULTON, B.Sc., A.M.Inst.C.E. Communicated by Professor A. H. GIBSON, D.Sc.
3. A Description of the Systematic Anatomy of a Foetal Sea-Leopard (*Stenorhynchus leptonyx*), with Remarks upon the Microscopic Anatomy of some of the Organs. By HAROLD A. HAIG, M.B., B.S., M.R.C.S. Communicated by Professor ARTHUR ROBINSON, M.D., M.R.C.S. (*With Lantern Illustrations.*)

The following Candidates for Fellowship were balloted for, and declared duly elected :—ROBERT JOHN HARVEY-GIBSON, M.A., F.L.S., Professor of Botany, University of Liverpool, 22 Falkner Square, Liverpool, and JOHN NOBLE JACK, Professor of Agriculture to the County Council of Sussex, Kingscote, The Avenue, Lewes, Sussex.

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#### EIGHTH ORDINARY MEETING.

*Monday, March 2, 1914.*

Professor Sir E. A. Schäfer, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. The Electrolytic Treatment of Lead Poisoning. By Professor Sir THOMAS OLIVER, M.D., LL.D., F.R.C.P., and Mr T. M. CLAGUE. (*With a Demonstration and with Lantern Illustrations.*)
2. The Aborigines of Tasmania. Part III.—The Hair of the Head, compared with that of other Ulotrichi, and with Australians and Polynesians. By Principal Sir WILLIAM TURNER, K.C.B.

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#### NINTH ORDINARY MEETING.

*Monday, March 16, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. Stalk-eyed Crustacea *Malacostraca* of the Scottish National Antarctic Expedition. By the Rev. T. R. R. STEBBING, M.A., F.R.S. Communicated by Dr J. H. ASHWORTH.

2. Note on the Atmospheric Electrical Potential Gradient in Industrial Districts. By Mr DANIEL W. STEUART and Mr INGVAR JÖRGENSEN. Communicated by JAMES A. S. WATSON, B.Sc.

3. A Chemical Examination of the Organic Matter in Oil-Shales. By JOHN B. ROBERTSON, M.A., B.Sc. Communicated by Dr J. S. FLETT, F.R.S. (*With Lantern Illustrations.*)

Mr WILLIAM FRASER signed the Roll, and was duly admitted a Fellow of the Society.

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#### TENTH ORDINARY MEETING.

*Monday, May 4, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. Description and Exhibition of a Four-Dimensional Model. By Dr D. M. Y. SOMMERVILLE.
2. Changes of Electrical Resistance accompanying Longitudinal and Transverse Magnetisations in Iron and Steel. By Dr C. G. KNOTT.
3. Rocks from Gough Island, S. Atlantic: Scottish National Antarctic Expedition. By Dr ROBERT CAMPBELL. Communicated by the PRESIDENT. (*With Lantern Illustrations.*)

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#### ELEVENTH ORDINARY MEETING.

*Monday, May 25, 1914.*

Dr B. N. Peach, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. On the Inheritance of Certain Characters of the Wool of Sheep. By A. D. DARBISHIRE, M.A., and Mr M. W. GRAY. (*With Lantern Illustrations.*)
2. On a New Species of *Sclerocheilus*, with a Revision of the Genus. By Dr J. H. ASHWORTH.

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#### TWELFTH ORDINARY MEETING.

*Monday, June 1, 1914.*

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The Council awarded :—

1. THE NEILL PRIZE for the biennial period 1911-1912, 1912-1913 to WILLIAM SPEIRS BRUCE, LL.D., in recognition of the scientific results of his Arctic and Antarctic explorations.
2. THE KEITH PRIZE for the biennial period 1911-1912, 1912-1913 to Mr JAMES RUSSELL, for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the *Proceedings* and *Transactions* of the Society, the last paper having been issued within the period.

The above prizes will be presented at the Ordinary Meeting on July 6, 1914.

The following Communications were read :—

1. The Analytical Study of the Mechanism of Writing. By JAMES DREVER, M.A., B.Sc. Communicated by Dr ALEXANDER MORGAN. (*With Exhibition of Apparatus and Lantern Illustrations.*)
2. The Pinna-Trace in the Ferns. By R. C. DAVIE, M.A., B.Sc. Communicated by Professor ISAAC BAYLEY BALFOUR, F.R.S. (*With Lantern Illustrations.*)
3. Abnormal Echinoids in the Collection of the Royal Scottish Museum. By Dr JAMES RITCHIE and JAMES A. TODD, M.A., B.Sc. Communicated by WILLIAM EAGLE CLARKE. (*With Lantern Illustrations.*)

The following signed the roll, and were admitted Fellows of the Society: Dr J. W. PARE, Dr ALEX. C. CUMMING, Mr JAMES B. RITCHIE, Mr BASIL A. PILKINGTON, and Mr THEODORE E. SALVESEN.

## THIRTEENTH ORDINARY MEETING.

Monday, June 15, 1914.

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. Obituary Notice of ALBERT C. L. G. GÜNTHER, M.A., M.D., Ph.D., F.R.S. By Professor W. C. M'INTOSH, F.R.S.
2. The Fossil Osmundaceæ, Part V. By Dr R. KIDSTON, F.R.S., F.G.S., and Professor D. T. Gwynne-VAUGHAN, M.A. (*With Lantern Illustrations.*)
3. The Hall and the Transverse Thermomagnetic Effects and their Temperature Coefficients. By F. UNWIN, M.Sc. Communicated by Professor F. G. BAILY. (*With Lantern Illustrations.*)
4. Some Factorable Continuants. By Professor W. H. METZLER, Ph.D.
5. Atlantic Sponges collected by the Scottish National Antarctic Expedition. By Miss JANE STEPHENS. Communicated by Dr W. S. BRUCE.

The following Candidates for Fellowship were balloted for, and duly declared elected :—  
 ALEXANDER GIBB, A.M. Inst.C.E., and ROBERT DURWARD CLARKSON, B.Sc., M.D., F.R.C.P.E.  
 Mr PETER RAMSAY signed the Roll, and was duly admitted a Fellow of the Society.

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## FOURTEENTH ORDINARY MEETING.

Monday, July 6, 1914.

Professor James Geikie, LL.D., D.C.L., F.R.S., F.G.S., President, in the Chair.

The following Prizes were presented :—

1. THE NEILL PRIZE for the biennial period 1911–1912, 1912–1913 to WILLIAM SPEIRS BRUCE, LL.D., in recognition of the scientific results of his Arctic and Antarctic explorations.
2. THE KEITH PRIZE for the biennial period 1911–1912, 1912–1913 to Mr JAMES RUSSELL, for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the *Proceedings* and *Transactions* of the Society, the last paper having been issued within the period.

## NEILL PRIZE AWARD, 1911–13.

The Neill Prize for the period 1911–1913 is awarded to Dr W. S. Bruce, a distinguished traveller and naturalist. Dr Bruce, as the Fellows of the Royal Society of Edinburgh well know, has spent his life in the exploration of Arctic and Antarctic seas and lands. He began his work more than twenty years ago by a voyage to certain islands of the Antarctic Ocean; in recent years he has especially explored the Archipelago of Spitzbergen; and in 1902 to 1904 he led the Scottish National Antarctic Expedition through its adventurous voyage to a successful issue.

Dr Bruce's many voyages, and especially the expedition of the *Scotia*, have led to the advancement of knowledge in many departments of science. With the science of geography itself, with the actual survey of new lands and seas, other Societies than this are peculiarly concerned. But as far as living memory goes back, the Royal Society of Edinburgh has been proud to encourage, with all its sympathy, and to help with all the means in its power, those discoveries, biological and physical, which follow and reward the explorations of the scientific traveller.

In the *Transactions* of our Society there have appeared, during several recent years, a long series of papers based on the results of the *Scotia* Expedition; in which papers new organisms from almost every group of the Animal Kingdom have been described, and in which important questions of physics, of meteorology, and of oceanography have been discussed. These many writings, by many hands, bear witness to the wisdom with which a great expedition was planned, to the enthusiasm with which its leader animated his band of men, and to the foresight and untiring industry which watched for and laid hold of the opportunities of discovery.

## KEITH PRIZE AWARD, 1911–13.

Mr Russell's work, published mainly in our *Transactions*, dates from the early years of the century. In his first paper he discussed the problem of magnetic shielding in hollow iron cylinders, distinguishing the cases in which a transverse field existed alone, or had superposed upon it either a circular or a longitudinal field. This work was experimental, but its results were compared with those of the usual approximate theory. An investigation was also given of the inductions produced by mutually perpendicular fields, and the effects were co-ordinated for the first time and shown to be consistent with the results of the theory of molecular magnetism.

Phenomena were discussed with regard to which considerable difference of opinion existed, and decisive results were obtained.

Mr Russell's work was next directed to an investigation of the effect of an oscillating magnetic field upon iron magnetised by a non-oscillating-field, and a careful discrimination was made, for the first time, between the cases in which the former field was first established, or conversely. Co-directed and perpendicularly directed fields were used, in the latter case possible disturbances due to the establishment of the oscillating field by means of a current flowing in the iron itself being for the first time avoided. Interesting and important results were obtained, and were applied to the formation of new views regarding the action of certain magnetic detectors used in wireless telegraphy.

The preceding investigation led to a similar one in which mechanical vibrations were employed, and Mr Russell's anticipation that similar results would be obtained was verified. Iron, nickel, and steel were investigated, both in the annealed and the tempered conditions, and general conclusions were obtained, while other interesting problems were suggested.

One fact—the dependence of the neutral point in a hysteresis loop upon the intensity of the vibrational disturbance—was further investigated in a subsequent paper, and the apparently discordant results of other observers were harmonised. In a connected investigation on the effect of load and vibration upon magnetism in nickel, Mr Russell supplemented work of Ewing and Chree, upon iron and cobalt respectively, published in the *Philosophical Transactions*, and established the existence of a "cyclic" Villari reversal.

Mr Russell is an experimenter of great skill and resource. A visit to his private laboratory reveals how one man can do the work of three. And he is an accurate and acute reasoner. In his lengthy series of experimental inquiries, he has co-ordinated old, disconnected, or even seemingly discordant results, and has established new facts and new views. Throughout his whole work his aim has been to co-ordinate and explain highly complicated phenomena as the very direct results of the ideas of the molecular theory of magnetism based upon a simple view, given in his first paper, of the structural condition of a magnetic metal demagnetised by decreasing reversals. This is most noticeable in his latest paper which was communicated within the period of the present award. When the theory of magnetism in a medium crystallised on the cubic system is extended to an averagely random collocation of crystals, Mr Russell's work will, with other work, form a touchstone.

The following Communications were read :—

1. Obituary Notice of JOHN STURGEON MACKAY, M.A., LL.D. By Dr GEORGE PHILIP, George Watson's College.
2. Temperature Observations in Loch Earn.—Part II. By E. M. WEDDERBURN, D.Sc., and A. W. YOUNG, M.A. (*With Lantern Illustrations.*)
3. Contributions to the Geology of South Georgia. By D. FERGUSON, M.I.M.E., with reports based on his collections by Professor J. W. GREGORY, D.Sc., F.R.S., and G. W. TYRRELL, A.R.C.Sc., F.G.S. (*With Lantern Illustrations.*)

The following Candidates for Fellowship were balloted for, and duly declared elected :—

ALFRED FRANK TREDGOLD, L.R.C.P., M.R.C.S., Hon. Consulting Physician to National Association for the Feeble-minded; FRANCIS JOHN LEWIS, D.Sc., F.L.S., Professor of Biology, University of Alberta; ARCHIBALD M'KENDRICK, F.R.C.S.E., D.P.H., L.D.S.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING  
Ending the 131st Session, 1913-1914.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 26, 1914, at 4.30 p.m.,

Professor JAMES GEIKIE, President, in the Chair,

the Minutes of the last Statutory Meeting, October 27, 1913, were read, approved, and signed.

On the motion of Dr KNOTT, seconded by Dr HORNE, Dr J. R. MILNE and Mr A. G. BURGESS were appointed Scrutineers, and the ballot for the New Council commenced.

The General Secretary announced that the Council had granted leave of absence to Mr G. A. STEWART, Librarian, and Mr W. J. BEATON, Assistant Librarian, so as to enable them to join His Majesty's forces. The Council had also felt it advisable to close the Society's Rooms on Saturdays at one o'clock.

The TREASURER's Accounts for the past year, 1913-1914, were submitted.

Professor BOWER moved the approval of the Treasurer's Report, and also votes of thanks to the Treasurer and the Auditors, who were reappointed. This was agreed to.

The Scrutineers reported that the following Council had been duly elected :—

Professor JAMES GEIKIE, LL.D., D.C.L., F.R.S., F.G.S., President.	}	Vice-Presidents.
Professor T. HUDSON BEARE, M.Inst.C.E.,		
Professor F. O. BOWER, M.A., D.Sc., F.R.S.,		
Professor SIR THOMAS R. FRASER, M.D., LL.D., F.R.C.P.E., F.R.S.,		
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S.,		
Professor SIR E. A. SCHÄFER, M.R.C.S., LL.D., F.R.S.,		
The Right Hon. SIR J. H. A. MACDONALD, K.C.B., P.C., LL.D., D.L., F.R.S., M.Inst.E.E.,		
CARGILL G. KNOTT, D.Sc., General Secretary.		
ROBERT KIDSTON, LL.D., F.R.S., F.G.S.,	Secretaries to Ordinary Meetings.	
Professor ARTHUR ROBINSON, M.D., M.R.C.S.,		
JAMES CURRIE, M.A., Treasurer,		
JOHN S. BLACK, M.A., LL.D., Curator of Library and Museum.		

ORDINARY MEMBERS OF COUNCIL.

JAMES GORDON GRAY, D.Sc.	LEONARD DOBBIN, Ph.D.
Professor R. A. SAMFSON, M.A., D.Sc., F.R.S.	ERNEST MACLAGAN WEDDERBURN, M.A., LL.B.
Professor D'ARCY W. THOMPSON, C.B., B.A., F.L.S.	W. B. BLAIKIE, LL.D.
Professor E. T. WHITTAKER, Sc.D., F.R.S.	JOHN HORNE, LL.D., F.S.S., F.G.S.
Principal A. P. LAURIE, M.A., D.Sc.	R. STEWART MACDOUGALL, M.A., D.Sc.
Professor J. GRAHAM KERR, M.A., F.R.S.	W. A. TAIT, D.Sc., M.Inst.C.E.

Society's Representative on } WILLIAM ALLAN CARTER, M.Inst.C.E.  
George Heriot's Trust, }

**A B S T R A C T**  
OF  
THE ACCOUNTS OF JAMES CURRIE, ESQ.  
*As Treasurer of the Royal Society of Edinburgh.*

SESSION 1913-1914.

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**I. ACCOUNT OF THE GENERAL FUND.**

CHARGE.

1. Arrears of Contributions at 1st October 1913 . . . . .	£106 1 0
2. Contributions for present Session:—	
1. 141 Fellows at £2, 2s. each . . . . .	£296 2 0
127 Fellows at £3, 3s. each . . . . .	400 1 0
	<hr/>
	£696 3 0
<i>Less</i> —Subscription for present Session, included in 1913 Accounts . . . . .	3 3 0
	<hr/>
	£693 0 0
2. Fees of Admission and Contributions of eleven new Resident Fellows at £5, 5s. each . . . . .	57 15 0
3. Fees of Admission of eight new Non-Resident Fellows at £26, 5s. each . . . . .	210 0 0
4. Commutation Fees in lieu of future Contributions of two Fellows . . . . .	45 3 0
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3. Contribution for 1914-1915 paid in advance . . . . .	1005 18 0
4. Interest received—	
Interest, less Tax £23, 4s. 10 <sup>6</sup> d. . . . .	£369 1 7
Annuity from Edinburgh and District Water Trust, less Tax £3, 2s. 2d. . . . .	49 7 10
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	418 9 5
5. Transactions and Proceedings sold . . . . .	140 1 1
6. Annual Grant from Government. . . . .	600 0 0
7. Income Tax repaid for year to 5th April 1914 . . . . .	25 18 11
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<b>Amount of the Charge</b>	<b>£2298 10 5</b>

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DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING :—	
Inhabited House Duty . . . . .	£ 0 6 3
Insurance . . . . .	9 0 9
Coal, etc., to 24th August 1914 . . . . .	20 18 6
Gas to 12th March 1914 . . . . .	0 4 10
Electric Light to 18th September 1914 . . . . .	7 19 8
Water 1913-14 . . . . .	4 4 0
	<hr/>
	£42 14 0
Carry forward . . . . .	<hr/>
	£42 14 0

	Brought forward	£	42	14	0
2. SALARIES :—					
General Secretary . . . . .	£100	0	0		
Librarian . . . . .	120	0	0		
Assistant Librarian . . . . .	49	3	4		
Office Keeper . . . . .	86	14	0		
Treasurer's Clerk . . . . .	25	0	0		
	—————				
			380	17	4
3. EXPENSES OF TRANSACTIONS :—					
Neill & Co., Ltd., Printers . . . . .	£430	10	11		
Do. (for illustrations) . . . . .	2	16	0		
M'Farlane & Erskine, Lithographers . . . . .	109	0	6		
Hislop & Day, Engravers . . . . .	6	18	2		
Orrick & Son, Bookbinders . . . . .	108	11	0		
John Fowler & Co., Engravers . . . . .	8	9	0		
Alex. Ritchie & Son, Lithographers . . . . .	44	3	0		
	—————				
			710	8	7
4. EXPENSES OF PROCEEDINGS :—					
Neill & Co., Ltd., Printers . . . . .	£281	13	7		
Do. (for illustrations) . . . . .	1	1	0		
Hislop & Day, Engravers . . . . .	12	4	9		
Alex. Ritchie & Son, Lithographers . . . . .	9	0	0		
M'Farlane & Erskine do. . . . .	2	10	0		
	—————				
			306	9	4
5. BOOKS, PERIODICALS, NEWSPAPERS, ETC. :—					
Otto Schulze & Co., Booksellers . . . . .	£109	15	8		
James Thin, do. . . . .	64	0	9		
R. Grant & Son, do. . . . .	7	6	4		
W. Green & Son, Ltd., do. . . . .	1	8	6		
International Catalogue of Scientific Literature . . . . .	17	0	0		
Robertson & Scott, News Agents . . . . .	2	0	6		
Egypt Exploration Funds Subscription . . . . .	4	4	0		
Ray Society do. . . . .	1	1	0		
Palaeontographical Society do. . . . .	1	1	0		
Orrick & Son, Bookbinders . . . . .	24	6	6		
T. & A. Constable, Printers . . . . .	0	16	0		
	—————				
			233	0	3
6. EXPENSES IN CONNECTION WITH NAPIER TERCENTENARY RECEPTION :—					
E. Sawers, Purveyor . . . . .	£15	2	4		
H. Dambmann . . . . .	6	5	0		
Gillies & Wright, Joiners . . . . .	5	7	10		
A. Coutie & Son, do. . . . .	4	10	0		
G. Waterston & Sons, Ltd., Stationers . . . . .	3	18	6		
Tait & Francis, Florists . . . . .	1	10	0		
Attendants, Extra Cleaning, Posts, etc. . . . .	3	12	6		
	—————				
			40	6	2
7. OTHER PAYMENTS :—					
Neill & Co., Ltd., Printers . . . . .	£68	5	0		
E. Sawers, Purveyor . . . . .	35	17	10		
S. Duncan, Tailor (uniforms) . . . . .	11	18	0		
Lantern Exhibitions, etc., at Lectures . . . . .	10	10	0		
Lindsay, Jamieson & Haldane, C.A., Auditors . . . . .	6	6	0		
Post Office Telephone Rent . . . . .	10	0	0		
A. Cowan & Sons, Ltd. . . . .	10	9	6		
G. Waterston & Sons, Ltd. . . . .	6	16	6		
Gillies & Wright, Joiners . . . . .	21	16	8		
R. Graham, Slater . . . . .	8	5	4		
Mackenzie & Moncur, Ltd. . . . .	2	16	3		
Oliver Typewriter Co., Ltd. . . . .	24	6	0		
Burn Bros., Plumbers . . . . .	2	7	3		
Petty Expenses, Postages, Carriage, etc. . . . .	89	8	2		
	—————				
			309	2	6
8. INTEREST PAID ON BORROWED MONEY :—					
Makerstoun Magnetic Meteorological Observation Fund . . . . .	5	5	2		
Carry forward . . . . .	£2028	3	4		

	Brought forward	£2028 3 4
9. IRRECOVERABLE ARREARS of Contributions written off		2 2 0
10. ARREARS of CONTRIBUTIONS outstanding at 1st October 1914:—		
Present Session	£59 17 0	
Previous Sessions	61 19 0	
		121 16 0
	<b>Amount of the Discharge</b>	<b>£2152 1 4</b>
<b>Amount of the Charge</b>		<b>£2298 10 5</b>
<b>Amount of the Discharge</b>		<b>2152 1 4</b>
<b>Excess of Receipts over Payments for 1913-1914</b>		<b>£146 9 1</b>
Deduct Floating Balance due by the Society at 1st October 1913		123 16 2
<b>Floating Balance in favour of the Society at 1st October 1914</b>		<b>£22 12 11</b>
<i>Being</i> —		
Balance due by Union Bank of Scotland, Ltd., on Account		
Current	£237 5 1	
Balance in hands of Librarian	6 6 6	
		£243 11 7
Deduct Loan from the Makerstoun Magnetic Observation Fund		220 18 8
		<b>£22 12 11</b>

**II. ACCOUNT OF THE KEITH FUND***To 1st October 1914.***CHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913		£62 10 7
2. INTEREST RECEIVED:—		
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1914, less Tax £1, 11s. 11d.	£25 6 3	
On £211, 4s. North British Railway Company 3 per cent. Lien Stock for year to 30th June 1914, less Tax 7s. 7d.	5 19 1	31 5 4
3. INCOME TAX repaid for year to 5th April 1914		1 18 8
		<b>£95 14 7</b>

**DISCHARGE.**

1. James Russell—Money Portion of Prize 1911-13		£49 19 1
2. Alexander Kirkwood & Son, Engravers, for Gold Medal		16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1914		29 15 6
		<b>£95 14 7</b>

**III. ACCOUNT OF THE NEILL FUND***To 1st October 1914.***CHARGE.**

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913		£49 3 0
2. INTEREST RECEIVED:—		
On £355 London, Chatham and Dover Railway 4½ per cent. Arbitration Debenture Stock for year to 30th June 1914, less Tax 19s. 4d.		15 0 2
3. INCOME TAX repaid for year to 5th April 1914		0 18 8
		<b>£65 1 10</b>

## DISCHARGE.

1. Dr Wm. S. Bruce—Money Portion of Prize 1911-13 . . . . .	£15 19 0
2. Alexander Kirkwood & Son, Engravers, for Gold Medal . . . . .	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1914 . . . . .	33 2 10
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	£65 1 10

**IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND***To 1st October 1914.*

## CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913 . . . . .	£165 19 1
2. INTEREST received:— On £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 for year to 30th June 1914, less Tax 17s. 4d. . . . .	13 14 8
3. INCOME TAX repaid for year to 5th April 1914 . . . . .	0 17 0
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	£180 10 9

## DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1914 . . . . .	£180 10 9
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**V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL  
OBSERVATION FUND***To 1st October 1914.*

## CHARGE.

1. BALANCE due by General Fund at 1st October 1913 . . . . .	£220 13 6
2. INTEREST received on Balances due by General Fund at Deposit Receipt Rates to 1st October 1914 . . . . .	5 5 2
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	£225 18 8

## DISCHARGE.

1. Donation to the Funds of the Napier Tercentenary Celebration . . . . .	£5 0 0
2. BALANCE due by General Fund at 1st October 1914 . . . . .	220 18 8
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	£225 18 8

**VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND***To 1st October 1914.*

(Instituted by Dr R. H. GUNNING of Edinburgh and Rio de Janeiro.)

## CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913 . . . . .	£41 15 4
2. INTEREST received on £1000 North British Railway Company 3 per cent. Consolidated Lien Stock for year to 30th June 1914, less Tax £1, 16s. 2d. . . . .	28 3 10
3. INCOME TAX repaid for year to 5th April 1914 . . . . .	1 15 0
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	£71 14 2

## DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1914 . . . . .	<u>£71 14 2</u>
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**STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH**

*As at 1st October 1914.*

**1. GENERAL FUND—**

1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at 75 per cent., the selling price at 1st October 1914 . . . . .	£1567 17 0
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at 75 per cent., do. . . . .	6389 15 8
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 154 per cent., do. . . . .	1347 10 0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at 100½ per cent., do. . . . .	1820 1 1
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 106½ per cent., do. . . . .	37 5 6
6. Arrears of Contributions, as per preceding Abstract of Accounts . . . . .	121 16 0
	<u>£11,284 5 3</u>

Add Floating Balance in favour of the Society, as per preceding Abstract of Accounts . . . . .

22 12 11
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AMOUNT . . . . .	<u>£11,306 18 2</u>
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Exclusive of Library, Museum, Pictures, etc., Furniture of the Society's Rooms at George Street, Edinburgh.

**2. KEITH FUND—**

1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at 75 per cent., the selling price at 1st October 1914 . . . . .	£672 14 4
2. £211, 4s. three per cent. Lien Stock of do. at 75 per cent., do. . . . .	158 8 0
3. Balance due by Union Bank of Scotland, Ltd., on Account Current . . . . .	29 15 6
AMOUNT . . . . .	<u>£860 17 10</u>

**3. NEILL FUND—**

1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 106½ per cent., the selling price at 1st October 1914 . . . . .	£378 1 6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . . . . .	33 2 10
AMOUNT . . . . .	<u>£411 4 4</u>

**4. MAKDOUGALL-BRISBANE FUND—**

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at 94½ per cent., the selling price at 1st October 1914 . . . . .	£344 18 6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . . . . .	180 10 9
AMOUNT . . . . .	<u>£525 9 3</u>

**5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—**

Balance due by General Fund at 1st October 1914 . . . . .

<u>£220 18 8</u>
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6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—	
1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at 75 per cent., the selling price at 1st October 1914 . . .	£750 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . . .	71 14 2
AMOUNT . . .	<u>£821 14 2</u>

EDINBURGH, 15th October 1914.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1913-1914, and have found them to be correct. The securities of the various Investments at 1st October 1914, as noted in the foregoing Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE, C.A.,  
*Auditors.*

## THE COUNCIL OF THE SOCIETY,

*January 1915.*

## PRESIDENT.

JAMES GEIKIE, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh.

## VICE-PRESIDENTS.

T. HUDSON BEARE, M.Inst.C.E., Professor of Engineering in the University of Edinburgh.  
 FREDERICK O. BOWER, M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow.  
 SIR THOMAS R. FRASER, M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh.  
 BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Palaeontologist of the Geological Survey of Scotland.  
 SIR EDWARD ALBERT SCHÄFER, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh.  
 THE RIGHT HON. SIR J. H. A. MACDONALD, P.C., K.C., K.C.B., F.R.S., M.Inst.E.E., Lord President of the Second Division of the Court of Session.

## GENERAL SECRETARY.

CARGILL G. KNOTT, D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh.

## SECRETARIES TO ORDINARY MEETINGS.

ROBERT KIDSTON, LL.D., F.R.S., F.G.S.  
 ARTHUR ROBINSON, M.D., M.R.C.S., Professor of Anatomy in the University of Edinburgh.

## TREASURER.

JAMES CURRIE, M.A.

## CURATOR OF LIBRARY AND MUSEUM.

JOHN SUTHERLAND BLACK, M.A., LL.D.

## COUNCILLORS.

JAMES GORDON GRAY, D.Sc., Lecturer on Physics in the University of Glasgow. RALPH A. SAMPSON, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, and Professor of Astronomy in the University of Edinburgh. D'ARCY W. THOMPSON, C.B., B.A., F.L.S., Professor of Natural History in the University College, Dundee. EDMUND T. WHITTAKER, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh. A. P. LAURIE, M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh.	JOHN GRAHAM KERR, M.A., F.R.S., Professor of Zoology in the University of Glasgow. LEONARD DOBBIN, Ph.D., Lecturer on Chemistry in the University of Edinburgh. ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., W.S., D.Sc. W. B. BLAIKIE, LL.D. JOHN HORNE, LL.D., F.R.S., F.G.S. R. STEWART MACDOUGALL, M.A., D.Sc. W. A. TAIT, D.Sc., M.Inst.C.E.
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By a Resolution of the Society, January 19, 1880, PRINCIPAL SIR WILLIAM TURNER, K.C.B., D.C.L., F.R.S., having filled the office of President, is also a Member of Council.

## SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

WILLIAM ALLAN CARTER, M.Inst.C.E.

OFFICE, LIBRARY, ETC., 22, 24 George Street, Edinburgh. TEL. NO., 2881.

**ALPHABETICAL LIST OF THE ORDINARY FELLOWS  
OF THE SOCIETY,**

*Corrected to January 1, 1915.*

N.B.—*Those marked \* are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.

K.	"	"	"	Keith Medal.
N.	"	"	"	Neill Medal.
V. J.	"	"	"	the Gunning Victoria Jubilee Prize.
C.	"	"	"	contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Electio[n.				Service on Council, etc.
1898	C.	* Abercromby, the Hon. John, LL.D., 62 Palmerston Place, Edinburgh		
1898		Adami, Prof. J. G., M.A., M.D. Cantab., F.R.S., Professor of Pathology in M'Gill University, Montreal		
1911		* Adams, Archibald Campbell, A.M.Inst.Mech.E., A.M.Inst.E.E., Consulting Engineer, 1 Old Smithills, Paisley		
1896		* Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh		
1871		Agnew, Sir Stair, K.C.B., M.A., formerly Registrar - General for Scotland, 22 Buckingham Terrace, Edinburgh	5	1882-85, 1886-89, 1891-93, 1895-98.
1875	C. K.	Aitken, John, LL.D., F.R.S., Ardenlea, Falkirk		
1895	V. J.	* Alford, Robert Gervase, M.Inst.C.E., Three Gables, Woodburn Park Road, Tunbridge Wells, Kent		
1889		Alison, John, M.A., Head Master, George Watson's College, Edinburgh		
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London		
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California	10	
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.		
1893		Anderson, J. Macvicar, Architect, 6 Stratton Street, London		
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square, Edinburgh		
1905		Anderson, William, F.G.S., P.O. Box 635, Sydney, New South Wales, Australia		
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 29 Lutton Place, Edinburgh	15	
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.		
1905	C.	* Andrew, George, M.A., B.A., H.M.I.S., Balwherie, Strathearn Road, Broughty Ferry		
1881		Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork		
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nylstroom, Smoke Lane, Reigate		
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester	20	
1893		* Archer, Walter E., 17 Sloane Court, London, S.W.		
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, Syracuse University, Syracuse, N.Y., U.S.A.		
1907		* Archibald, James, M.A., Head Master, St Bernard's School, 1 Leamington Terrace, Edinburgh		
1911	C.	* Ashworth, James Hartley, D.Sc., Lecturer on Invertebrate Zoology, University of Edinburgh, 4 Cluny Terrace, Edinburgh		1912-14.
1907		Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt	25	

# Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.			Service on Council, etc.
1896	C.	* Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909-12.
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Garden, Inverleith House, Edinburgh	1888-91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, 26 Barton Road, Cambridge	
1892	C.	* Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Surgeon General, Indian Medical Service, Madras, India	30
1889		Barbour, A. H. F., M.A., M.D., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 729 Great Western Road, Glasgow	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1910		* Barclay, Lewis Bennett, C.E., 13 Cargill Terrace, Edinburgh	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII. Sanatorium, Midhurst	35
1914		* Barkla, Charles Glover, D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, Littledean, 34 Priestfield Road, Edinburgh	
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, Sir William F., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 6 De Vesci Terrace, Kingstown, County Dublin	
1887		Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Duncan Street, Edinburgh	1909-12.
1895	C.	Barton, Edwin H., D.Sc., A.M.Inst.E.E., Fellow Physical Society of London, Professor of Experimental Physics, University College, Nottingham	40
1904		* Baxter, William Muirhead, Glenalmond, Sciennes Gardens, Edinburgh	
1913		Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), Medical Officer of Health and School Medical Officer, City of Carlisle, 15 Brunswick Street, Carlisle	
1888		Beare, Thomas Hudson, B.Sc., M.Inst.C.E., Professor of Engineering in the University of Edinburgh (VICE-PRESIDENT)	1907-1909. V.P 1909-
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town	45
1892		Beck, Sir J. H. Meiting, Kt., M.D., M.R.C.P.E., Drostdy, Tulbagh, Cape Province, South Africa	
1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		Begg, Ferdinand Faithfull, 5 Whittington Avenue, London, E.C.	
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Witton Gilbert, Durham	50
1900		* Bennett, James Bower, C.E., 5 Hill Street, Edinburgh	
1887		Bernard, J. Mackay, of Dunsinan, B.Sc., Dunsinan, Perth	
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens, Edinburgh	
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline	55
1880	C.	Birch, De Burgh, C.B., M.D., Professor of Physiology in the University of Leeds, 8 Osborne Terrace, Leeds	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884	C.	Black, John S., M.A., LL.D. (CURATOR OF LIBRARY AND MUSEUM), 6 Oxford Terrace, Edinburgh	1891-94. Cur. 1906-
1897		* Blaikie, Walter Biggar, LL.D., The Loan, Colinton	
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge	1914-
1898	C.	* Blyth, Benjamin Hall, M.A., V.P.Inst.C.E., 17 Palmerston Place, Edinburgh	60
1894		* Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol	
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886		Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow (VICE-PRESIDENT)	1887-90, 1893-96, 1907-09. V.P 1910-

Date of Election.			Service on Council, etc.
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., A.Inst.C.E., A.Inst.M.E., M.Inst.E.E., etc., 4 Albert Square, Manchester	65
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Veterinary College, Edinburgh	1907-10.
1886		Bramwell, Byrom, M.D., F.R.C.P.E., LL.D., 23 Drumsheugh Gardens, Edinburgh	1890-93.
1907		* Bramwell, Edwin, M.B., F.R.C.P.E., F.R.C.P. Lond., 24 Walker Street, Edinburgh	
1912		Bridger, Adolphus Edward, M.D. (Edin.), F.R.C.P. (Edin.), B.Sc. (Paris), B.L. (Paris), Foley Lodge, Langham Street, London, W.	70
1895		Bright, Charles, M.Inst.C.E., M.Inst.E.E., F.R.A.S., F.G.S., Consulting Engineer to the Commonwealth of Australia, The Grange, Leigh, Kent, and Members' Mansions, Victoria Street, London, S.W.	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Dunmow, Essex	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, South African College, Cape Town	
1864	C.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh	75
	K. B.		
1913	C.	* Brown, Alexander Russell, M.A., B.Sc., Science Master, Buckhaven Junior Student Centre, Norfield, Buckhaven	
1898		* Brown, David, F.C.S., F.I.C., J.P., Willowbrae House, Willowbrae Road, Edinburgh	
1911		* Brown, David Rainy, Chemical Manufacturer (J. F. Macfarlan & Co.), 93 Abbeyhill, Edinburgh	
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	80
1909	B. C.	* Brownlee, John, M.A., M.D., D.Sc., Ruchill Hospital, Bilsland Drive, Glasgow	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh	
1906	C. N.	* Bruce, William Speirs, LL.D., Director of the Scottish Oceanographical Laboratory, Edinburgh, Antarctica, Joppa, Midlothian	1909-12.
1898	K. C.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow	1911-14.
1870	C. K.	Buchanan, John Young, M.A., F.R.S., 26 Norfolk Street, Park Lane, London, W.	1878-81, 1884-86.
1887	C.	Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace, Edinburgh	
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne	
1902		* Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies' College, 64 Strathearn Road, Edinburgh	
1894	C. K.	* Burgess, James, C.I.E., LL.D., Hon. A.R.I.B.A., F.R.G.S., Hon. M. Imp. Russ. Archeol. Soc., and Amer. Or. Soc., M. Soc. Asiat. de Paris, M.R.A.S., H. Corr. M. Batavian Soc. of Arts and Sciences, and Berlin Soc. Anthropol., H. Assoc. Finno-Ugrian Soc., 22 Seton Place, Edinburgh	1895-98, 1899-1902. V-P 1908-14.
1887		Burnet, Sir John James, Architect, 18 University Avenue, Hillhead, Glasgow	90
1888		Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	Cadell, Henry Moubrey, of Grange, B.Sc., Linlithgow	
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock	
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	95
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Anatomy Department, Middlesex Hospital Medical School, London, W.	
1904		* Campbell, Charles Duff, Scottish Liberal Club, Princes Street, Edinburgh	

Date of Election.		Service on Council, etc.
1899	C. * Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	100
1910	Carnegie, David, M.Inst.C.E., M.Inst.Mech.E., M.I.S.Inst., 33-35 Charterhouse Square, London, E.C.	
1905	C. * Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901	Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905	Carter, Joseph Henry, F.R.C.V.S., Stone House, Church Street, Burnley, Lancashire	
1898	* Carter, Wm. Allan, M.Inst.C.E., 32 Great King Street, Edinburgh (Society's Representative on George Heriot's Trust)	105
1898	Carus-Wilson, Cecil, F.R.G.S., F.G.S., Waldegrave Park, Strawberry Hill, Middlesex, and Sandaeres Lodge, Parkstone-on-Sea, Dorset	
1908	Cavanagh, Thomas Francis, M.D., The Hospital, Bella Coola, B.C., Canada	
1882	Cay, W. Dyce, M.Inst.C.E., 39 Victoria Street, Westminster, London	
1899	Chatham, James, Actuary, 7 Belgrave Crescent, Edinburgh	
1912	Chaudhuri, Banawari Lal, B.A.(Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	110
1874	Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains	
1891	* Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, Craiglea Drive, Edinburgh	
1911	* Clark, William Inglis, D.Sc., 29 Lauder Road, Edinburgh	
1903	* Clarke, William Eagle, F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road, Edinburgh	
1909	Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	115
1913	* Cleghorn, Alexander, M.Inst.C.E., Marine Engineer, 14 Hatfield Drive, Kelvinside, Glasgow	
1875	Clouston, Sir T. S., M.D., LL.D., F.R.C.P.E., 26 Heriot Row, Edinburgh	
1904	C. Coker, Ernest George, M.A., D.Sc., Professor of Mechanical Engineering and Applied Mechanics, City and Guilds Technical College, Finsbury, Leonard Street, City Road, London, E.C.	
1904	Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S.,	
	C. Professor of Organic Chemistry in the University College, Gower Street, London	120
1904	* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909	C. * Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh	
1886	Connan, Daniel M., M.A.	
1872	Constable, Archibald, LL.D., 11 Thistle Street, Edinburgh	
1894	Cook, John, M.A., LL.D., formerly Principal, Central College, Bangalore, Director of Meteorology in Mysore, and Fellow, University of Madras, India, 9 Cobden Crescent, Edinburgh	125
1891	* Cooper, Charles A., LL.D., 41 Drumsheugh Gardens, Edinburgh	
1905	* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont, Stirlingshire	
1914	* Coutts, William Barron, M.A., B.Sc., 33 Dalhousie Terrace, Edinburgh	
1911	* Cowan, Alexander C., Papermaker, Valleyfield House, Penicuik, Midlothian	
1908	Craig, James Ireland, M.A., B.A., Controller of the Department of General Statistics, 14 Abdin Street, Cairo : The Koubbeh Gardens, near Cairo, Egypt	130
1875	Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh	
1907	* Cramer, William, Ph.D., Lecturer in Physiological Chemistry in the University of Edinburgh, Physiological Department, The University, Edinburgh	
1903	Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town	
1887	Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road, Edinburgh	
1870	Crichton-Browne, Sir Jas., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President and Treasurer of the Royal Institution of Great Britain, 45 Hans Place, S.W., and Royal Courts of Justice, Strand, London	135
1886	Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	

Date of Election.		Service on Council, etc.
1914	* Cumming, Alexander Charles, D.Sc., Lecturer in Chemistry, University, Edinburgh, 16 Kilmaurs Terrace, Edinburgh	
1898	* Currie, James, M.A. Cantab. (TREASURER), Larkfield, Goldenacre, Edinburgh	Treas. 1906-
1904	* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	
1885	Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	140
1912	* Darbshire, Arthur Dukinfield, M.A., Lecturer in Genetics at the University of Edinburgh	
1884	Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone House, Bow, North Devon	
1894	* Denny, Sir Archibald, Bart., LL.D., Cardross Park, Cardross, Dumbartonshire	
1869	Dewar, Sir James, Kt., M.A., LL.D., D.C.L., D.Sc., F.R.S., V.P.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	1872-74.
V. J.		
1905	* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh	145
1906	* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane	
1904	Dickinson, Walter George Burnett, F.R.C.V.S., Boston, Lincolnshire	
1884	Dickson, the Right Hon. Charles Scott, K.C., LL.D., 22 Moray Place, Edinburgh	
1888	C. Dickson, Henry Newton, M.A., D.Sc., 160 Castle Hill, Reading	
1876	C. Dickson, J. D. Hamilton, M.A., Senior Fellow and formerly Tutor, St Peter's College, Cambridge	150
1885	C. Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, Wesley Avenue, Los Angeles, California, U.S.A.	
1897	* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh	
1904	* Dobbie, Sir James Johnston, Kt., M.A., D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London. 4 Vicarage Gate, Kensington, London, W.	1905-08.
1881	C. Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh	1904-07
1867	C. Donaldson, Sir James, M.A., LL.D., Principal of the University of St Andrews	1913- 1870-73.
1905	* Donaldson, Rev. Wm. Galloway, F.R.G.S., F.E.I.S., The Manse, Forfar	155
1882	C. Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1901	* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow	
1866	Douglas, David, 22 Drummond Place, Edinburgh	
1910	* Douglas, Loudon MacQueen, Author and Lecturer, 3 Lauder Road, Edinburgh	
1908	C. Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Lister House, Wrexham, North Wales	160
1901	* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1904	* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh	
1903	Dunstan, John, M.R.C.V.S., Inversnaid, Liskeard, Cornwall	
1892	C. Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent	165
1899	* Duthie, George, M.A., Inspector-General of Education, Salisbury, Rhodesia	
1906	C. * Dyson, Sir Frank Watson, Kt., M.A., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich	1907-10.
1893	Edington, Alexander, M.D., Howick, Natal	
1904	* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow	
1904	* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	170
1875	Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A.	
1913	* Elliot, George Francis Scott, M.A. (Cantab.), B.Sc., F.R.G.S., F.L.S., Drumwhill, Mossdale	
1906	C. Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow	
1897	C. * Erskine-Murray, James Robert, D.Sc., 4 Great Winchester Street, London, E.C.	
1884	Evans, William, F.F.A., 38 Morningside Park, Edinburgh	175
1879	C. N. Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian	1882-85, 1904-07. V-P 1907-12.
1902	* Ewen, John Taylor, B.Sc., M.I.Mech.E., H.M. Inspector of Schools, 104 King's Gate Aberdeen	

Date of Election.		Service on Council, etc.
1878	C. Ewing, Sir James Alfred, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S., Director of Naval Education, Admiralty, Froghole, Edenbridge, Kent	1888-91.
1900	C. Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Guy's Hospital (Bacteriological Department), London	
1910	* Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh	180
1875	Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	
1907	C. Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow.	
1888	C. Fawsitt, Charles A., Coney Park, Bridge of Allan	
1883	C. Felkin, Robert W., M.D., F.R.G.S., 47 Bassett Road, North Kensington, London, W.	
1899	* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	185
1907	* Fergus, Edward Oswald, 12 Clairmont Gardens, Glasgow	
1904	* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1888	Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow	
1898	* Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens, Edinburgh	
1899	* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, 23 Dundonald Road, Glasgow, W.	190
1911	Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Woodburn, Rutherglen, Glasgow	
1906	* Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street, Edinburgh	
1900	C. N. * Flett, John S., M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Scotland, 33 George Square, Edinburgh	
1872	C. Forbes, Professor George, M.A., M.Inst.C.E., M.Inst.E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W.	
1904	Forbes, Norman Hay, F.R.C.S.E., L.R.C.P. Lond., M.R.C.S. Eng., Correspondent Memb. Soc. d'Hydrologie médicale de Paris, Druminnor, Church Stretton, Salop	195
1892	* Ford, John Simpson, F.C.S., 4 Nile Grove, Edinburgh	
1910	* Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh	
1896	* Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh	
1867	C. K. B. Fraser, Sir Thomas R., Kt., M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens, Edinburgh. (VICE-PRESIDENT)	{ 1870-73, 1877-79, 1883-86, 1894-97. V.P 1911-
1914	* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 17 Eildon Street, Edinburgh	200
1891	* Fullarton, J. H., M.A., D.Sc., 23 Porchester Gardens, London, W.	
1891	* Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen	
1907	* Galbraith, Alexander, Superintendent Engineer, Cunard Line, Liverpool, 93 Trinity Road, Bootle, Liverpool	
1888	C. Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh	
1901	Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	205
1899	Gatehouse, T. E., A.M. Inst.C.E., M.Inst.M.E., M.Inst.E.E., Fairfield, 128 Tulse Hill, London, S.W.	
1867	Gayner, Charles, M.D., F.L.S.	
1909	C. Geddes, Auckland C., M.D., Professor of Anatomy, M'Gill University, Montreal, Canada	
1880	C. Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	
1861	C. B. Geikie, Sir Archibald, O.M., K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Late Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey	1869-72, 1874-76, 1879-82.
		210

Date of Election.			Service on Council, etc.
1871	C. B.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., formerly Professor of Geology in the University of Edinburgh (PRESIDENT), Kilmorie, Colinton Road, Edinburgh	1882-85, 1888-91, 1897-99. V-P 1892-97. 1900-05. P. 1913-
1914		Gemmell, John Edward, M.B., C.M., Hon. Surgeon Hospital for Women and Maternity Hospital; Hon. Gynaecologist, Victoria Central Hospital, Liscard, 28 Rodney Street, Liverpool.	
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh	
1914		* Gibb, Alexander, A.M.Inst.C.E., St Martin's Abbey, by Perth	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	215
1912	C.	* Gibson, Arnold Hartley, D.Sc., Professor of Engineering, University College, Dundee	
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws	
1890		Gibson, George A., M.A., LL.D., Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow	
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London	1905-08. 1912-13.
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	220
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street, Edinburgh	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Administrator, Government House, Darwin Northern Territory, Australia	
1909		* Gladstone, Hugh Stewart, M.A., M.B.O.U., F.Z.S., 40 Lennox Gardens, London, S.W.	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer on Embryology and Senior Demonstrator of Anatomy, Middlesex Hospital, London, 22 Regent's Park Terrace, London, N.W.	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	225
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh	
1899		* Goodwin, Thomas S., M.B., C.M., F.C.S., 25 Worple Road, Isleworth, and Derwent Lodge, London Road, Spring-grove, Isleworth, Middlesex	
1913	C.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), B.A. (Cantab.), Lecturer in Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich	230
1891		* Graham, Richard D., 11 Strathearn Road, Edinburgh	
1898	C.	* Gray, Albert A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow	
1910		Gray, Bruce M'Gregor, C.E., A.M.Inst.C.E., Westbourne Grove, Selby, Yorkshire	
1909	C.	* Gray, James Gordon, D.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow	1913- 235
1910		* Green, Charles Edward, Publisher, Gracemount House, Liberton	
1886		Greenfield, W. S., M.D., F.R.C.P.E., LL.D., Emeritus Professor of General Pathology in the University of Edinburgh, Kirkbrae, Elie, Fife	
1897		Greenlees, Thomas Duncan, M.D. Edin., Rostrevor, Kirtleton Avenue, Weymouth, Dorset	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	1908-11.
1906		Greig, Edward David Wilson, M.D., B.Sc., Captain, H.M. Indian Medical Service, Byculla Club, Bombay, India	240
1905		Greig, Robert Blyth, LL.D., F.Z.S., Board of Agriculture for Scotland, 29 St Andrew Square, Edinburgh	
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh	
1907	C.	* Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	

Date of Election.			Service on Council, etc.
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Department of Pharmacology, University Museum, Oxford	245
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	
1911	C.	* Guy, William, F.R.C.S., L.R.C.P., L.D.S. Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1910	B. C.	Gwynne-Vaughan, D. T., F.L.S., Professor of Botany, 14 London Road, Reading	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham	
1905	B. C.	* Halm, Jacob E., Ph.D., Chief Assistant Astronomer, Royal Observatory, Cape Town, Cape of Good Hope	250
1899		Hamilton, Allan M'Lane, M.D., LL.D., 36 East 40th Street, New York, U.S.A.	
1876	C.	Hannay, J. Ballantyne, Sorbie, 10 Balgillo Terrace, Broughty Ferry	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia	
1914		Harrison, Edward Philip, Ph.D., Professor of Physics, Presidency College, University of Calcutta, The Observatory, Alipore, Calcutta	
1888	C.	Hart, D. Berry, M.D., F.R.C.P.E., 5 Randolph Cliff, Edinburgh	255
1869		Hartley, Sir Charles A., K.C.M.G., M.Inst.C.E., 26 Pall Mall, London	
1914	C.	Harvey-Gibson, Robert John, M.A., F.L.S., D.L. for the County Palatine of Lancaster, M.R.S.G.S., Professor of Botany, University of Liverpool, 22 Falkner Square, Liverpool	
1881		Harvie-Brown, J. A., of Quarter, LL.D., F.Z.S., Dunipace House, Larbert, Stirlingshire	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	* Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh	260
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P., M.R.C.S., 3 St Peter's Square, Manchester	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	265
1881	C. N.	Herdman, W. A., D.Sc., F.R.S., Past Pres. L.S., Professor of Natural History in the University of Liverpool, Croxteth Lodge, Ullet Road, Liverpool	
1908		* Hewat, Archibald, F.F.A., F.I.A., 13 Eton Terrace, Edinburgh	
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, 33 George Square, Edinburgh	
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkely Gardens, Kensington, London, W.	270
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Ordnance College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa., U.S.A.	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland, 12 Keith Crescent, Blackhall, Midlothian	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904	C.	* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh	275
1897		Houston, Alex, Cruikshanks, M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow, 11 Cambridge Drive, Glasgow	
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-on-Tyne	

1902-05,  
1906-07,  
1914-  
V-P  
1907-1913.

Date of Election.		Service on Council, etc.
1899	Howie, W. Lamond, F.C.S., 26 Neville Court, Abbey Road, Regent's Park, London, N.W.	
1883	C. Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Director of the Welsh National Museum; Crowland, Llandaff, Wales	280
1910	Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt	
1886	Hunt, Rev. H. G. Bonavia, Mus.D. Dub., Mus.B. Oxon., The Vicarage, Burgess Hill, Sussex	
1911	Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., Resident Engineer Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire	
1887	C. Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	
1887	C. Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 54 Harley Street, London	285
1908	Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W.	
1912	* Inglis, Robert John Mathieson, A.M. Inst.C.E., Engineer, Northern Division, North British Railway, Tantah, Peebles	
1904	C. Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	
1904	* Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace, Edinburgh	
1914	Jack, John Noble, Professor of Agriculture to the County Council of Sussex, Kingscote, The Avenue, Lewes, Sussex	290
1875	Jack, William, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow	
1894	Jackson, Sir John, C.V.O., LL.D., 48 Belgrave Square, London	
1889	James, Alexander, M.D., F.R.C.P.E., 14 Randolph Crescent, Edinburgh	
1901	* Jardine, Robert, M.D., M.R.C.S. F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow	
1912	C. * Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York	295
1906	C. * Jehu, Thomas James, M.A., M.D., F.G.S., Professor of Geology in the University of Edinburgh	
1900	* Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian	
1895	Johnston, Col. Henry Halcro, C.B., Late A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney	
1903	C. * Johnston, Thomas Nicol, M.B., C.M., Pogbie, Humbie, East Lothian	
1874	Jones, Francis, M.Sc., Lecturer on Chemistry, 17 Whalley Road, Whalley Range, Manchester	300
1905	Jones, George William, M.A., B.Sc., LL.B., Scottish Tutorial Institute, Edinburgh and Glasgow, 25 North Bridge: Coraldene, Kirk Brae, Liberton, Edinburgh	
1888	Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1907	* Kemp, John, M.A., Sea Bank School, North Berwick	
1912	Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assistant Professor of Neurology, Cornell University, New York, 20 West 50th Street, New York, U.S.A.	
1909	Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.	305
1908	* Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square, Edinburgh	
1903	C. N. * Kerr, John Graham, M.A., F.R.S., Professor of Zoology in the University of Glasgow	1904-07, 1913-
1891	Kerr, Joshua Law, M.D., The Chequers, Mittagong, Sydney, Australia	
1913	* Kerr, Walter Hume, M.A., B.Sc., Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh	
1908	Kidd, Walter Aubrey, M.D., 12 Montpelier Row, Blackheath, London	310
1886	C. N. Kidston, Robert, LL.D., F.R.S., F.G.S. (SECRETARY), 12 Clarendon Place, Stirling	1891-94, 1903-06. Sec. 1909-
1907	* King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas; Junior Inspector of Schools, La Maisonneuve, Clarkston, Glasgow	
1880	King, W. F., Lonend, Russell Place, Trinity, Leith	
1883	Kinnear, the Right Hon. Lord, P.C., one of the Senators of the College of Justice, 2 Moray Place, Edinburgh	
1878	Kintore, the Right Hon. the Earl of, P.C., G.C.M.G., M.A. Cantab., LL.D. Cambridge, Aberdeen and Adelaide, Keith Hall, Inverurie, Aberdeenshire	315

# Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.		Service on Council, etc.
1901	* Knight, Rev. G. A. Frank, M.A., 52 Sardinia Terrace, Hillhead, Glasgow	
1907	* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, St James' School, Glasgow, The Shieling, Uddingston, by Glasgow	
1880	C. K. Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (formerly Professor of Physics, Imperial University, Japan) (GEN. SECRETARY), 42 Upper Gray Street, Edinburgh	1894-97, 1898-01, 1902-05. Sec. 1905-1912. Gen. Sec. 1912-
1886	C. Laing, Rev. George P., 17 Buckingham Terrace, Edinburgh	
1878	Lang, P.R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	320
1910	* Lauder, Alexander, D.Sc., F.I.C., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh	
1885	C. Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh	1908-11, 1913 -
1894	C. * Laurie, Malcolm, B.A., D.Sc., F.L.S., 19 Merchiston Park, Edinburgh	
1910	C. * Lawson, A. Ainsworth, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia	
1905	* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	325
1910	C. * Lee, Gabriel W., D.Sc., Palaeontologist, Geological Survey of Scotland, 33 George Square, Edinburgh	
1903	* Leighton, Gerald Rowley, M.D., Local Government Board, 125 George Street, Edinburgh	
1874	C. K. Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1910	Levie, Alexander, F.R.C.V.S., D.V.S.M., Veterinary Surgeon, Lecturer on Veterinary Science, Veterinary Infirmary, 12 Derwent Street, Derby	
1914	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	330
1905	* Lightbody, Forrest Hay, 56 Queen Street, Edinburgh	
1889	Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1912	* Lindsay, John George, M.A., B.Sc. (Edin.), Science Master, Royal High School, 33 Lauriston Gardens, Edinburgh	
1912	* Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903	Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India	335
1903	* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	
1898	* Lothian, Alexander Veitch, M.A., B.Sc., Training College, Cowcaddens, Glasgow	
1884	Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888	Lowe, D. F., M.A., LL.D., formerly Head Master of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900	Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A.	340
1894	* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887	M'Aldowie, Alexander M., M.D., Glengarriff, Leckhampton, Cheltenham	
1907	MacAlister, Donald Alexander, A.R.S.M., F.G.S., 26 Thurloe Square, South Kensington, London, S.W.	
1883	M'Bride, P., M.D., F.R.C.P.E., 10 Park Avenue, Harrogate, and Hill House, Withypool, Dunster, Somerset	
1903	* M'Cormick, Sir W. S., M.A., LL.D., Secretary to the Carnegie Trust for the Universities of Scotland, 13 Douglas Crescent, Edinburgh	345
1905	* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	
1897	C. * Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm	
1904	* Macdonald, John A., M.A., B.Sc., King Edward VII. School, Johannesburg, Transvaal	
1886	Macdonald, the Right Hon. Sir J. H. A. (Lord Kingsburgh) P.C., K.C., K.C.B., LL.D., F.R.S., M.Inst.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place, Edinburgh	1889-92.

Date of Election.			Service on Council, etc.
1904		Macdonald, William, B.Sc., M.Sc., Agriculturist, Editor <i>Transvaal Agricultural Journal</i> , Department of Agriculture, Pretoria Club, Pretoria, Transvaal	350
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh	
1901	C.	* MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh	1914-
1910		Macewen, Hugh Allan, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A.	355
1897		* MacGillivray, Angus, C.M., M.D., D.Sc., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	
1903		* M'Intosh, Donald C., M.A., D.Sc., 3 Glenisla Gardens, Edinburgh	
1911		M'Intosh, John William, A.R.C.V.S., 14 Templar Street, Myatts Park, London, S.E.	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, Pres. Ray. Society, 2 Abbotsford Crescent, St Andrews	1885-88.
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	360
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 2 Coates Place, Edinburgh	
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	{ 1875-78, 1885-88, 1893-94, 1900-02. V-P 1894-1900.
1912	C.	M'Kendrick, Anderson Gray, M.B., Major, Indian Medical Service, Officiating Statistical Officer to the Government of India, The Pasteur Institute, Kasauli, India	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Glasgow	365
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	
1911		* M'Kenzie, Kenneth John, M.A., Master of Method to Leith School Board, 24 Dudley Gardens, Leith	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1904		* Mackenzie, W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Local Government Board for Scotland, 4 Clarendon Crescent, Edinburgh	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	370
1904		* Mackintosh, Donald James, M.V.O., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	
1869	C.	Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent, Edinburgh	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., M.Inst.E.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kervisland Terrace, Hillhead, Glasgow	
1913		* M'Lellan, Dugald, M.Inst.C.E., District Engineer, Caledonian Railway, 42 Ormidale Terrace, Murrayfield, Edinburgh	42 375
1907		* Maenair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	
1898	C.	Mahalanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV., 37 Lower Chitpore Road, Calcutta, India	
1908		Mallik, Devendranath, B.A., B.Sc., Professor of Physics and Mathematics, Patna College, Bankipur, Bengal, India	
1912		Maloney, William Joseph, M.D.(Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	380
1913		Marchant, Rev. James, F.R.A.S., Director, National Council for Promotion of Race-Regeneration; Literary Adviser to House of Cassell; 42 Great Russell Street, London, W.C.	
1880	C.	Marsden, R. Sydney, M.D., C.M., D.Sc., D.P.H., Hon. L.A.H. Dub., M.R.I.A., F.I.C., M.O.H., Rowallan House, Cearns Road, and Town Hall, Birkenhead	
1909	C.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica and Therapeutics, Medical School, Dundee, Arnsheen, Westfield Terrace, West Newport, Fife	

# Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.			Service on Council, etc.
1882	C.	Marshall, D. H., M.A., Professor, Union and Alwington Avenue, Kingston, Ontario, Canada	
1901	C.	* Marshall, F. H. A., Sc.D., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	385
1903		Martin, Nicholas Henry, F.L.S., F.C.S., Ravenswood, Low Fell, Gateshead	
1912		* Martin, Sir Thomas Carlaw, LL.D., J.P., Director, Royal Scottish Museum, 4 Gordon Terrace, Edinburgh	
1913		Masson, George Henry, M.D., D.Sc., M.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London	390
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Langley Mount, Watford, Herts	
1906		* Mathieson, Robert, F.C.S., Rillbank, Innerleithen	
1902		Matthews, Ernest Romney, A.M.Inst.C.E., F.G.S., Chadwick Professor of Municipal Engineering in the University of London, University College, Gower Street, London, W.C.	
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1888		Methven, Cathcart W., M.Inst.C.E., F.R.I.B.A., Durban, Natal, S. Africa	395
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y., U.S.A.	
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, 84 Grange Crescent, Sharrow, Sheffield	400
1905		* Milne, Archibald, M.A., B.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh	
1905		* Milne, C. H., M.A., Head Master, Daniel Stewart's College, 4 Campbell Road, Murrayfield, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Lecturer on Natural Philosophy, 11 Melville Crescent, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Meloyne, Malone Park, Belfast	405
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India, 103 Trinity Road, Edinburgh	
1897		Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., Cruach, Lochgilphead	
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square, Edinburgh	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay, India	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony	415
1914		Mort, Spencer, M.D., Ch.B., F.R.C.S.E., Medical Superintendent, Edmonton Infirmary, London, N.	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, 26 Park Street, Wellesley, Calcutta, India	
1892	C.	Mossman, Robert C., Acting Editor, British Rainfall Organization's Publications, 63A Burntwood Lane, Wandsworth Common, London, S.W.	
1874	C. K.	Muir, Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Mowbray Hall, Rosebank, Cape Colony	1885-88. V-P
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers	1888-91.
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Date of Election.			Service on Council, etc.
1907	Muirhead, James M. P., J.P., F.R.S.L., F.S.S., c/o Dunlop Rubber Co., Ltd., 3 Wallace Street, Fort, Bombay		
1887	Mukhopādhyay, Asūtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta, India		
1891	C. * Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank, Largs, Ayrshire		{ 1894-97, 1900-03, V-P 1903-08.
1896	* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh		
1907	C. * Murray, James, Hill Farm Bungalow, Froxfield, Hants	425	
1907	Musgrave, James, M.D., F.R.C.S. Edin. and Eng., Bute Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews		
1902	Mylne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts		
1888	Napier, A. D. Leith, M.D., C.M., M.R.C.P., 28 Angas Street, Adelaide, Australia		
1897	Nash, Alfred George, B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I.		
1906	* Newington, Frank A., M.Inst.C.E., M.Inst.E.E., 7 Wester Coates Road, Edinburgh	430	
1898	Newman, Sir George, M.D., D.P.H. Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London : Grim's Wood, Harrow Weald, Middlesex		
1884	Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh		{ 1885-87, 1892-95, 1897-1900.
1880	C. Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh		
1878	Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham		
1906	* O'Connor, Henry, A.M.Inst.C.E., 1 Drummond Place, Edinburgh	435	
1888	Ogilvie, F. Grant, C.B., M.A., B.Sc., LL.D., Secretary of the Board of Education for the Science Museum and the Geological Survey, and Director of the Science Museum, 15 Evelyn Gardens, London, S.W.		
1888	Oliphant, James, M.A., 11 Heathfield Park, Willesden Green, London		
1886	Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.		
1895	C. Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne		
1914	* Oswald, Alfred, Lecturer in German, Glasgow Provincial Training College, Nordheim, Bearsden, Glasgow	440	
1908	Page, William Davidge, F.C.S., F.G.S., M.Inst.M.E., 10 Clifton Dale, York		
1905	Pallin, William Alfred, F.R.C.V.S., Major in the Army Veterinary Corps, c/o Messrs Holt & Co., 3 Whitehall Place, London		
1914	Pare, John William, M.B., C.M., M.D., L.D.S., Lecturer in Dental Anatomy, National Dental Hospital, 64 Brook Street, Grosvenor Square, London, W.		
1892	Parker, Thomas, M.Inst.C.E., Severn House, Iron Bridge, Salop		
1901	* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	445	
1886	C. Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., F.R.S., Professor of Physiology in the University of Glasgow, University, Glasgow		{ 1894-97 1904-06, 1909-12.
1892	* Paulin, Sir David, Actuary, 6 Forbes Street, Edinburgh		
1881	C. N. Peach, Benjamin N., LL.D., F.R.S., F.G.S. (VICE-PRESIDENT), formerly District Superintendent and Acting Palaeontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh		{ 1905-08, 1911-1912. V-P 1912-
1907	* Pearce, John Thomson, B.A., B.Sc., School House, Tranent		
1914	Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon	450	
1904	* Peck, James Wallace, M.A., Chief Inspector, National Health Insurance, Scotland, 83 Princes Street, Edinburgh		
1889	Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh		
1887	C. B. Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee, Rosemount, Forthill Road, Broughty Ferry		1904-07 1908-11.
1893	Perkin, Arthur George, F.R.S., 8 Montpellier Terrace, Hyde Park, Leeds		
1913	* Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin	455	
1889	Philip, Sir R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square, Edinburgh		
1907	Philips, Charles E. S., Castle House, Shooter's Hill, Kent		
1914	* Pilkington, Basil Alexander, 20 Queen's Avenue, Blackhall, Midlothian		

# Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.		Service on Council, etc.
1905	* Pinkerton, Peter, M.A., D.Sc., Rector, High School, Glasgow, 44 St James's Street, Hillhead, Glasgow	
1908	C. * Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Bacteriological Laboratory, Nairobi, British East Africa	460
1911	* Pirie, James Simpson, Civil Engineer, 28 Scotland Street, Edinburgh	
1906	Pitchford, Herbert Watkins, F.R.C.V.S., Bacteriologist and Analyst, Natal Government, The Laboratory, Pietermaritzburg, Natal	
1886	Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1888	Prain, Sir David, Lt.-Col., Indian Medical Service (Retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. K. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902	* Preller, Charles Du Riche, M.A., Ph.D., A.M.Inst.C.E., 61 Melville Street, Edinburgh	465
1892	* Pressland, Arthur J., M.A. Camb., Edinburgh Academy	
1875	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1908	C. * Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles	
1903	* Pullar, Laurence, Dunbarney, Bridge of Earn, Perthshire	
1911	Purdy, John Smith, M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Chief Health Officer for Tasmania, Islington, Hobart, Tasmania	470
1898	* Purves, John Archibald, D.Sc., 13 Albany Street, Edinburgh	
1897	* Rainy, Harry, M.A., M.B., C.M., F.R.C.P. Ed., 16 Great Stuart Street, Edinburgh	
1899	* Ramage, Alexander G., 8 Western Terrace, Murrayfield, Edinburgh	
1884	Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1914	* Ramsay, Peter, M.A., B.Sc., Head Mathematical Master, George Watson's College, 63 Comiston Drive, Edinburgh	475
1911	* Rankin, Adam A., Vice-President, British Astronomical Association, West of Scotland Branch, 324 Crow Road, Broomhill, Glasgow, W.	
1891	* Rankine, John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh	
1904	Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900	Raw, Nathan, M.D., M.R.C.P. (London), B.S., F.R.C.S., D.P.H., 66 Rodney Street, Liverpool	
1883	C. Readman, J. B., D.Sc., F.C.S., Belmont, Hereford	480
1889	Redwood, Sir Boverton, Bt., D.Sc. (Hon.), F.I.C., F.C.S., A.Inst.C.E., The Cloisters, 18 Avenue Road, Regent's Park, London, N.W.	
1902	Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London	
1902	Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants	
1913	Reid, Harry Avery, F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand	
1908	C. * Rennie, John, D.Sc., Lecturer on Parasitology, and Assistant to the Professor of Natural History, University of Aberdeen, 60 Desswood Place, Aberdeen	485
1914	Renshaw, Graham, M.B., M.R.C.S., L.R.C.P., L.S.A., Surgeon, Bridge House, Sale, Manchester	
1913	* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., General Manager and Chief Engineer, Electricity Supply, Dundee and District, The Cottage, Craigie, Broughty Ferry	
1908	Richardson, Linsdall, F.L.S., F.G.S., Organising Inspector of Technical Education for the Gloucestershire Education Committee, 10 Oxford Parade, Cheltenham	
1875	Richardson, Ralph, W.S., 10 Magdala Place, Edinburgh	
1914	* Ritchie, James Bonnyman, B.Sc., Science Master, Kelvinside Academy, Glasgow	490
1906	C. * Ritchie, William Thomas, M.D., F.R.C.P.E., 9 Atholl Place, Edinburgh	
1898	C. Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1880	Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	
1900	* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	
1896	* Robertson, Robert, M.A., 25 Mansionhouse Road, Edinburgh	495
1902	C. * Robertson, Robert A., M.A. B.Sc., Lecturer on Botany in the University of St Andrews	

Date of Election.		Service on Council, etc.
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 2 Mayfield Gardens, Edinburgh
1910		* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh (SECRETARY)
1881		Rosebery, the Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.R.A.S., Professor of Mathematics and Physics, University of Western Australia, Perth, Western Australia
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, Dunaura, Heugh Street, Falkirk
1902	C. K.	* Russell, James, 22 Glenorchy Terrace, Edinburgh
1880		Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane, Edinburgh
1904		Sachs, Edwin O., Architect, Chairman of the British Fire Prevention Committee, Vice-President of the International Fire Service Council, 8 Waterloo Place, Pall Mall, London, S.W.
1906		Saleeby, Caleb William, M.D., 13 Greville Place, London
1914		* Salvesen, Theodore Emile, 37 Inverleith Place, Edinburgh
1912		* Sampson, Ralph Allen, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh
1903		* Samuel, John S., 8 Park Avenue, Glasgow
1903		* Sarolea, Charles, Ph.D., D.Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, 21 Royal Terrace, Edinburgh
1891		Sawyer, Sir James, Kt., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham
1900	C.	* Schäfer, Sir Edward Albert, M.R.C.S., LL.D., F.R.S. (VICE-PRESIDENT), Professor of Physiology in the University of Edinburgh
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.
1880		Scott, J. H., M.B., C.M., M.R.C.S., Professor of Anatomy in the University of Otago, New Zealand
1889	C.	Scougal, A. E., M.A., LL.D., formerly H.M. Senior Chief Inspector of Schools and Inspector of Training Colleges, 1 Wester Coates Avenue, Edinburgh
1902		Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.
1871		Simpson, Sir A. R., M.D., Emeritus Professor of Midwifery in the University of Edinburgh, 52 Queen Street, Edinburgh
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh
1911	C.	Simpson, Sutherland, M.D., D.Sc.(Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India
1903		* Skinner, Robert Taylor, M.A., Governor and Head Master, Donaldson's Hospital, Edinburgh
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth
1891	C. K.	* Smith, Alexander, B.Sc., Ph.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A.
1882	C.	Smith, C. Michie, C.I.E., B.Sc., F.R.A.S., formerly Director of the Kodaikānal and Madras Observatories, Winsford, Kodaikānal, South India
1885		Smith, George, F.C.S., 5 Rosehall Terrace, Falkirk
1911		* Smith, Stephen, B.Sc., Goldsmith, 31 Grange Loan, Edinburgh
1907	C.	Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia
1880		Smith, William Robert, M.D., D.Sc., LL.D., Professor of Forensic Medicine and Toxicology in King's College, University of London, and Principal of the Royal Institute of Public Health, 36 Russell Square, London, W.C.
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Medical Officer of Health, Coventry
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palaeontology in the University of Oxford
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 31 Cameron Street, Dunfermline

Date of Election.		Service on Council, etc.
1889	C. Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpius Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C. * Sommerville, Duncan McLaren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1882	Sorley, James, 82 Onslow Gardens, London	
1896	* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive, Edinburgh	535
1874	C. Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace, Edinburgh	
1906	Squance, Thomas Coke, M.D., F.R.M.S., F.S.A.Scot., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, 15 Grange Crescent, Sunderland	1885-87.
1891	* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh	
1914	* Steggall, John Edward Aloysius, M.A., Professor of Mathematics at University College, Dundee, in St Andrews University, Woodend, Perth Road, Dundee	
1912	C. Stephenson, John, M.B., D.Sc. (Lond.), Indian Medical Service, Professor of Biology, Government College, Lahore, India.	540
1910	* Stephenson, Thomas, F.C.S., Editor of the <i>Prescriber</i> , Examiner to the Pharmaceutical Society, 9 Woodburn Terrace, Edinburgh	
1886	C. Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884	Stevenson, David Alan, B.Sc., M.Inst.C.E., 84 George Street, Edinburgh	
1888	C. Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh	
1902	* Stockdale, Herbert Fitton, Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	545
1889	Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.
1906	Story, Fraser, Professor of Forestry, University College, Bangor, North Wales	
1907	* Strong, John, M.A., Rector of Montrose Academy, Peel Place, Montrose	
1903	Sutherland, David W., M.D., M.R.C.P., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India	
1905	Swithenbank, Harold William, Denham Court, Denham, Bucks	550
1912	* Syme, William Smith, M.D. (Edin.), 10 India Street, Glasgow	
1885	C. Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Professor of Anatomy in Queen's College, Belfast	1892-
1904	* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C. Tait, William Archer, D.Sc., M.Inst.C.E., 38 George Square, Edinburgh	1914-
1895	Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah, U.S.A.	
1890	C. Tanakadate, Aikiyu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	
1870	Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899	* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	
1892	Thackwell, J. B., M.B., C.M., 423A Battersea Park Road, London, S.W.	
1885	C. Thompson, D'Arcy W., C.B., B.A., F.L.S., Professor of Natural History in University College, Dundee	1892-95, 1896-99, 1907-10, 1912-
1907	* Thompson, John Hannay, M.Sc. (Durh.), M.Inst.C.E., M.Inst.Mech.E., Engineer to the Dundee Harbour Trust, Sorbie, 10 Balgillo Terrace, Broughty Ferry	
1905	* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887	Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Piteullen, Perth	
1911	* Thomson, Frank Wyvill, M.A., M.B., C.M., D.P.H., D.T.M., Lt.-Col. I.M.S. (Retired), Bonsyde, Linlithgow	
1896	* Thomson, George Ritchie, M.B., C.M., General Hospital, Johannesburg, Transvaal	565
1903	Thomson, George S., F.C.S., Ferma Albion, Maroulesci, Roumania	
1906	* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C. Thomson, J. Arthur, M.A., LL.D., Regius Professor of Natural History in the University of Aberdeen	1906-08.
1906	C. Thomson, James Stuart, F.L.S., Zoological Department, University, Manchester	
1880	Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, 18 Lansdowne Road, London, W.	570
1899	* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C. Thomson, Robert Black, M.B., Edin., Professor of Anatomy, South African College, Cape Town	

Date of Election.		Service on Council, etc.
1870	Thomson, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	
1882	Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town	
1876	C. Thomson, William, Royal Institution, Manchester	575
1911	* Tosh, James Ramsay, M.A., D.Sc. (St Ands.), Thursday Island, Queensland, Australia	
1914	Tredgold, Alfred Frank, L.R.C.P., M.R.C.S., Hon. Consulting Physician to National Association for the Feeble-minded, 6 Dapdune Crescent, Guildford, Surrey	
1888	Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan, Edinburgh	
1905	* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh	
1906	C. * Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P., Lecturer on Medical Physics, Surgeons' Hall, Physician in charge of Radium Treatment, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh	580
		1866-68, 1895-97. 1913- Sec. 1869-91. V-P 1891-95, 1897-1903. P.
1861	K. N. C. Turner, Sir William, K.C.B., M.B., F.R.C.S.L. and E., LL.D., D.C.L., D.Sc., F.R.S., Late Pres. R.S.E., Knight of the Royal Prussian Order <i>Pour le Mérite</i> , Principal and Vice-Chancellor of the University of Edinburgh, 6 Eton Terrace, Edinburgh	1908-1913.
1895	Turton, Albert H., M.I.M.M., 171 George Road, Erdington, Birmingham	
1898	C. * Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, Duns, Berwickshire	1907-10.
1889	Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire	
1910	Vincent, Swale, M.D. Lond., D.Sc. Edin., etc., Professor of Physiology, University of Manitoba, Winnipeg, Canada	585
1911	C. * Walker, Henry, M.A., D.Sc., Head Physics Master, Kilmarnock Academy and Technical School, 30 M'Lellan Drive, Kilmarnock	
1891	C. B. * Walker, James, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh	1903-05, 1910-13.
1873	C. Walker, Robert, M.A., LL.D., University, Aberdeen	
1902	* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C. Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	590
1898	Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891	* Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London	
1907	Waters, E. Wynston, Medical Officer, H.B.M. Administration, E. Africa, Malindi, British East Africa Protectorate, via Mombasa	
1901	C. * Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	
1911	* Watson, James A. S., B.Sc., etc., Assistant in Agriculture, University of Edinburgh, 15 Dick Place, Edinburgh	595
1900	* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	
1910	* Watson, William John, M.A., LL.D. Aberdeen, B.A. Oxon., Professor of Celtic Languages and Literature, University, Edinburgh, 17 Merchiston Avenue, Edinburgh	
1907	* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace, Edinburgh	1912-14.
1911	Watt, James, W.S., F.F.A., 24 Rothesay Terrace, Edinburgh	
1911	* Watt, Rev. Lauchlan Maclean, B.D., Minister of St Stephen's Parish, 7 Royal Circus, Edinburgh	600
1896	Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynaecology, Rush Medical College, 1748 Harrison Street, Chicago, Ill., U.S.A.	
1907	B. C. * Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., 7 Dean Park Crescent, Edinburgh	1913-
1903	C. * Wedderburn, J. H. Maclagan, M.A., D.Sc., 95 Mercer Street, Princeton, N.J., U.S.A.	
1904	Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma	
1896	Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., D.C.L., Professor of Philosophy in the University of Michigan, Ann Arbor, U.S.A.	605
1909	C. * Westergaard, Reginald Ludovic Andreas Emil. Ph.D., Professor of Technical Mycology, Heriot-Watt College, Hafnia, Liberton, Edinburgh	
1896	C. White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	

Date of Election.		Service on Council, etc.
1911	* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh	
1912	* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh, 35 George Square, Edinburgh	1912-
1879	Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen	610
1908	* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen	
1910	C. * Williamson, William, 9 Plewlands Terrace, Edinburgh	
1900	Wilson, Alfred C., F.C.S., Woewood Croft, Stockton-on-Tees	
1911	* Wilson, Andrew, M.Inst.C.E., 51 Queen Street, Edinburgh	
1902	* Wilson, Charles T. R., M.A., F.R.S., 21 Grange Road, Cambridge, Sidney Sussex College, Cambridge	615
1895	Wilson-Barker, David, R.N.R., F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenwich, Kent	
1882	Wilson, George, M.A., M.D., LL.D.	
1891	* Wilson, John Hardie, D.Sc., University of St Andrews, 39 South Street, St Andrews	
1902	Wilson, William Wright, F.R.C.S.E., M.R.C.S., Cottesbrook House, Acock's Green, Birmingham	
1908	C. * Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith	620
1886	Woodhead, German Sims, M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge	1887-90.
1884	Woods, G. A., M.R.C.S., 1 Hammerton Road, Bromley, Kent	
1911	* Wrigley, Ruric Whitehead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1890	Wright, Johnstone Christie, Conservative Club, Edinburgh	
1896	* Wright, Sir Robert Patrick, Chairman of the Board of Agriculture for Scotland, Kingarth, Colinton, Midlothian	625
1882	Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow	
1892	Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896	C. * Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton	
1904	Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal	

## LIST OF HONORARY FELLOWS OF THE SOCIETY

At January 1, 1914.

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X).

Elected,

1897 Émile Hilaire Amagat, Membre de l'Institut, St Satur, Cher, France.  
 1900 Arthur Auwers, Bellevue-Strasse 55, Berlin-Lichterfelde, Germany.  
 1900 Adolf Ritter von Baeyer, Universität, München, Germany.  
 1905 Waldemar Christofer Brögger, K. Frederiks Universitet, Christiania, Norway.  
 1905 Moritz Cantor, Gaisbergstrasse 15, Heidelberg, Germany.  
 1902 Jean Gaston Darboux, Secrétariat de l'Institut, Paris, France.  
 1910 Hugo de Vries, Universiteit, Amsterdam, Holland.  
 1905 Paul Ehrlich, K. Institut für Experimentelle Therapie, Sandhofstrasse 44, Frankfurt-a.-M., Germany.  
 1908 Emil Fischer, Universität, Berlin, Germany.  
 1910 Karl F. von Goebel, Universität, München, Germany.  
 1905 Paul Heinrich von Groth, Universität, München, Germany.  
 1888 Ernst Haeckel, Universität, Jena, Germany.  
 1913 George Ellery Hale, Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.  
 1883 Julius Hann, Universität, Wien, Austria.  
 1913 Émil Clement Jungfleisch, Collège de France, Paris, France.  
 1910 Jacobus Cornelius Kapteyn, Universiteit, Groningen, Holland.

## Elected

1897 Gabriel Lippmann, Université, Paris, France.  
 1895 Carl Menger, Wien ix., Fuchstallerg, 2, Austria.  
 1910 Elie Metchnikoff, Institut Pasteur, Paris, France.  
 1910 Albert Abraham Michelson, University, Chicago, U.S.A.  
 1897 Fridtjof Nansen, K. Frederiks Universitet, Christiania, Norway.  
 1908 Henry Fairfield Osborn, Columbia University and American Museum of Natural History, New York, N.Y., U.S.A.  
 1910 Wilhelm Ostwald, Gross-Bothen, bei Leipzig, Germany.  
 1908 Ivan Petrovitch Pavlov, Wedenskaja Strasse 4, St Petersburg, Russia.  
 1910 Frederick Ward Putnam, Peabody Museum of Harvard University, Cambridge, Mass., U.S.A.  
 1889 Georg Hermann Quincke, Bergstrasse 41, Heidelberg, Germany.  
 1913 Santiago Ramón y Cajal, Universidad, Madrid, Spain.  
 1908 Magnus Gustaf Retzius, Högskolan, Stockholm, Sweden.  
 1908 Augusto Righi, Regia Università, Bologna, Italy.  
 1913 Vito Volterra, Regia Università, Rome, Italy.  
 1905 Wilhelm Waldeyer, Universität, Berlin, Germany.  
 1905 Wilhelm Wundt, Universität, Leipzig, Germany.  
 1913 Charles René Zeiller, École Nationale Supérieure des Mines, Paris, France.

Total, 33.

## BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X).

1900 Sir David Ferrier, Kt., M.A., M.D., LL.D., F.R.S., Emer. Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.  
 1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Math.D., F.R.S., Chief Professor of Mathematics in the Imperial College of Science and Technology, London, formerly Sadlerian Professor of Pure Mathematics in the University of Cambridge, Imperial College of Science, London, S.W.  
 1910 Sir James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool, Trinity College, Cambridge.  
 1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., 1 Queen Anne Street, Cavendish Square, London, W.  
 1913 Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.  
 1908 Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S., 29 Thurloe Place, S. Kensington, London, S.W.  
 1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, St John's College, Cambridge.  
 1900 Archibald Liversidge, M.A., LL.D., F.R.S., Em.-Professor of Chemistry in the University of Sydney, Fieldhead, Combe Warren, Kingston, Surrey.  
 1908 Sir James A. H. Murray, LL.D., D.C.L., D.Litt., Ph.D., Litt.D., F.S.A., Corresp. Member of the Institute of France, etc., Editor of the Oxford English Dictionary, Oxford.  
 1905 Sir William Ramsay, K.C.B., LL.D., F.R.S., formerly Professor of Chemistry in the University College, London, 19 Chester Terrace, Regent's Park, London, N.W.  
 1886 The Rt. Hon. Lord Rayleigh, O.M., P.C., J.P., D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France, Terling Place, Witham, Essex.  
 1908 Charles Scott Sherrington, M.A., M.D., LL.D., F.R.S., Waynflete Professor of Physiology in the University of Oxford, Physiological Laboratory, Oxford.  
 1913 Sir William Turner Thiselton-Dyer, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew; The Ferns, Witcombe, Gloucester.  
 1905 Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.  
 1909 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Imperial College of Science and Technology, South Kensington, London, S. W., Wlinfole Salcombe, South Devon.

Total, 15.

## ORDINARY FELLOWS OF THE SOCIETY ELECTED

*During Session 1913-14.*

(Arranged according to their date of election.)

*17th November 1913.*

EDWARD PHILIP HARRISON, Ph.D.

*1st December 1913.*

WILLIAM BARRON COUTTS, M.A., B.Sc.	JOHN EDWARD GEMMELL, M.B., C.M. (Edin.).
WILLIAM FRASER.	ALFRED OSWALD.
JOHN WILLIAM PARE, M.B., C.M. (Edin.), M.D., L.D.S. (Eng.).	

*19th January 1914.*

SPENCER MORT, M.D., Ch.B., F.R.C.S.E.	JOSEPH PEARSON, D.Sc., F.L.S.
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*16th February 1914.*

ROBERT JOHN HARVEY-GIBSON, M.A., F.L.S., D.L. for the County Palatine of Lancaster, M.R.S.G.S.
JOHN NOBLE JACK.

*25th May 1914.*

ALEXANDER CHARLES CUMMING, D.Sc.	GRAHAM RENSHAW, M.B., M.R.C.S., L.R.C.P., L.S.A.
BASIL ALEXANDER PILKINGTON.	JAMES BONNYMAN RITCHIE, B.Sc.
PETER RAMSAY, M.A., B.S.C.	THEODORE EMILE SALVESSEN.

*15th June 1914.*

ALEXANDER GIBB, A.M.Inst.C.E.

*6th July 1914.*

FRANCIS JOHN LEWIS, D.Sc., F.L.S.
ARCHIBALD M'KENDRICK, F.R.C.S.E., D.P.H., L.D.S.
ALFRED FRANK TREDGOLD, L.R.C.P., M.R.C.S.

## ORDINARY FELLOWS DECEASED AND RESIGNED

*During Session 1913-14.*

## DECEASED.

JOHN GIBSON, Ph.D.	JAMES A. MACDONALD, M.A., B.Sc.
J. W. INGLIS, M.Inst.C.E.	JOHN STURGEON MACKAY, M.A., LL.D.
Lieut. GEORGE JOHNSTONE, R.N.R.	Sir JOHN MURRAY, K.C.B., LL.D., Ph.D., D.Sc., F.R.S.
JOHN MACALLAN, F.I.C.	R. TRAILL OMOND.

## RESIGNED.

Rev. JOHN H. BURN.	CHARLES B. BOOG WATSON.
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## FOREIGN HONORARY FELLOW DECEASED.

EDUARD SUESS.

## BRITISH HONORARY FELLOWS DECEASED.

Sir ROBERT STAWELL BALL, Kt., LL.D., F.R.S., M.R.I.A.	ALBERT C. L. G. GÜNTHER, Ph.D., F.R.S.
Col. ALEXANDER ROSS CLARKE, C.B., R.E., F.R.S.	GEORGE WILLIAM HILL.
Sir DAVID GILL, K.C.B., LL.D., F.R.S.	ALFRED RUSSEL WALLACE, O.M., LL.D., D.C.L., F.R.S.

List of Library Exchanges, Presentations, etc.

1. TRANSACTIONS AND PROCEEDINGS OF LEARNED SOCIETIES, ACADEMIES, ETC., RECEIVED BY EXCHANGE OF PUBLICATIONS, AND LIST OF PUBLIC INSTITUTIONS ENTITLED TO RECEIVE COPIES OF THE TRANSACTIONS AND PROCEEDINGS OF THE ROYAL SOCIETY OF EDINBURGH. (*For convenience certain Presentations are included in this List.*)

T.P. prefixed to a name indicates that the Institution is entitled to receive *Transactions* and *Proceedings*. P. indicates *Proceedings*.

AFRICA (BRITISH CENTRAL).

ZOMBA.—*Scientific Department.* Meteorological Observations, Fol. (*Presented by H.M. Acting Commissioner and Consul-General.*)

AMERICA (NORTH). (*See CANADA, UNITED STATES, AND MEXICO.*)

AMERICA (SOUTH).

T.P. BUENOS AIRES (ARGENTINE REPUBLIC).—*Museo Nacional.* Anales.

P. *Sociedad Physis.* Boletin.

*Oficina Meteorologica Argentina.* Anales. (*Presented.*)

CORDOBA—

T.P. *Academia Nacional de Ciencias de la Republica Argentina.* Boletin.

T.P. *National Observatory.* Annals.—Maps.

T.P. LA PLATA (ARGENTINE REPUBLIC).—*Museo de La Plata.*

LIMA (PERU). *Cuerpo de Ingenieros de Minas del Peru.* Boletin. (*Presented.*)

P. MONTEVIDEO (URUGUAY).—*Museo Nacional.* Anales (Flora Uruguay).

T.P. PARÁ (BRAZIL).—*Museu Paraense de Historia Natural e Ethnographia.* Boletin.

P. QUITO (ECUADOR).—*Observatorio Astronomico y Meteorologico.*

RIO DE JANEIRO (BRAZIL)—

T.P. *Observatorio.* Annuario.—Boletin Mensal.

P. *Museu Nacional.* Revista (Archivos).

SANTIAGO (CHILI)—

T.P. *Société Scientifique du Chili.* Actes.

P. *Deutscher Wissenschaftlicher Verein.*

P. SAN SALVADOR.—*Observatorio Astronómico y Meteorológico.*

VALPARAISO (CHILI).—*Servicio Meteorológico.* Anuario. (*Presented.*)

AUSTRALIA.

*Australasian Association for the Advancement of Science.*—Reports. (*Presented.*)

## ADELAIDE—

P. *University Library.*  
 P. *Royal Society of South Australia.* Transactions and Proceedings.—Memoirs.  
 P. *Royal Geographical Society of Australasia (South Australian Branch).*  
     Proceedings.  
*Observatory.* Meteorological Observations. 4to. (*Presented.*)

## BRISBANE—

T.P. *University of Queensland.*  
 P. *Royal Society of Queensland.* Transactions.  
 P. *Royal Geographical Society (Queensland Branch).* Queensland Geographical Journal.  
 P. *Government Meteorological Office.*  
 P. *Water Supply Department.*  
 P. GEELONG (VICTORIA).—*Gordon Technical College.*  
 T.P. HOBART.—*Royal Society of Tasmania.* Proceedings.

## MELBOURNE—

*Commonwealth Bureau of Census and Statistics.* Official Year Book.  
     By G. H. Knibbs. (*Presented.*)  
*National Museum.* Memoirs. (*Presented.*)  
 T.P. *University Library.*  
 P. *Royal Society of Victoria.* Proceedings.—Transactions.

## PERTH, W.A.—

P. *Geological Survey.* Annual Progress Reports.—Bulletins.  
*Government Statistician's Office.* Monthly Statistical Abstract. (*Presented.*)

## SYDNEY—

T.P. *University Library.* Calendar.—Reprints of Papers from Science Laboratories.  
 T.P. *Department of Mines and Agriculture (Geological Survey), N.S.W.*  
     Records.—Annual Reports.—Palaeontology. Mineral Resources.  
 T.P. *Linnean Society of New South Wales.* Proceedings.  
 T.P. *Royal Society of New South Wales.* Journal and Proceedings.  
 P. *Australian Museum.* Records.—Reports.—Memoirs.—Catalogues.  
*N.S.W. Government.* Fisheries Report. (*Presented.*)

## AUSTRIA.

## CRACOW—

T.P. *Académie des Sciences.* Rozprawy Wydziału matematyczno-przyrodniczego (Proceedings, Math. and Nat. Sciences Cl.).—Rozprawy Wydziału filologicznego (Proc., Philological Section).—Rozprawy Wydziału historyczno-filozoficznego (Proc., Hist.-Phil. Section).—Sprawozdanie Komisji do badania historii sztuki w Polsce (Proc., Commission on History of Art in Poland).—Sprawozdanie Komisji fizjograficznej (Proc., Commission on Physiography).—Geological Atlas of Galicia; Text, Maps.—Bulletin International, etc.

## GRATZ—

T.P. *Naturwissenschaftlicher Verein für Steiermark.* Mittheilungen.  
 P. *Chemisches Institut der K. K. Universität.*  
 P. LEMBERG.—*Société Scientifique de Chevtchenko.*

## PRAGUE—

P. *Deutscher Nat.-Med. Verein für Böhmen "Lotos."*—"Lotos."

T.P. *K. K. Sternwarte.* Magnetische und Meteorologische Beobachtungen. Astronomische Beobachtungen.

T.P. *K. Böhmische Gesellschaft.* Sitzungsberichte : Math.-Naturw. Classe ; Phil.-Hist.-Philol. Classe.—Jahresbericht,—and other publications.

T.P. *Ceská Akademie Císaře Františka Josefa pro Vědy Slovesnost a Umění.* Almanach. — *Vestnik* (Proceedings). — *Rozpravy* (Transactions) : Phil.-Hist. Class ; Math.-Phys. Cl. ; Philol. Cl.—Historický Archiv.—*Bulletin International*, Résumé des Travaux présentés,—and other publications of the Academy.

P. SARAJEVO (BOSNIA).—*The Governor-General of Bosnia-Herzegovina.* Ergebnisse der Meteorologischen Beobachtungen.

## TRIESTE—

P. *Società Adriatica di Scienze Naturali.*

P. *Museo Civico di Storia Naturale.*

P. *Osservatorio Marittimo.* Rapporto Annuale.

## VIENNA—

T.P. *Kais. Akademie der Wissenschaften.* Denkschriften : Math.-Naturwissenschaftliche Classe ; Philosophisch-Historische Classe — Sitzungsberichte der Math.-Naturwissenschaftlichen Classe ; Abtheil. I., II.A, II.B, III. ; Philosoph.-Historische Classe.—Almanach.—Mittheilungen der Erdbeben Commission.

T.P. *K. K. Geologische Reichsanstalt.* Abhandlungen.—Jahrbücher.—Verhandlungen.

T.P. *Oesterreichische Gesellschaft für Meteorologie.* Meteorologische Zeitschrift.

T.P. *K. K. Zoologisch-Botanische Gesellschaft.* Verhandlungen.—Abhandlungen.

P. *K. K. Naturhistorisches Hofmuseum.* Annalen.

K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Jahrbücher. 4to.—Allgemeiner Bericht und Chronik. 8vo. (*Presented.*)

K. K. Militär Geographisches Institut. Astronomisch-Geodätischen Arbeiten. — Astronomische Arbeiten. 4to.—Längenbestimmungen. 4to.—Die Ergebnisse der Triangulierungen. 4to. (*Presented.*)

*Zoologisches Institut der Universität und der Zoologischen Station in Triest.* Arbeiten. (*Purchased.*)

## BELGIUM.

## BRUSSELS—

T.P. *Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique.* Mémoires.—Bulletins.—Annuaire.—Biographie Nationale.

T.P. *Musée Royal d'Histoire Naturelle.* Mémoires.

T.P. *Musée du Congo.* Annales.—Botanique. Zoologie. Ethnographie et. Anthropologie. Linguistique, etc.

T.P. *L'Observatoire Royal de Belgique, Uccle.* Annuaire.—Annales Astronomiques —Annales Météorologiques.—Annales.—Physique du Globe.—Bulletin Climatologique.—Observations Météorologiques.

BRUSSELS—*continued*—

T.P. *Société Scientifique.* *Annales.*  
 P. *Société Belge d'Astronomie.* *Ciel et Terre.* (*Purchased.*)  
 T.P. GHENT.—*University Library.*  
 T.P. LOUVAIN.—*University Library.*

BOSNIA-HERZEGOVINA. (*See AUSTRIA.*)

## BULGARIA.

P. SOFIA.—*Station Centrale Météorologique de Bulgarie.* *Bulletin Mensuel.*—  
*Bulletins Annuaires.*

## CANADA.

EDMONTON (ALBERTA).—*Department of Agriculture.* *Annual Report.*—  
*(Presented.)*

P. HALIFAX (NOVA SCOTIA).—*Nova Scotian Institute of Science.* *Proceedings and Transactions.*  
 T.P. KINGSTON.—*Queen's University.*

## MONTREAL—

P. *Natural History Society.* *Proceedings.*  
 P. *Canadian Society of Civil Engineers.* *Transactions.*—*Annual Reports.*

## OTTAWA—

T.P. *Royal Society of Canada.* *Proceedings and Transactions.*  
 T.P. *Geological Survey of Canada.* *Annual Reports.*—*Palaeozoic Fossils.*—*Maps, Memoirs, and other Publications.*

P. *Literary and Scientific Society.* *Transactions.*

T.P. QUEBEC.—*Literary and Philosophical Society.* *Transactions.*

## TORONTO—

T.P. *University.* *University Studies.* (*History. Psychological Series. Geological Series. Economic Series. Physiological Series. Biological Series. Physical Science Series. Papers from the Chemical Laboratory.*) etc.

T.P. *Canadian Institute.* *Transactions.*

P. *Royal Astronomical Society of Canada.* *Journal.*—*Astronomical Handbook.*

CAPE COLONY. (*See UNION OF SOUTH AFRICA.*)

## CEYLON.

## COLOMBO—

T.P. *Museum.* *Spolia Zeylanica.* *Annual Report.*

## CHINA.

## HONG KONG—

P. *Royal Observatory.* *Monthly Meteorological Bulletin.*—*Report.*

## DENMARK.

## COPENHAGEN—

T.P. *Académie Royale de Copenhague.* Mémoires : Classe des Sciences.—Oversigt.  
 P. *Naturhistorisk Forening.* Videnskabelige Meddelelser.  
 P. *Danish Biological Station.* Report.  
*Conseil Permanent International pour l'Exploration de la Mer.* Publications de circonstance.—Rapports et Procès-Verbaux de Réunions.—Bulletin des Résultats acquis pendant les croisières périodiques.—Bulletin Statistique. (*Presented.*)  
*Kommissionen for Havundersogelser.* Meddelelser : Série Fiskeri. Série Plankton. Série Hydrografi.—Skrifter. (*Presented.*)  
*University (Zoological Museum).* Reports of the Danish Ingolf-Expedition. (*Presented.*)

## EGYPT.

T.P. CAIRO.—*School of Medicine.* Records.  
*Ministry of Finance* (Survey Dept.: Archaeological Survey of Nubia). Bulletin, Reports, Papers. (*Presented.*)

## ENGLAND AND WALES.

## BIRMINGHAM—

P. *Philosophical Society.* Proceedings.  
*University.* Calendar. (*Presented.*)

## CAMBRIDGE—

T.P. *Philosophical Society.* Transactions and Proceedings.  
 T.P. *University Library.*—*Observatory.* Report.—Observations.  
 T.P. CARDIFF.—*University College of South Wales.*  
 COVENTRY.—Annual Report of the Health of the City. (*Presented by Dr Snell.*)  
 P. ESSEX.—*Essex Field Club.* The Essex Naturalist.  
 T.P. GREENWICH.—*Royal Observatory.* Astronomical, Magnetical, and Meteorological Observations.—Photo-heliographic Results and other Publications.  
 T.P. HARPENDEN (HERTS).—*Rothamsted Exp. Station.* (*Lawes Agricultural Trust.*)

## LEEDS—

T.P. *Philosophical and Literary Society.* Reports.  
 P. *Yorkshire Geological and Polytechnic Society.* Proceedings.

## LIVERPOOL—

T.P. *University College Library.*  
 P. *Biological Society.* Proceedings and Transactions.  
 P. *Geological Society.* Proceedings.

## LONDON—

P. *Admiralty.* Nautical Almanac and Astronomical Ephemeris.—Health of the Navy (Annual Report).  
 T.P. *Anthropological Institute.* Journal.

LONDON—*continued*—

T.P. *Athenæum Club.*  
*British Antarctic Expedition, 1907-09.* Reports on Scientific Investigations.  
*(Presented.)*

T.P. *British Association for the Advancement of Science.* Reports.

T.P. *British Museum (Copyright Office).* Reproductions from Illuminated Manuscripts.

T.P. *British Museum. Natural History Department.* Catalogues, Monographs, Lists, etc. *National Antarctic Expedition, 1901-04.* Publications.

T.P. *Chemical Society.* Journal. Abstract of Proceedings.

P. *Faraday Society.* Transactions.

T.P. *Geological Society.* Quarterly Journal.—Geological Literature.—Abstract of Proceedings.

T.P. *Geological Survey of the United Kingdom.* Summary of Progress. Memoirs.

P. *Geologists' Association.* Proceedings.

T.P. *Hydrographic Office.*

T.P. *Imperial Institute.*

T.P. *Institution of Civil Engineers.* Minutes of Proceedings, etc.

T.P. *Institution of Electrical Engineers.* Journal.

P. *Institution of Mechanical Engineers.* Proceedings.

T.P. *International Catalogue of Scientific Literature.* (*Purchased.*)

T.P. *Linnean Society.* Journal: Zoology; Botany.—Transactions: Zoology; Botany.—Proceedings.

P. *Mathematical Society.* Proceedings.

P. *Meteorological Office.* Report of the Meteorological Committee to the Lords Commissioners of H.M. Treasury.—Reports of the International Meteorological Committee.—Hourly Readings.—Weekly Weather Reports.—Monthly and Quarterly Summaries.—Meteorological Observations at Stations of First and Second Order, and other Publications. Geophysical Journal.—Geophysical Memoirs.

*Mineralogical Society of Great Britain and Ireland.* Mineralogical Magazine and Journal. (*Presented.*)

*National Antarctic Expedition, 1901-04.* (*Presented.*)

*Optical Society.* Transactions. (*Purchased.*)

P. *Pharmaceutical Society.* Journal.—Calendar.

P. *Physical Society.* Proceedings.

T.P. *Royal Astronomical Society.* Monthly Notices.—Memoirs.

T.P. *Royal College of Surgeons.*

T.P. *Royal Geographical Society.* Geographical Journal.

T.P. *Royal Horticultural Society.* Journal.

T.P. *Royal Institution.* Proceedings.

P. *Royal Meteorological Society.* Quarterly Journal.

T.P. *Royal Microscopical Society.* Journal.

P. *Royal Photographic Society.* Photographic Journal.

T.P. *Royal Society.* Philosophical Transactions.—Proceedings.—Year-Book.—*National Antarctic Expedition, 1901-04,* Publications; and other Publications.

T.P. *Royal Society of Arts.* Journal.

LONDON—*continued*—

T.P. *Royal Society of Literature.* Transactions.—Reports.

T.P. *Royal Society of Medicine.* Proceedings.

T.P. *Royal Statistical Society.* Journal.

T.P. *Society of Antiquaries.* Proceedings.—Archæologia; or Miscellaneous Tracts relating to Antiquity.

*Society of Chemical Industry.* Journal. (*Presented.*)

*Society for Psychical Research.* Journal.—Proceedings. (*Presented by W. C. Crawford, Esq.*)

*Solar Physics Committee.* Annual Report, and other Publications. (*Presented.*)

T.P. *United Service Institution.*

T.P. *University College.* Calendar.

T.P. *University.*

T.P. *Zoological Society.* Transactions.—Proceedings.

T.P. The Editor of *Nature*.—*Nature*.

T.P. The Editor of *The Electrician*.—*Electrician*.

T.P. The Editor of *Science Abstracts*.—*Science Abstracts*.

## MANCHESTER—

T.P. *Literary and Philosophical Society.* Memoirs and Proceedings.

T.P. *University.*—Publications—Medical Series. Public Health Series. Anatomical Series. Physical Series. Biological Series. Lectures. *Manchester Museum (University of Manchester)*. Annual Reports—Notes from the Museum.

P. *Microscopical Society.* Transactions and Annual Report.

## NEWCASTLE-ON-TYNE—

P. *Natural History Society of Northumberland, Durham, etc.* Transactions.

T.P. *North of England Institute of Mining and Mechanical Engineers.* Transactions.—Annual Reports.

*Cullercoats Dove Marine Laboratory.* Annual Report. (*Presented.*)

P. *Literary and Philosophical Society.*

*University of Durham Philosophical Society.* Proceedings. (*Presented.*)

P. NORWICH.—*Norfolk and Norwich Naturalists' Society.* Transactions.

## OXFORD—

T.P. *Bodleian Library.*

P. *Ashmolean Society.* Proceedings and Report.

P. *Radcliffe Observatory.* Results of Astronomical and Meteorological Observations.

*University Observatory.* Astrographic Catalogue. (*Presented.*)

P. PENZANCE.—*Royal Geological Society of Cornwall.* Transactions.

T.P. PLYMOUTH.—*Marine Biological Association.* Journal.

## RICHMOND (SURREY)—

T.P. *Kew Observatory.*

P. SCARBOROUGH.—*Philosophical Society.*

T.P. SHEFFIELD.—*University College.*

SOUTHPORT.—*Meteorological Observatory.* Results of Observations. Joseph Baxendell, Meteorologist. (*Presented.*)

T.P. TEDDINGTON (MIDDLESEX).—*National Physical Laboratory.* Collected Researches.—Annual Reports.

P. TRURO.—*Royal Institution of Cornwall.* Journal.

YORK—

T.P. *Yorkshire Philosophical Society.* Reports.

## FINLAND.

HELSINGFORS—

*Academiae Scientiarum Fennicæ.* Annales. Sitzungsberichte.—Documenta Historica. (Presented.)

*Hydrographisch Biologisch Untersuchungen.* (Presented.)

T.P. *Societas Scientiarum Fennica (Société des Sciences de Finlande).* Acta Societatis Scientiarum Fennicæ.—Öfversigt.—Meteorologisches Jahrbuch. —Bidrag till Kändedom om Finlands Natur och Folk.

T.P. *Societas pro Fauna et Flora Fennica.* Acta.—Meddelanden.

P. *Société de Géographie de Finlande.* Fennia.—Meddelanden.

## FRANCE.

BESANÇON.—*Université Observatoire National.* Bulletin Chronométrique et Bulletin Météorologique. (Presented.)

BORDEAUX—

T.P. *Société des Sciences Physiques et Naturelles.* Mémoires.—Observations Pluviométriques et Thermométriques.—Procès-Verbaux des Séances.

P. *Société de Géographie Commerciale.* Bulletin.  
*L'Observatoire.* Catalogue Photographique du Ciel.

P. CHERBOURG.—*Société Nationale des Sciences Naturelles et Mathématiques.* Mémoires.

P. CONCARNEAU.—*College de France (Laboratoire de Zoologie et de Physiologie Maritime).* Travaux Scientifiques.

P. DIJON.—*Académie des Sciences.* Mémoires.

LILLE—

T.P. *Société des Sciences.*

T.P. *Société Géologique du Nord.* Annales.—Mémoires.

P. *Université de France.* Travaux et Mémoires.

LYONS—

T.P. *Académie des Sciences, Belles Lettres et Arts.* Mémoires.

T.P. *Société d'Agriculture, Histoire Nat. et Arts.* Annales.

T.P. *Université.* Annales, Nouv. Série:—I. Sciences, Médecine. II. Droit, Lettres.

P. *Société Botanique.* Annales.—Nouveaux Bulletins.

P. *Société Linnéenne.* Annales.

MARSEILLES—

T.P. *Faculté des Sciences.* Annales.

P. *Société Scientifique Industrielle.* Bulletin.

T.P. MONTPELLIER.—*Académie des Sciences et Lettres.* Mémoires: Section des Sciences; Section des Lettres; Section de Médecine. Bulletin Mensuel.

T.P. NANTES.—*Société Scientifique des Sciences Naturelles de l'Ouest de la France.*  
    *Bulletin.*

T.P. NICE.—*L'Observatoire.* Annales.

PARIS—

T.P. *Académie des Sciences.* Comptes Rendus. — *Observatoire d'Abbadia:*  
    Observations, 4to, and other Publications.

T.P. *Académie des Inscriptions et Belles-Lettres.* Comptes Rendus.

T.P. *Association Française pour l'Avancement des Sciences.* Comptes Rendus.

T.P. *Bureau International des Poids et Mesures.* — Procès-Verbaux des Séances.  
    —Travaux et Mémoires.

T.P. *Bureau des Longitudes.* Annaire.

P. *L'École des Ponts et Chaussées.*

T.P. *Ministère de la Marine (Service Hydrographique.)* Annales Hydro-  
graphiques. Expédition de Charcot, 1903–05. (*See Presentation List.*)

T.P. *École des Mines.* Annales des Mines.

T.P. *École Normale Supérieure.* Annales.

T.P. *École Polytechnique.* Journal.

P. *École Libre des Sciences Politiques.*

T.P. *Institut Océanographique.* Annales.

T.P. *Ministère de l'Instruction Publique.* Expédition de Charcot, 1908–10. (*See  
Presentation List.*)

T.P. *Musée Guimet.* Revue de l'Histoire des Religions.—Annales.—Bibliothèque  
d'Études.

T.P. *Muséum d'Histoire Naturelle.* Nouvelles Archives.—Bulletin.

T.P. *L'Observatoire.* Rapport Annuel sur l'État de l'Observatoire.—Annales.—  
Mémoires.—Carte Photographique du Ciel. Fol.—Catalogue Photo-  
graphique du Ciel. 4to.  
*L'Observatoire d'Astronomie Physique de Meudon.* Annales. (*Presented.*)

T.P. *Société Nationale d'Agriculture.* Bulletins.—Mémoires.

P. *Société d'Anthropologie.* Bulletin et Mémoires.

T.P. *Société Nationale des Antiquaires.* Mémoires.—Bulletin.

T.P. *Société de Biologie.* Comptes Rendus.

T.P. *Société d'Encouragement pour l'Industrie Nationale.* Bulletin.

T.P. *Société Française de Physique.* Journal de Physique.—Annaire.—Procès-  
Verbaux.

T.P. *Société de Géographie.* La Géographie.

T.P. *Société Géologique de France.* Bulletins.—Mémoires (Paléontologie).

P. *Sociétés des Jeunes Naturalistes et d'Études Scientifiques.* Feuilles des  
Jeunes Naturalistes.

T.P. *Société Mathématique.* Bulletin.

P. *Société Philomathique.* Bulletin.

T.P. *Société Zoologique.* Bulletin.—Mémoires.

P. *Revue Générale des Sciences Pures et Appliquées.*

T.P. RENNES.—*Société Scientifique et Médicale de l'Ouest.* Bulletin.  
TOULOUSE—

T.P. *Université.—Faculté des Sciences.—L'Observatoire.* Annales.

P. *Académie des Sciences.* Mémoires.

## GERMANY.

## BERLIN—

*Carte Géologique internationale de l'Europe.* Livres I.-VIII. (complete).  
(Presented.)

T.P. *K. Akademie der Wissenschaften.* Abhandlungen.—Sitzungsberichte.  
T.P. *Physikalische Gesellschaft.* Fortschritte der Physik: 1<sup>te</sup> Abtheil; Physik der Materie. 2<sup>te</sup> Abtheil; Physik des Aethers. 3<sup>e</sup> Abtheil; Kosmische Physik.—Verhandlungen.  
T.P. *Deutsche Geologische Gesellschaft.* Zeitschrift.—Monatsberichte.  
P. *Deutsche Meteorologische Gesellschaft.* Zeitschrift.  
P. *Königl. Preussisches Meteorologisches Institut.*  
P. *Gesellschaft Naturforschender Freunde.* Sitzungsberichte.—Archiv für Biontologie.  
P. *Kgl. Technische Hochschule.* Programm.  
T.P. *Zoologisches Museum.* Mitteilungen.

## BONN—

P. *Naturhistorischer Verein der Preussischen Rheinlande und Westfalens.* Verhandlungen.  
*Niederrheinische Gesellschaft für Natur- und Heilkunde.* Sitzungsberichte.  
(Presented.)

T.P. BREMEN.—*Naturwissenschaftlicher Verein.* Abhandlungen.  
P. BRUNSWICK.—*Verein für Naturwissenschaft.* Jahresberichte.  
P. CARLSRUHE.—*Technische Hochschule.* Dissertations.  
P. CASSEL.—*Verein für Naturkunde.* Berichte.  
T.P. CHARLOTTENBURG.—*Physikalisch-Technische Reichsanstalt.* Abhandlungen.  
P. CHEMNITZ.—*Naturwissenschaftliche Gesellschaft.* Berichte.  
T.P. DANTZIC.—*Naturforschende Gesellschaft.* Schriften.  
*Westpreussischer Botanisch-Zoologischer Verein.* Bericht. (Presented.)

## ERLANGEN—

T.P. *University.* Inaugural Dissertations.  
*Physikalisch-Medicinische Societät.* Sitzungsberichte.  
T.P. FRANKFURT-AM-MAIN.—*Senckenbergische Naturforschende Gesellschaft.* Abhandlungen.—Berichte.  
P. FRANKFURT-AM-ODER.—*Naturwissenschaftlicher Verein.* Helios.  
P. FREIBURG-I BR.—*Naturforschende Gesellschaft.* Berichte.

## GIESSEN—

T.P. *University.* Inaugural Dissertations.  
P. *Oberhessische Gesellschaft für Natur- und Heilkunde.* Berichte.  
T.P. GÖTTINGEN.—*K. Gesellschaft der Wissenschaften.* Abhandlungen, Neue Folge: Math.-Phys. Classe; Phil.-Hist. Classe.—Nachrichten: Math.-Phys. Cl.; Phil.-Hist. Cl.; Geschäftliche Mittheilungen.—(Gelehrte Anzeigen. Purchased.)

## HALLE—

T.P. *K. Leopold-Carolinisch-Deutsche Akademie der Naturforscher.* Nova Acta (Verhandlungen).—Leopoldina.  
T.P. *Naturforschende Gesellschaft.* Abhandlungen.  
P. *Verein für Erdkunde.* Mittheilungen.

**HALLE**—*continued*—

P. *Naturwissenschaftlicher Verein für Sachsen und Thüringen.*  
 P. *Deutsche Mathematiker Vereinigung.* Jahresbericht.

**HAMBURG**—

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*Maryland Weather Service.* Reports. (*Presented.*)

*Peabody Institute.* Annual Reports. (*Presented.*)

## BERKELEY (CALIFORNIA)—

T.P. *University of California.*—University Chronicle.—Reports of Agricultural College.—Publications (Zoology, Botany, Geology, Physiology, Pathology and American Archaeology and Ethnology).—Memoirs.

*Academy of Pacific Coast History.* Publications.

## BOSTON—

T.P. *Bowditch Library.*

T.P. *Boston Society of Natural History.* Memoirs.—Proceedings.—Occasional Papers.

T.P. *American Academy of Arts and Sciences.* Memoirs.—Proceedings.

P. BROOKLYN.—*Institute of Arts and Sciences.* Museum Reports.—Bulletins.

P. BUFFALO.—*Society of Natural Sciences.* Bulletin.

CALIFORNIA. (*See SAN FRANCISCO, SACRAMENTO, BERKELEY, MOUNT HAMILTON, MOUNT WILSON and STANFORD.*)

## CAMBRIDGE—

T.P. *Harvard University. — Museum of Comparative Zoology.* Memoirs.—Bulletins—Annual Reports.

T.P. *Astronomical Observatory, Harvard College.* Annals.—Annual Reports.—Observatory Circulars.

P. CHAPEL HILL (NORTH CAROLINA).—*E. Mitchell Scientific Society.* Journal.

P. CHARLOTTESVILLE. *Philosophical Society, University of Virginia.* Bulletin; Scientific Series and Humanistic Series.—Proceedings.

## CHICAGO—

T.P. *Field Museum of Natural History.* Publications: Geological Series; Botanical Series; Zoological Series; Ornithological Series; Anthropological Series.—Annual Reports.

P. *University of Chicago.*

T.P. *Yerkes Observatory (University of Chicago).* Publications.

P. *Academy of Sciences.* Bulletins.—Special Publications.—Bulletins of the Natural History Survey.

## CINCINNATI—

P. *Observatory (University).* Publications.—University Record.

P. *Society of Natural History.* Journal.

T.P. CLEVELAND (OHIO).—*Geological Society of America.* Bulletins.

T.P. CLINTON (IOWA).—*Litchfield Observatory, Hamilton College.*

COLORADO SPRINGS.—*Colorado College.* Colorado College Studies. (*Presented.*)

P. CONNECTICUT.—*Connecticut Academy of Arts and Sciences.* Transactions.—Memoirs.

P. DAVENPORT.—*Academy of Natural Sciences.* Proceedings.

P. DENVER (COLORADO).—*Scientific Society of Colorado.* Proceedings.

T.P. DES MOINES (IOWA).—*Iowa Academy of Sciences.* Proceedings.

P. GARRISON, N.Y.—Editor, *American Naturalist.*

T.P. GRANVILLE (OHIO).—*Denison University and Scientific Association.* Bulletin of the Scientific Laboratories.

P. INDIANOPOLIS.—*Indiana Academy of Sciences.* Proceedings.

## IOWA CITY—

P. *Geological Survey.* Annual Reports.

P. *State University. Laboratories of Natural History.* Bulletins.—Contributions from the Physical Laboratories.

IOWA. (*See DES MOINES.*)

## ITHACA (N.Y.)—

P. The Editor, *Physical Review.* (Cornell University.)

ITHACA (N.Y.)—*continued*—

P. The Editors, *Journal of Physical Chemistry*. (Cornell University.)

T.P. LAWRENCE (KANSAS).—*University of Kansas*. Science Bulletin (University Quarterly).

P. LINCOLN (NEBRASKA).—*University of Nebraska*. Agricultural Experiment Station. Bulletins.

## MADISON—

T.P. Wisconsin University. Washburn Observatory. Observations.

P. Wisconsin Academy of Sciences, Arts, and Letters. Transactions.

P. Geological and Natural History Survey. Bulletins.

P. MASSACHUSETTS.—*Tufts College Library*. Tufts College Studies.

P. MERIDEN (CONN.).—Meriden Scientific Association.

MICHIGAN. (*See ANN ARBOR*.)

MINNEAPOLIS (MINN.)—

T.P. { University of Minnesota. Studies.—Bulletin of the School of Mines.  
Geological and Natural History Survey of Minnesota. Reports.

P. Botanical Survey.

MISSOURI. (*See ST. LOUIS and ROLLA*.)

P. MOUNT HAMILTON (CALIFORNIA).—*Lick Observatory*. Bulletins.—Publications.

T.P. MOUNT WILSON (CALIFORNIA).—*Solar Observatory*. Contributions.—Reports.

T.P. NEWHAVEN (CONN.).—*Yale College*. Astronomical Observatory of Yale University. Transactions.—Reports.

P. NEW ORLEANS.—Academy of Sciences.

## NEW YORK—

T.P. American Mathematical Society. Bulletins.—Transactions.

T.P. American Museum of Natural History. Bulletins.—Memoirs.—American Museum Journal.—Annual Reports.—Anthropological Papers.—Guide Leaflets.—Handbook Series.—Monograph Series.

P. American Geographical Society. Bulletin.

P. American Institute of Electrical Engineers. Proceedings.

NEW YORK. (*See also ALBANY*.)

## PHILADELPHIA—

T.P. American Philosophical Society for Promoting Useful Knowledge. Proceedings.—Transactions.

T.P. Academy of Natural Sciences. Proceedings.—Journal.

T.P. University of Pennsylvania. Publications:—Philology, Literature, and Archæology, Mathematics, etc. Contributions from the Zoological and Botanical Laboratories. University Bulletins.—Theses.—Calendar.

T.P. Geological Survey of Pennsylvania.

P. Wagner Free Institute of Science. Transactions.

P. Geographical Society. Bulletin.

P. Commercial Museum.

P. PORTLAND (MAINE).—Society of Natural History. Proceedings.

P. PRINCETON, N.J.—University. Annals of Mathematics.—University Observatory. Contributions.

P. ROCHESTER.—Academy of Science. Proceedings.

T.P. ROLLA (MISS.).—Bureau of Geology and Mines. Biennial Reports, etc.

T.P. [SALEM.—*Essex Institute.*

SAINT LOUIS—

T.P. *Academy of Sciences.* Transactions.  
 P. *Missouri Botanical Garden.* Annual Reports.  
 P. *Washington University.* University Studies.  
 T.P. SAN FRANCISCO (CALIFORNIA).—*Academy of Sciences.* Proceedings.—Memoirs.—Occasional Papers.

STANFORD (CALIFORNIA).—*University.* Publications. (*Presented.*)

P. TOPEKA.—*Kansas Academy of Science.* Transactions.  
 T.P. URBANA.—*University of Illinois.* Bulletins of State Geological Survey, State Laboratory of Natural History, and Engineering Experiment Station.

WASHINGTON—

T.P. *U.S. National Academy of Sciences.* Memoirs.  
 T.P. *Bureau of Ethnology.* Annual Reports.—Bulletins.  
 T.P. *U.S. Coast and Geodetic Survey.* Annual Reports, etc.  
 T.P. *U.S. Commission of Fish and Fisheries.* Reports.—Bulletins.  
 T.P. *U.S. Naval Observatory.* Reports.—Observations.  
 T.P. *U.S. Geological Survey.* Bulletins.—Annual Reports.—Monographs.—Geologic Atlas of the United States.—Mineral Resources.—Professional Papers.—Water Supply and Irrigation Papers.  
*Geological Society of America.* (*See CLEVELAND.*)  
 T.P. *Weather Bureau.* (*Department of Agriculture.*) Monthly Weather Review.—Bulletins.—Reports.—Bulletin of the Mount Weather Observatory (now embodied in Monthly Weather Review).  
 T.P. *Smithsonian Institution.* Miscellaneous Collections.—The same (Quarterly Issue).—Contributions to Knowledge.—Reports.—Annals of the Astrophysical Observatory.—Harriman Alaska Expedition, Vol. XIV. 4to.  
 T.P. *Surgeon-General's Office.* Index Catalogue of the Library. 4to.  
 T.P. *Carnegie Institution of Washington.* Year-Books.—Publications. Classics of International Law.—*Carnegie Foundation for the Advancement of Teaching.* Annual Report.—Bulletin.  
 T.P. *American Association for the Advancement of Science.* Proceedings.  
 P. *U.S. National Museum.* Bulletins.—Reports.—Proceedings.—Contributions from the U.S. National Herbarium.  
 P. *Department of Agriculture.* (*Division of Economic Ornithology and Mammalogy.*) Bulletin.  
 P. *U.S. Patent Office.*  
*Washington Academy of Sciences, Journal of the.* (*Purchase.*)  
*Bureau of Standards. Department of Commerce and Labour.* Bulletins. (*Presented.*)—Technologic Papers.  
 WISCONSIN. (*See MADISON.*)

VICTORIA. (*See AUSTRALIA.*)

**List of Periodicals and Annual Publications added to the  
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*Periodicals not found in this List will be found in Exchange List.*

*Annuals (Works of Reference), see end of List.*

*Acta Mathematica.*

*American Journal of Science and Arts.*

\* ——— *Naturalist.*

\* ——— *Journal of Mathematics.*

\* ——— *Chemical Journal.*

\* ——— *Journal of Philology.*

*Anatomischer Anzeiger.*

——— ——— *Ergänzungshefte.*

*Annalen der Chemie (Liebig's).*

\* ——— *der Physik.*

\* ——— *der Physik. (Beiblätter.)*

*Annales de Chimie.*

——— *d'Hygiène Publique et de Médecine Légale.*

——— *de Physique.*

——— *des Sciences Naturelles. Zoologie et Paléontologie.*

——— *des Sciences Naturelles. Botanique.*

*Annali dell' Islam. (Presented.)*

*Annals and Magazine of Natural History (Zoology, Botany, and Geology).*

——— *of Botany.*

\* ——— *of Mathematics. (Princeton, N.J.)*

*Anthropologie (L').*

*Arbeiten-Zoologisches Institut der Universität und der Zoologischen Station in Triest.*

\* *Archiv för Matematik og Naturvidenskab.*

\* *Archiv für Biontologie.*

*Archives de Biologie.*

——— *de Zoologie Expérimentale et Générale.*

\* ——— *des Sciences Biologiques.*

——— *des Sciences Physiques et Naturelles.*

——— *Italiennes de Biologie.*

\* *Arkiv för Matematik, Astronomi och Fysik. (Stockholm.)*

\* ——— *för Kemi, Mineralogi och Geologi. ,,*

\* ——— *för Botanik. ,,*

\* ——— *för Zoologi. ,,*

*Astronomie (L').*

*Astronomische Nachrichten.*

*Astrophysical Journal.*

*Athenæum.*

*Bericht über die Wissenschaftlichen Leistungen in der Naturgeschichte der niederen*

*Thiere. Begründet von R. Leuckart.*

*Bibliothèque Universelle et Revue Suisse.*

——— ——— *See Archives des Sciences Physiques et Naturelles.*

\* Received by exchange.

Biologisches Centralblatt.  
Blackwood's Magazine.  
Bollettino delle Pubblicazioni Italiane. (*Presented.*)  
Bookman.  
Botanische Zeitung.  
Botanisches Centralblatt.  
— Beiheft.  
British Rainfall.  
Bulletin Astronomique.  
— des Sciences Mathématiques.  
— Mensuel de la Société Astronomique de Paris. *See* L'Astronomie.  
Cambridge British Flora. By C. E. Moss.  
Catalogue of Scientific Papers, 1800-1900. Subject Index.  
Centralblatt für Bakteriologie und Parasitenkunde.  
— für Mineralogie, Geologie und Palæontologie.  
Ciel et Terre.  
Contemporary Review.  
Crelle's Journal. *See* Journal für Reine und Angewandte Mathematik.  
Dictionary, New English. Ed. by Sir J. A. H. Murray.  
Dingler's Polytechnisches Journal.  
Edinburgh Medical Journal.  
— Review.  
Egypt Exploration Fund. Publications.  
\* Electrician.  
Encyklopädie der Mathematischen Wissenschaften.  
Engineering.  
English Mechanic and World of Science.  
\* Essex Naturalist.  
Fauna und Flora des Golfes von Neapel.  
Flora.  
Fortnightly Review.  
\* Gazette Médicale d'Orient.  
\* Geographical Journal.  
\* Geographical Magazine (Scottish).  
\* Geographie (La).  
Geological Magazine.  
Göttingische Gelehrte Anzeigen.  
Indian Antiquary.  
— Engineering. (*Presented.*)  
Indian Journal of Medical Research. (*Presented.*)  
Intermédiaire (L') des Mathématiciens.  
International Catalogue of Scientific Literature.  
Internationale Revue der Gesamten Hydrobiologie und Hydrographie.  
Jahrbücher für Wissenschaftliche Botanik (Pringsheim).  
Jahresbericht über die Fortschritte der Chemie und verwandter Theile anderer  
Wissenschaft.  
Journal de Conchyliologie.

\* Received by exchange.

Journal des Débats.

\_\_\_\_\_ de Mathématiques Pures et Appliquées.

\_\_\_\_\_ de Pharmacie et de Chimie.

\* \_\_\_\_\_ de Physique.

\_\_\_\_\_ des Savants.

\_\_\_\_\_ für die Reine und Angewandte Mathematik (Crelle).

\_\_\_\_\_ für Praktische Chemie.

\_\_\_\_\_ of Anatomy and Physiology.

\_\_\_\_\_ of Botany.

\_\_\_\_\_ of Pathology and Bacteriology.

\* \_\_\_\_\_ of Physical Chemistry.

\* \_\_\_\_\_ of the Royal Society of Arts.

\_\_\_\_\_ of the Society of Chemical Industry. (*Presented.*)

\_\_\_\_\_ of the Washington Academy of Sciences.

Knowledge.

Manual of Conchology.

\* Mathematische und Naturwissenschaftliche Berichte aus Ungarn.

Mind.

Mineralogical Magazine. (*Presented.*)

Mineralogische und Petrographische Mittheilungen (Tschermak's).

Monist.

\* Nature.

\_\_\_\_\_ (La).

Neues Jahrbuch für Mineralogie, Geologie, und Palaeontologie

\_\_\_\_\_ Beilage.

Nineteenth Century.

Notes and Queries.

Nuova Notarisia (De Toni).

\* Nuovo Cimento ; Giornale di Fisica, Chimica e Storia Naturale.

\* Nyt Magazin för Naturvidenskaberne.

Observatory.

Optical Society, London, Transactions.

Page's Engineering Weekly. (*Presented.*)

Palaeontographical Society's Publications.

Petermann's Mittheilungen.

\_\_\_\_\_ Ergänzungsheft.

\* Pharmaceutical Journal.

Philosophical Magazine. (London, Edinburgh, and Dublin.)

\* Photographic Journal.

\* Physical Review.

Plankton-Expedition Ergebnisse.

Quarterly Journal of Microscopical Science.

\_\_\_\_\_ of Experimental Physiology.

Quarterly Review.

Ray Society's Publications.

Registrar-General's Returns (Births, Deaths, and Marriages). (*Presented.*)

Resultate der Wissenschaftliche Erforschung der Balatonsees.

\* Received by exchange.

Review of Neurology and Psychiatry.

\* Revue Générale des Sciences Pures et Appliquées.

\_\_\_\_ Philosophique de la France et de l'Étranger

\_\_\_\_ Politique et Littéraire. (Revue Bleue.)

\_\_\_\_ Scientifique. (Revue Rose.)

\* \_\_\_\_ Semestrielle des Publications Mathématiques.

Saturday Review.

Science.

\* Science Abstracts.

\_\_\_\_ Progress.

Scotsman.

Scottish Naturalist.

Symons's Meteorological Magazine.

Thesaurus Linguae Latinæ.

Times.

Zeitschrift für die Naturwissenschaften.

\_\_\_\_ für Krystallographie und Mineralogie.

\_\_\_\_ für Wissenschaftliche Zoologie.

Zoological Record.

Zoologische Jahrbücher. Abteilung für Anatomie und Ontogenie der Tiere.

\_\_\_\_ Abteilung für Systematik, Geographie und Biologie der Tiere.

\_\_\_\_ Abteilung für Allgemeine Zoologie und Physiologie der Tiere.

Zoologischer Anzeiger.

\_\_\_\_ Jahresbericht.

#### ANNUALS (WORKS OF REFERENCE).

Annuaire du Bureau des Longitudes.

Slater's Directory. (Scotland.)

County Directory. (Scotland.)

Edinburgh and Leith Directory.

English Catalogue of Books.

Medical Directory.

Minerva (Jahrbuch der Gelehrten Welt).

Minerva (Handbuch der Gelehrten Welt).

\* Nautical Almanac.

Oliver & Boyd's Almanac.

University Calendars:—St Andrews, Edinburgh, Aberdeen, Glasgow, London University College, Birmingham, Belfast, Sydney, N.S.W.; also Calendar of Royal Technical College, Glasgow.

Wer ist's?

Whitaker's Almanack.

Who's Who.

Who's Who in Science (International).

Willing's Press Guide.

Year-Book of Scientific and Learned Societies of Great Britain and Ireland.

Zoological Record.

\* Received by exchange.

## Additions to Library by Gift or Purchase.

Andoyer (H.). Nouvelles Tables Trigonométriques Fondamentales (Logarithmes). 4to. Paris, 1911. (*Presented by the Author.*)

Burckhardt (J.-Ch.). Table des Diviseurs pour tous les nombres du Deuxième Million. 4to. Paris, 1814. (*Purchased.*)

Cauchy (Augustin). Œuvres complètes. 11<sup>e</sup> Série, Tome XI. 4to. Paris, 1913. (*Presented by the "Académie des Sciences," Paris.*)

Charcot (Dr Jean). Expédition Antarctique Française. Deuxième (1908–1910). Documents Scientifiques. Sciences Physiques; Sciences Naturelles. (Several parts.) 4to. Paris, 1913. (*Presented by the "Ministère de l'Instruction Publique," Paris.*)

Coal Resources of the World. Vols. I.–III. and Atlas. Published under the auspices of the XII. International Geological Congress, Canada, 1913. 4to. Toronto, 1913. (*Purchased.*)

Commission of Conservation, Canada. Fourth Annual Report, 1913. (Committee on Waters and Water Powers)—Long Sault Rapids, St Lawrence River: An Inquiry into . . . Project to Develop Power therefrom. (Committee on Forests)—Forest Protection in Canada, 1912. 4to. Toronto and Ottawa, 1913. (*Presented by the High Commissioner for Canada.*)

Commission Polaire Internationale. Procès-verbal de la Session tenue à Rome en 1913. 8vo. Bruxelles, 1913. (*Presented.*)

Fergusson (John C.). Fergusson's "Percentage Unit" of Angular Measurement, with Logarithms. 4to. London, 1912. (*Presented by the Author.*)

Geikie (James). The Antiquity of Man in Europe. (Munro Lectures, Edinburgh University, 1913.) 8vo. Edinburgh, 1914. (*Presented by the Author.*)

— Mountains: their Origin, Growth, and Decay. 8vo. Edinburgh, 1913. (*Presented by the Author.*)

Glaisher (James). Factor Tables for the Fourth, Fifth, and Sixth Millions. 4to. London, 1879, 1880, 1883. (*Purchased.*)

Groningen, Academia Groningana MDCXIV–MCMXIV. 4to. Groningen, 1914. (*Presented by Sir Edward Schäfer.*)

India, Census of, 1911. Vol. I. Part I.—Report. Part II.—Tables. Fol. Calcutta, 1913. (*Presented by the Government of India.*)

Katanga, Rapport sur les Travaux de la Mission Scientifique du (1910–1912). 8vo. Bruxelles, 1913. (*Presented by the "Ministère des Colonies," Bruxelles.*)

Kristiania, Det. Kgl. Frederiks Universitets 100-aarsjubilæum, 1911. 4to. Kristiania, 1913. (*Presented by the University.*)

Laurie (A. P.). The Pigments and Mediums of the Old Masters. 8vo. London, 1914. (*Presented by the Author.*)

Louvain, Mémoires de l'Institut Géologique de l'Université de Louvain. Tome I. 4to. Louvain, 1913. (*Presented by the University.*)

McGowan (J. P.). Investigation into the Disease of Sheep called "Scrapie." (Edinburgh and East of Scotland College of Agriculture.) 8vo. Edinburgh, 1914. (*Presented by the Author.*)

Millar (W. J.). Comparisons of some Chance Groupings with Mendel's Laws. (Buteshire Natural History Society.) 8vo. Rothesay, 1914. (*Presented by the Author.*)

Moss (C. E.). Cambridge British Flora. Vol. II.—Salicaceæ to Chenopodiaceæ. Fol. Cambridge, 1914. (*Purchased.*)

Naperi De Arte Logistica. 4to. Edinburgh, 1839. (*Presented by the Misses Sang; originally presented to Dr Sang by the Editor, Mark Napier.*)

Napier (Mark). Memoirs of John Napier. 4to. Edinburgh, 1834. (*Presented by the Misses Sang; originally presented to Dr Sang by the Author.*)

Philip (Alexander). The Reform of the Calendar. 8vo. London, 1914. (*Presented by the Author.*)

Rayleigh (Lord). Scientific Papers. Vols. I-V. 8vo. Cambridge, 1899-1912. (*Presented by the Author.*)

Sang (Edward). Elementary Arithmetic. 8vo. Edinburgh, 1856. (*Presented by the Misses Sang.*)

— Higher Arithmetic. 8vo. Edinburgh, 1857. (*Presented by the Misses Sang.*)

— Solution of Algebraic Equations of all Orders. 8vo. Edinburgh, 1829. (*Presented by the Misses Sang.*)

— Letters to, from Lieut. Shortrede, in connection with the preparation of Shortrede's Logarithmic Tables. Parincha, India, 1836. (*Presented by the Misses Sang.*)

— Notice of Dioptric Light at Kirkcaldy. 8vo. Edinburgh, 1838. (*Presented by the Misses Sang.*)

— Fifty Years' Papers to the Royal Scottish Society of Arts. Jubilee Presentation from the Fellows (1882). (*Presented by the Misses Sang.*)

Smith (J. G.). Strathendrick and its Inhabitants from Early Times. 4to. Glasgow, 1896. (*Presented by Sir William Bilsland.*)

Solomon (J. I.). A Memorandum on the Pearl Fisheries of Ceylon. (Burma Shell Co. Ltd.) Fol. Rangoon, 1909. (*Presented by the Author.*)

Stuckney, (J. J.). Table of Compound Interest at 1/8 per cent. and of Anti-logarithms to Base 1·00125. 4to. London, 1914. (*Presented by the Author.*)

Uppsala, Zoologiska Bidrag fran (med understöd af R. Bünsows Zoologiska Fond.). Utgifna af A. Wirén. Bd. 1, 2. 8vo. Uppsala, 1911-13. (*Presented.*)

Versluys (J.). Contribution à la Théorie de l'Ecoulement de l'Eau Souterraine. 8vo. Amsterdam, 1914. (*Presented by the Author.*)

Zeiller (C. R.). Palæontological Pamphlets. 8vo. 4to. Paris, 1905-1912. (*Presented by the Author.*)

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